

A Visual Exploration and Design of Morse Vector Fields

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Vector field design on surfaces was originally motivated by applications in graphics such as texture synthesis and rendering. In this paper, we consider the idea of vector field design with a new motivation from computational topology. We are interested in designing and visualizing vector fields to aid the study of Morse functions, Morse vector fields, and Morse–Smale complexes. To achieve such a goal, we present MVF Designer, a new interactive design system that provides fine-grained control over vector field geometry, enables the editing of vector field topology, and supports a design process in a simple and efficient way using elementary moves, which are actions that initiate and advance our design process (see Figure 1).

Our system allows mathematicians to explore the complex configuration spaces of Morse functions, their gradients, and their associated Morse–Smale complexes. Understanding these spaces will help us expand further their applicability in topological data analysis and visualization. First, we will use MVF Designer to study topological invariants in flow classification. Second, we can use MVF Designer to characterize the set of Morse functions that give rise to the same persistence barcode. Finally, we can utilize MVF Designer to study inverse problems in topological data analysis. For instance, given a barcode, we will use MVF Designer as a tool to aid in the reverse engineering of Morse functions or Morse vector fields that give rise to the given barcode.

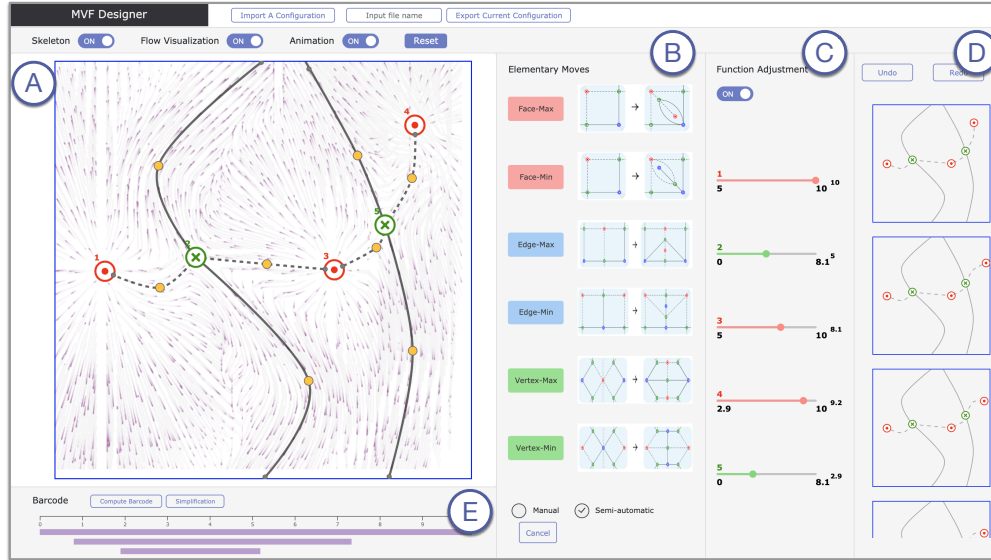


Figure 1: With MVF Designer, users can interact with and manipulate a Morse vector field at the structural and visual levels. The interface consists of several panels. (A) **Flow visualization** panel: Supports modifying the topology and geometry of the topological skeleton; visualizes the dynamics of the underlying vector field via animation. (B) **Elementary moves** panel: Provides a set of elementary moves as fundamental building blocks of a vector field. (C) **Function adjustment** panel: Allows modifying the function values at singularities. (D) **History** panel: Provides undo and redo features to remove or repeat single or multiple operations. (E) **Barcode** panel: Computes and displays persistence barcodes to guide vector field simplification.