# Bayes' Rule

Instructor: Shandian Zhe

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# Bayes' Rule

So far we've talked about conditional probabilities:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Bayes' Rule allows us to switch the order of $A$ and $B$, i.e., to compute $P(B|A)$.

# Bayes' Rule

How it works:

$$P(A \cap B) = P(A|B)P(B)$$

and

$$P(A \cap B) = P(B \cap A) = P(B|A)P(A).$$

Setting the two expressions equal:

$$P(A|B)P(B) = P(B|A)P(A),$$

Solving for $P(B|A)$:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

# In-Class Problem 1

You have two urns:

- Urn 1: 4 black balls, 3 white balls.
- Urn 2: 2 black balls, 2 white balls.

You pick one urn at random and then select a ball from the urn.

1. What is the probability the ball is white?
2. If you picked a black ball, what is the probability that you had picked the first urn?

# In-Class Problem 2

You have a system with:

- A main power supply with a 10% chance of failure.
- An auxiliary power supply:
  - 10% chance of failure if the main supply is running.
  - 15% chance of failure if the main supply fails.

1. What is the probability that the auxiliary power will fail?
2. If the auxiliary power fails, what is the probability that the main power also failed?

# Bayes' Rule in Machine Learning

Bayes' Rule is crucial in Machine Learning:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)},$$

where:

- $M$: "My model describes the real world."
- $D$: "This is the data I've collected."

# Bayes' Rule in Machine Learning

Bayes' Rule is crucial in Machine Learning:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)},$$

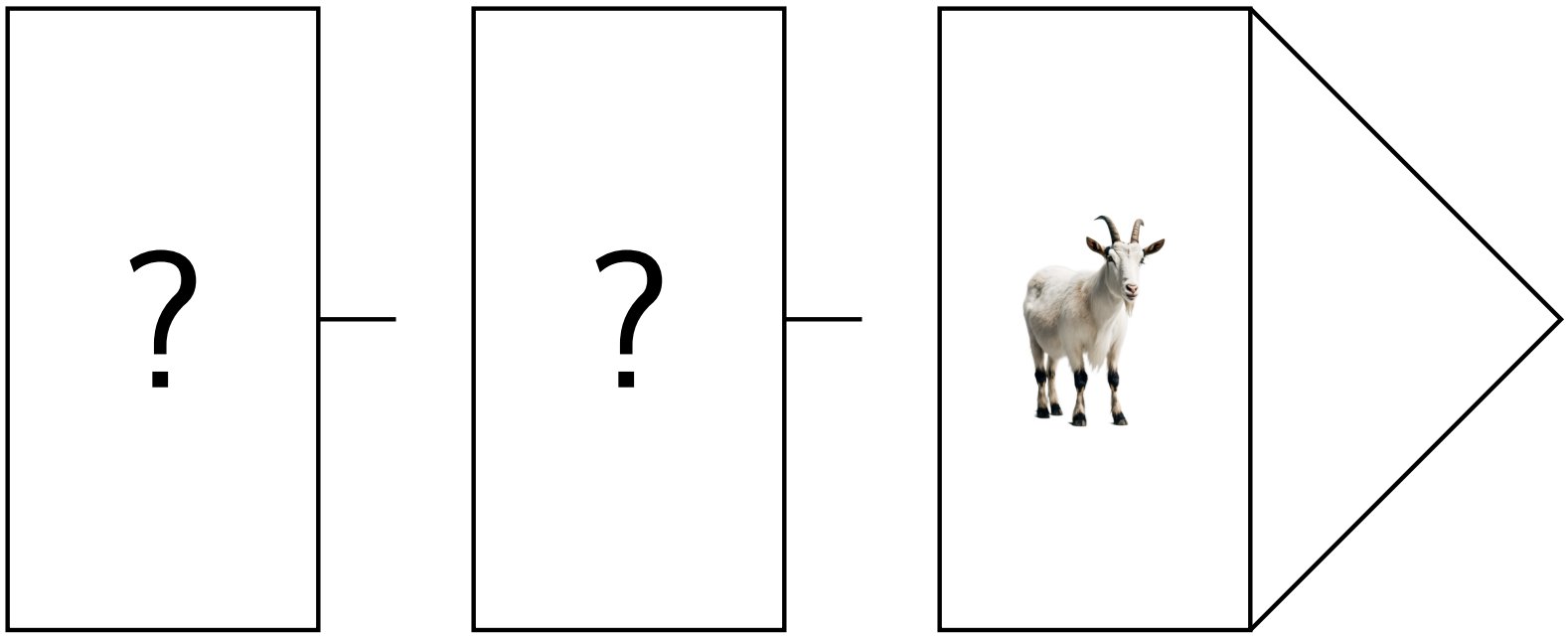
In English: $P(M|D)$: Probability my model describes the real world given the data.

# Brain Teaser: Monty Hall problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a **car**; behind the others, **goats**. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?[1]

[1] a question from reader Craig F. Whitaker's letter (copied from Wikipedia)

# Brain Teaser: Monty Hall problem



# Let Us Guess:

Before one sees anything, the chance that the car is behind each gate is $\frac{1}{3}$. Does seeing a goat in Gate 3 changes the odds?

# Let Us Solve It:

Events

- Define $A_1$ as *Car is behind Gate 1*
- Define $A_2$ as *Car is behind Gate 2*
- Define $A_3$ as *Car is behind Gate 3*
- Define $C$ as *Host opens Gate 3, which reveals a goat*

# Let Us Solve It:

Question:

- What is $p(A_1|C)$?
- What is $p(A_2|C)$?