Maximum likelihood estimation (MLE)

$$\nabla \ln p(\mathbf{t}|\mathbf{w}, \beta) = \sum_{n=1}^{N} \left\{ t_n - \mathbf{w}^{\mathrm{T}} \phi(\mathbf{x}_n) \right\} \phi(\mathbf{x}_n)^{\mathrm{T}}$$

$(t_1, t_2, \ldots, t_N)$ $\begin{pmatrix} \equiv \\ \equiv \\ \equiv \end{pmatrix}$

$$0 = t^{\mathrm{T}} \overline{\Phi} - \mathbf{w}^{\mathrm{T}} \overline{\Phi}^{\mathrm{T}} \overline{\Phi}$$

$$0 = \sum_{n=1}^{N} t_n \phi(\mathbf{x}_n)^{\mathrm{T}} - \mathbf{w}^{\mathrm{T}} \left(\sum_{n=1}^{N} \phi(\mathbf{x}_n)\phi(\mathbf{x}_n)^{\mathrm{T}} \right)$$

$t = \begin{pmatrix} t_1 \\ \vdots \\ t_N \end{pmatrix}$

$N \times 1$

$\overline{\Phi} = \begin{pmatrix} \phi(x_1)^{\mathrm{T}} \\ \phi(x_2)^{\mathrm{T}} \\ \vdots \\ \phi(x_N)^{\mathrm{T}} \end{pmatrix}$

$N \times M$

$||$

$t^{\mathrm{T}} \overline{\Phi}$

$\overline{\Phi}^{\mathrm{T}} \overline{\Phi}$

$$\mathbf{w}_{\mathrm{ML}} = \left(\mathbf{\Phi}^{\mathrm{T}} \mathbf{\Phi} \right)^{-1} \mathbf{\Phi}^{\mathrm{T}} \mathbf{t}$$

$\left[\phi(x_1), \ldots, \phi(x_N) \right] \begin{pmatrix} \equiv \\ \equiv \\ \equiv \end{pmatrix}$

Design matrix

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