Confidence Intervals

$\hat{\theta}_n$ - estimator $\longleftrightarrow$ θ

Prob$(L_n \le \theta \le U_n) \ge$ ☺ $\longleftarrow$ zero

$1 \le$ ☺ ≤ 1

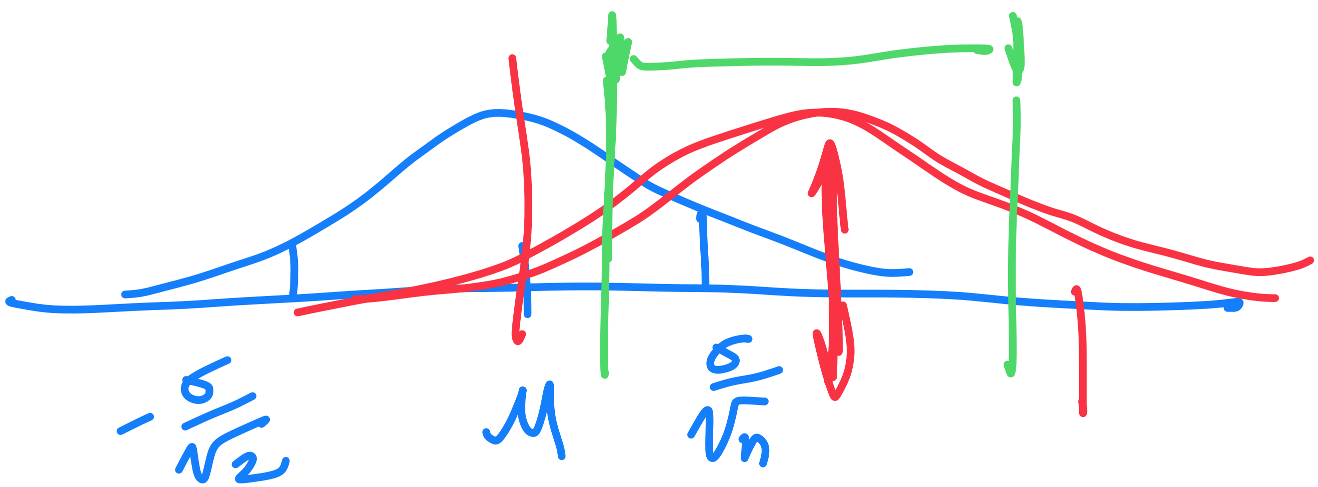
☺ $= 1 - \alpha$

Conf ~ Normal.

$(X_1 \ldots X_n) \sim N(\mu, \sigma^2)$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\mu \sim N\left(\bar{X}, \frac{\sigma^2}{n}\right)$



Z - statistic

$$Z = \frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$$

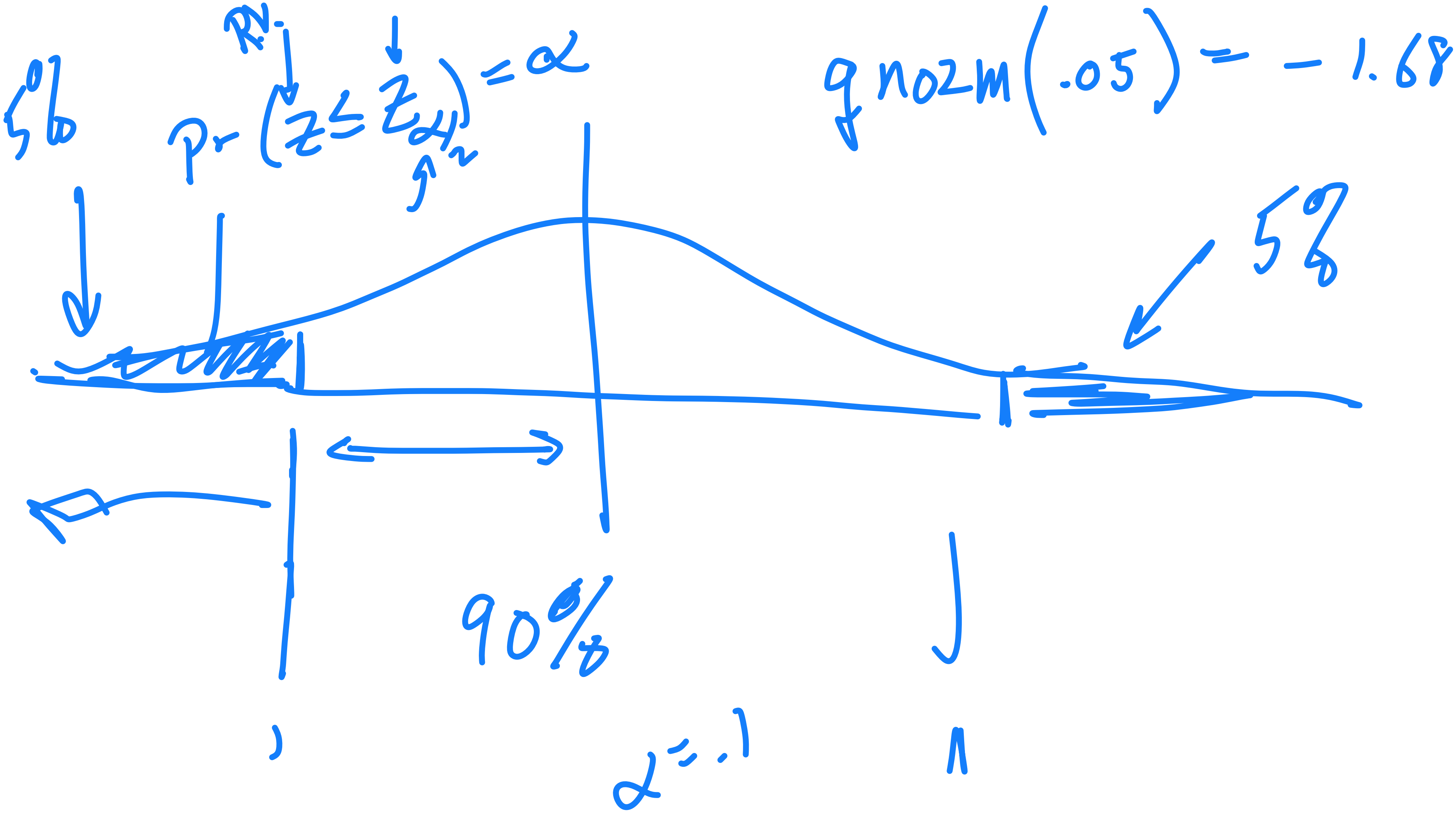
?

$Z_{\alpha/2}$

$\alpha/2$

$Z_{.025} = 1.96$, $Z_{.05} = 1.68$

$$\left(\bar{x} - 1.68 \frac{\mu}{\sqrt{n}}, \quad \bar{x} + 1.68 \frac{\mu}{\sqrt{n}} \right)$$


5%
R.V.
Pr(Z ≤ Z_{α/2}) = α
qnorm(.05) = -1.68
5%
90%
α = .1

Margin of Error for Polls.

"margin of error is $\pm 3\%$"

$95\% \leftrightarrow .05$

Ber.

$\sum_i X_i = Bin(n, p)$

Bin $\to$ $\frac{1}{n} N(np, np(1-p)) \to N\left(p, \frac{p(1-p)}{n}\right)$

n large

$\sigma = \sqrt{\frac{p(1-p)}{n}}$

$\bar{p} \pm z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$

1.96

$\sqrt{n}$

$\pm$ 3%

$$\bar{p} \pm 1.96 \sqrt{\frac{(.5)^2}{n^2}}$$

$$\underbrace{\qquad\qquad}_{=\ .03}$$

$$.03 = \left(\frac{1.96 \times 0.5}{n}\right)^2 \;\therefore\; n = \left(1.96 \times \frac{.5}{.03}\right)^2 = 1067$$