

# Confidence Intervals

$\hat{\theta}_n$  - estimator  $\leftrightarrow \theta$

$$\text{Prob}(L_n \leq \theta \leq U_n) \cong \beta \quad \leftarrow \text{zwei}$$

$$1 \leq \beta \leq 1$$

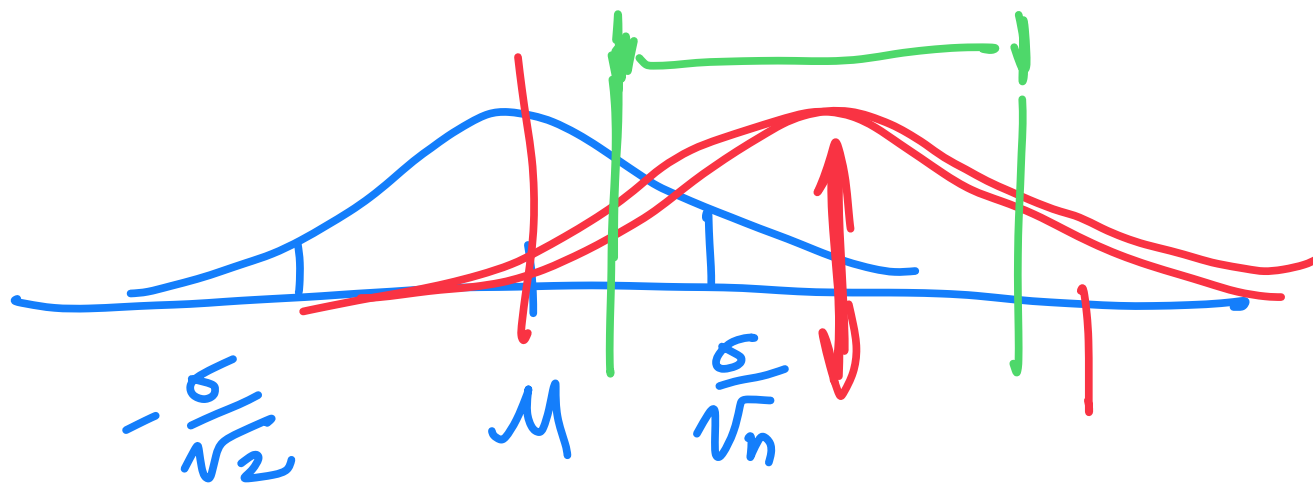
$$\beta = 1 - \alpha$$

Conf ~ Normal.

$$(x_1, \dots, x_n) \sim N(\mu, \sigma^2)$$

$$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$$

$$\mu \sim N(\bar{x}, \frac{\sigma^2}{n})$$



z - statistic

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

*(Note: In the original image, the  $\mu$  and  $\sigma^2$  are circled in yellow, and a yellow arrow points from the denominator to the  $\sigma^2$  term.)*

$z_{\alpha/2}$

$$z_{.025} = 1.96$$

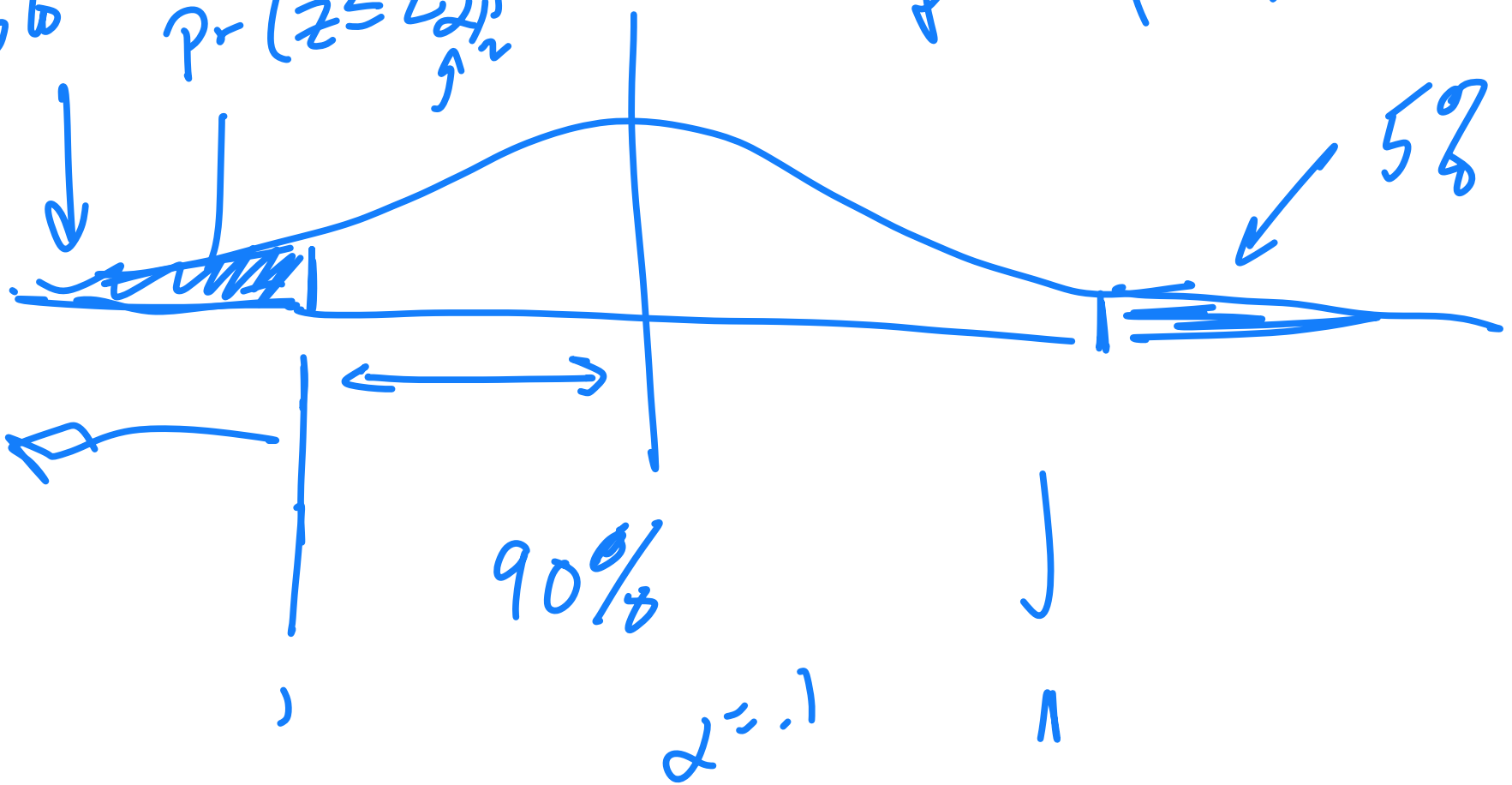
$$z_{.05} = 1.68$$

*(Note: In the original image, this value is circled in blue, and an arrow points from  $\alpha/2$  above to it.)*

$$\left( \bar{x} - 1.68 \frac{\sigma}{\sqrt{n}}, \quad \bar{x} + 1.68 \frac{\sigma}{\sqrt{n}} \right)$$

5%  
 $P_r(Z \leq Z_{\frac{\alpha}{2}}) = \alpha$

$$q_{NO2M}(.05) = -1.68$$



# Margin of Error for Polls.

"margin of error is  $\pm 3\%$ "

95%  $\leftrightarrow$  .05

$\sum_i X_i \xleftarrow{\text{Ber.}} = \text{Bin}(n, p)$

Bin.  
n large

$\rightarrow$

$\frac{1}{n} N(np, np(1-p))$

$\rightarrow N(p, \frac{p(1-p)}{n})$

$\uparrow$   
 $\hat{p}$

$\pm Z_{\alpha/2}$   
 $\downarrow$   
1.96

$\sqrt{\frac{p(1-p)}{n}}$   $\sqrt{\sigma}$

$\sigma = \sqrt{\frac{p(1-p)}{n}}$   
 $\uparrow$   
 $\sqrt{n}$

$$\bar{p} \pm 1.96 \sqrt{\frac{(0.5)^2}{n^2}}$$

± 3%

$$.03 = \left( \frac{1.96 \times 0.5}{n} \right) \Rightarrow n = \left( \frac{1.96 \times .5}{.03} \right) = 1067$$

2.03