Review

"Random sample"

"Statistic"

Sample mean & Normal RV's.

- Properties of statistics
- Parameters of a distribution

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$X_i \sim N(\mu, \sigma)$$

$$E[\bar{X}] = \mu$$

$$Var[\bar{X}] = \frac{\sigma^2}{n}$$

$$\Updownarrow$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$Z = \frac{\bar{X} - \mu}{\left(\sigma^2/n\right)^{\frac{1}{2}}}$$

$$Z \sim N(0,1)$$

How to Choose Parameters

θ parameter

$\hat{\theta} = T(X_1 \ldots X_n)$. estimate

$\bar{X} = \frac{1}{n} \sum X_i$ sample mean.

Estimators

"Bias" of an estimator

$E[\hat{\theta}] - \theta$

Unbiased estimator has zero bias

Unbiased Estimators

Example - Sample Mean.

Example - Sample Variance

$X \sim \sigma^2, \mu$

Sample Var $S^2 = \frac{1}{n-1} \sum_i (X_i - \bar{X})^2 = \frac{1}{n-1} \sum_i X_i^2 - 2 \sum_i X_i \bar{X} + \sum_i \bar{X}^2$

1 2

$$E[S^2] = \frac{1}{n-1} \sum_{i=1}^{n} \left(E[X_i^2] - 2E[X_i \bar{X}] + E[\bar{X}^2] \right)$$

3

$\operatorname{Var}[Y] = E[Y^2] - E[Y]^2$

$E[Y^2] = \operatorname{Var}[Y] + E[Y]^2$

1 $\operatorname{Var}[X_i] + E[X_i]^2 = \sigma^2 + \mu^2$

2 $\cancel{\operatorname{Cov}[X_i \bar{X}]} + E[X_i] E[\bar{X}] = \mu^2$

3 $\operatorname{Var}[\bar{X}] + E[\bar{X}]^2 = \frac{\sigma^2}{n} + \mu^2$

$$\frac{1}{n-1}\sum_i \left(\sigma^2 + \mu^2 - 2\mu^2 + \frac{\sigma^2}{n} + \mu^2 \right)$$

$$= \frac{n}{n-1}\left(\sigma^2 + \cancel{\mu^2} - 2\cancel{\mu^2} + \frac{\sigma^2}{n} + \cancel{\mu^2} \right)$$

$$= \frac{\cancel{n}}{n-1}\left(\frac{n\sigma^2}{\cancel{n}} - \frac{\sigma^2}{\cancel{n}} \right) = \left(\frac{n-1}{n-1}\right)\sigma^2 = \boxed{\sigma^2}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}(X_i - \bar{X})^2$$

$$\hat{S}^2 = \frac{1}{n_1}\Sigma(a-\bar{a})^2$$

$$E[S^2] - \sigma^2 = \sigma^2 - \sigma^2 = 0$$

unbiased

Example - Bernoulli

$p = \frac{1}{2}$ $(x_1 \ldots x_2)$

$$\frac{1}{n} \sum_i X_i = \hat{p}$$

$$E\left[\frac{1}{n}\sum_i x_i\right] = \frac{1}{n}\sum_i \underbrace{E[X_i]}^{p} = \frac{1}{n} np = p$$

Deciding Between Estimators.

$\hat{\theta}_1 = T_1[X_1 \ldots X_2]$

$\hat{\theta}_2 = T_2[X_1 \ldots X_2]$

unbiased - both

choose

$Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$