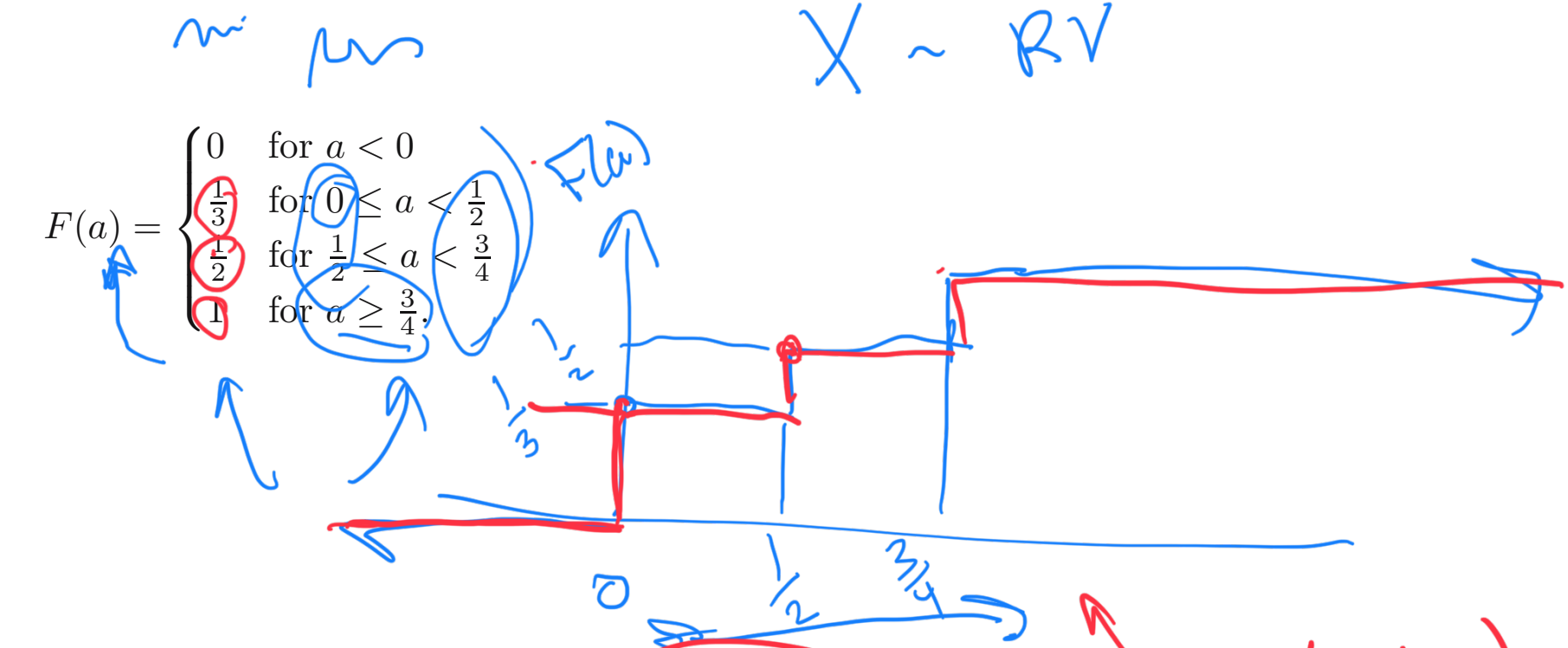
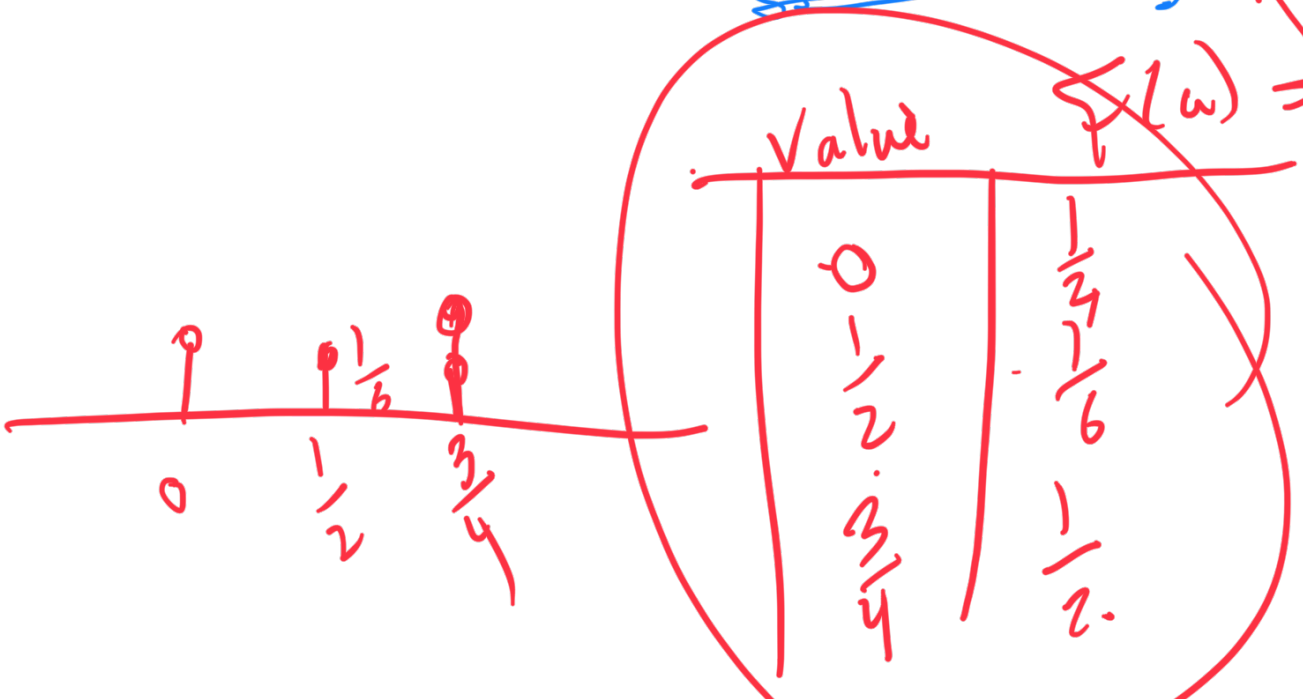
$$F(a) = \begin{cases} 0 & \text{for } a < 0 \\ \frac{1}{3} & \text{for } 0 \le a < \frac{1}{2} \\ \frac{1}{2} & \text{for } \frac{1}{2} \le a < \frac{3}{4} \\ 1 & \text{for } a \ge \frac{3}{4}. \end{cases}$$





Cont R.V.

$f(a)$ pdf

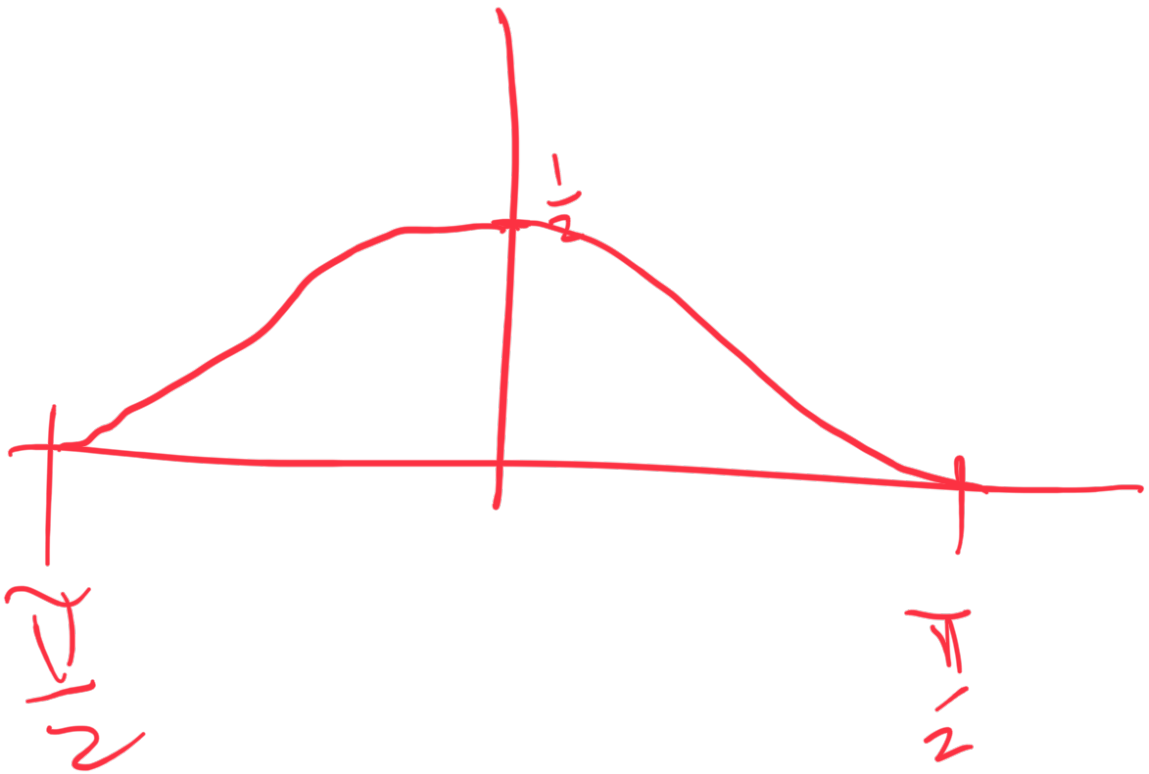
$$F(a) = \int_{-\infty}^{a} f(\alpha)\, d\alpha$$

$$f(x) = \begin{cases} \frac{1}{2} - 2x^2 & x \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$$

Is this a valid pdf?

$$\int_{-\infty}^{\infty} f(x)\, dx = 1 \qquad f(x) \geq 0 \quad \forall\, x$$

$$f(x) = \begin{cases} \frac{1}{2}\cos x & -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ 0 & \text{sonst} \end{cases}$$



$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$$

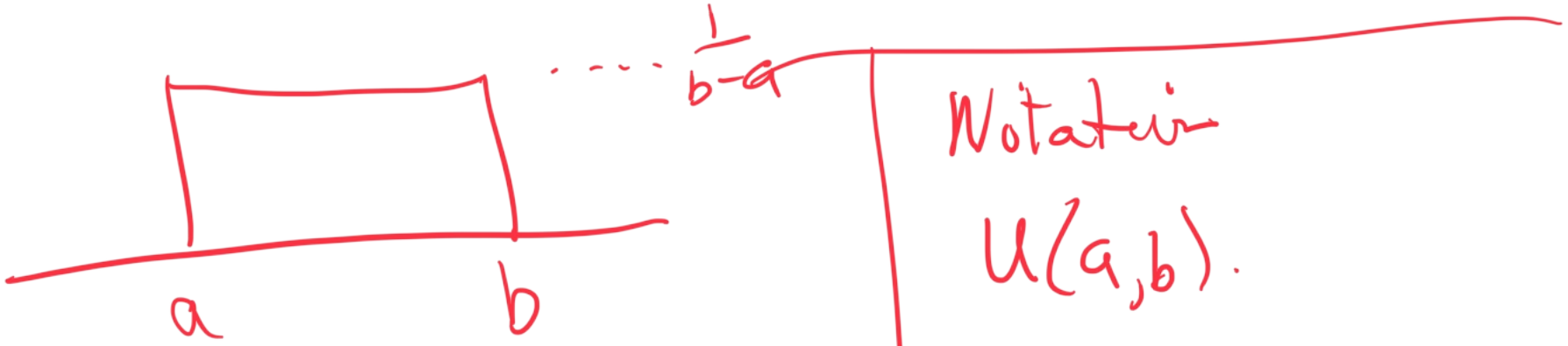
$$= \frac{1}{2}\left[\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1$$

$$= \frac{1}{2}\left(1 - (-1)\right) = \frac{1}{2}\,2$$

$$= 1$$

Uniform Dist.

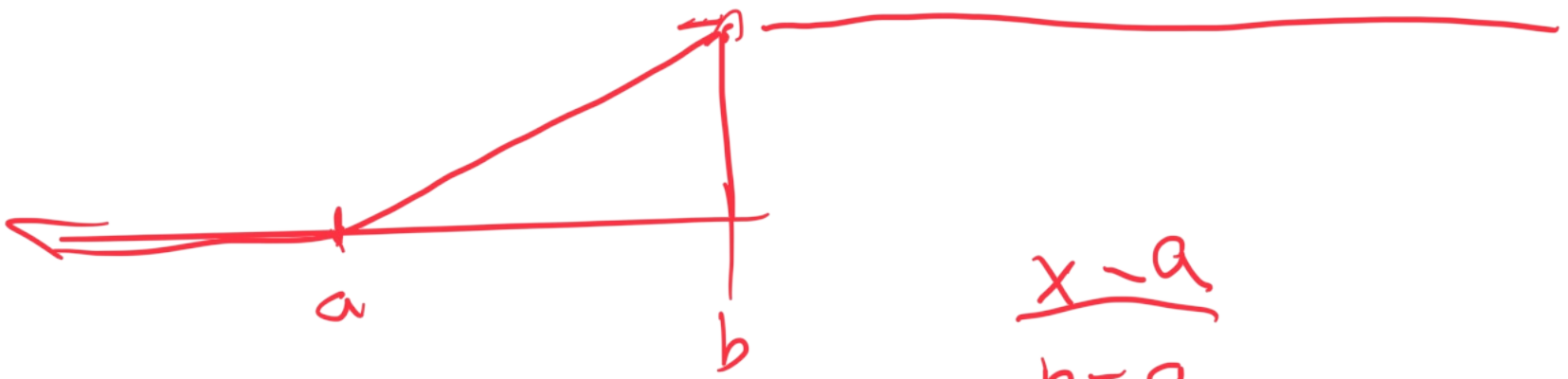
$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$



Notation

$U(a,b)$.

$$F(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x < b \\ 1 & x \ge b \\ 0 & x < a \end{cases}$$



$$\frac{x-a}{b-a}$$

Ex

Sister - airplane - arrival $U(5:00, 5:15)$

prob that plane lands between 5:05 - 5:07

$f(x)$, integrate

$F(x)$, $F(5:07) - F(5:05)$

Prob. per minute is $\frac{1}{15} = \frac{1}{b-a}$

$$F(x) = \frac{x - 300}{15}$$

$F(307) = \frac{7}{15}$

$F(305) = \frac{5}{15}$

$b - a = 315 - 300$

$Pr(5:05 \leq a \leq 5:07) = \frac{2}{15}$

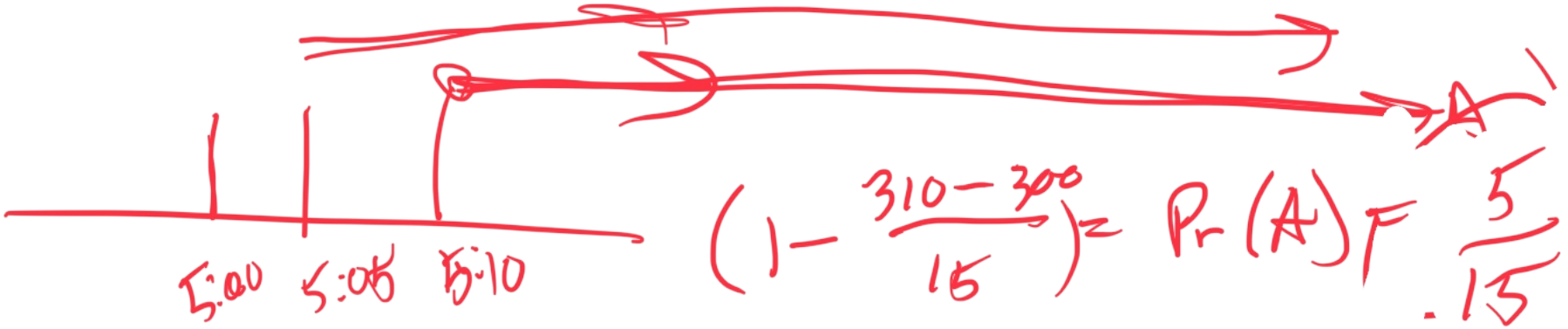
Pr that your sister lands after 5:10 given it's already 5:05

$$Pr(X \geq 5:10 \mid X \geq 5:05) = \frac{Pr(A \cap B)}{Pr(B)}$$

A: $X \geq 5:10$

B: $X \geq 5:05$

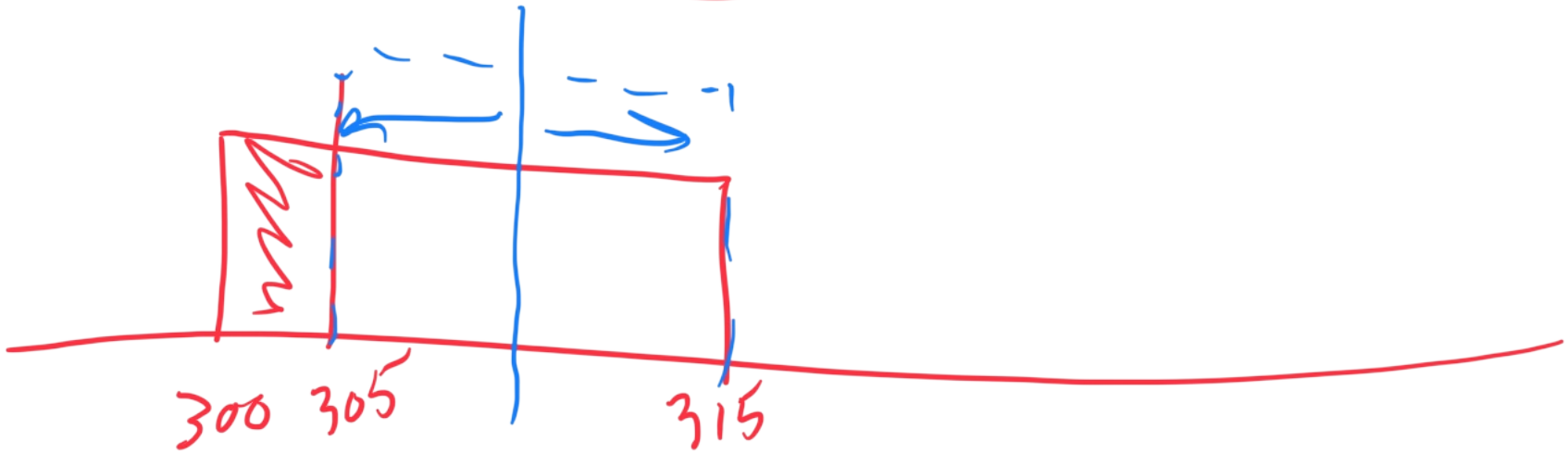
$$Pr(A \cap B) = P(A)$$



$$\left(1 - \frac{310-300}{15}\right) = Pr(A) \quad \text{or} \quad \frac{5}{15}$$

$$P(B) = 1 - \frac{305-300}{15} = \frac{10}{15}$$

$$Pr(A|B) = \frac{5/15}{10/15} = \frac{5}{10} = \frac{1}{2}$$

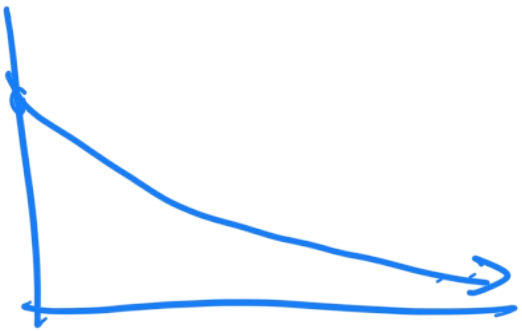


Exponential distribution

pdf: $\lambda e^{-\lambda x}$ $x \geq 0$ 0 otherwise

cdf: $1 - e^{-\lambda x}$

Notation: $X \sim Exp(\lambda)$



Events that occur randomly in the future

geometric dist

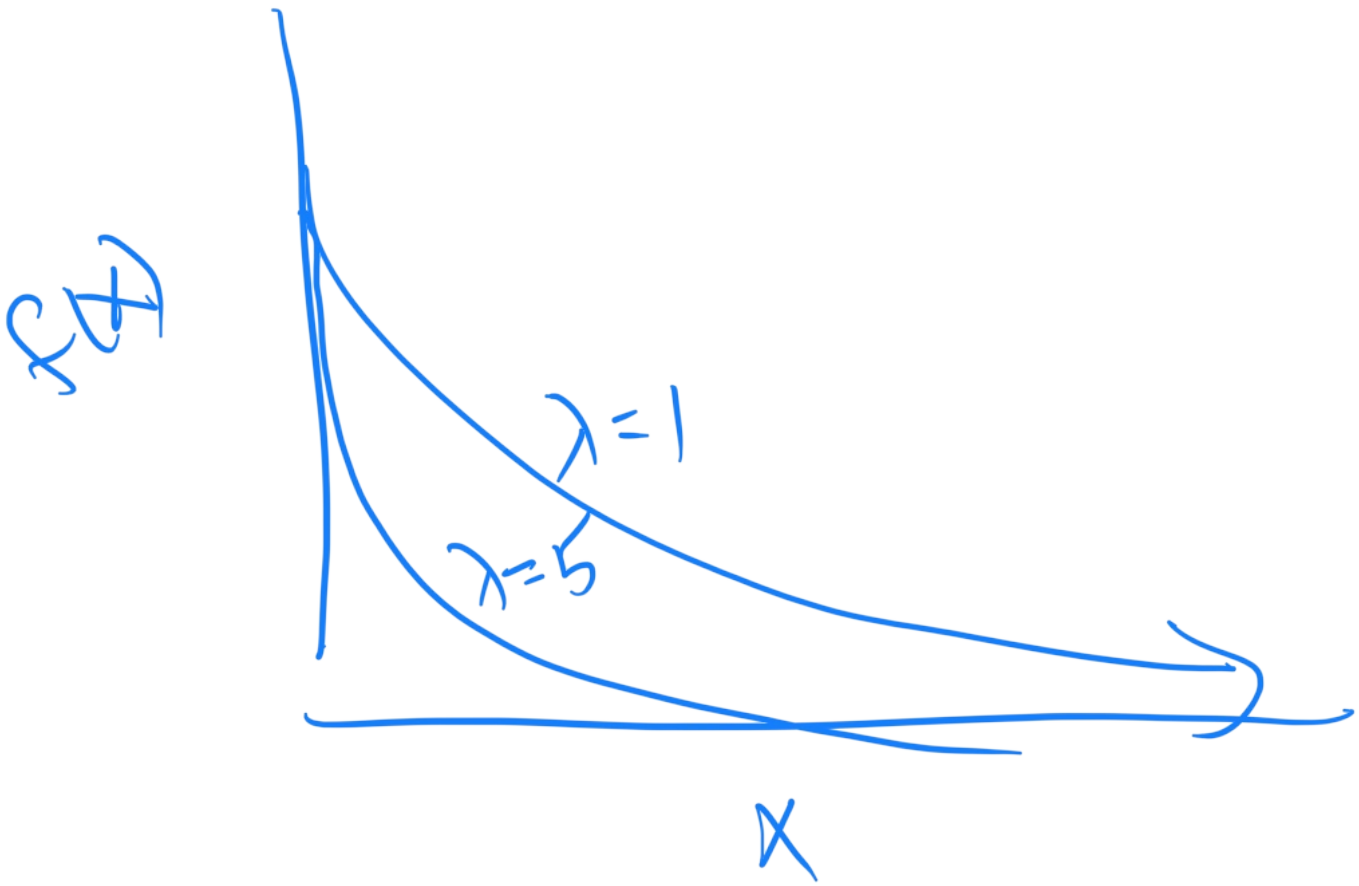
Poisson $X \sim \exp(\lambda = \frac{1}{2}\,\text{sec}^{-1})$.

Prob email takes more than 5 sec

Poisson

$$1 - F(5) = 1 - (1 - e^{-\frac{1}{2}5})$$

$$= e^{-5/2} = 0.082$$


f(x)
λ=1
λ=5
x

Exp → "Memoryless"

$$p(t+s \mid s \geq t)$$