Joint, Discrete RVs.

Repeated exp. → RV.

Collecting multiple measurements.

Studying relations between variables

— smoking ↔ health

Repeated experiments w/error.

Joint probability mass function

X, Y — RV's.

$\Pr\{X = a\} = f_X(a)$ $\quad \sum_i f_X(a_i)$ $\quad \forall a_i \in$

$\{a_0 \ldots a_n\}$

$f_X(a_i) \geq 0 \quad \forall a_i$

Joint Prob Mass function

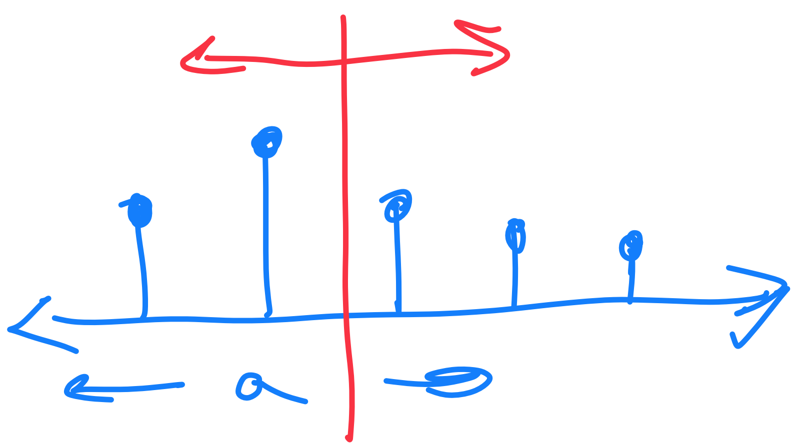
$f_{X,Y}(a,b) \equiv \Pr[X = a, Y = b] = \Pr(\{X = a\} \cap \{Y = b\})$

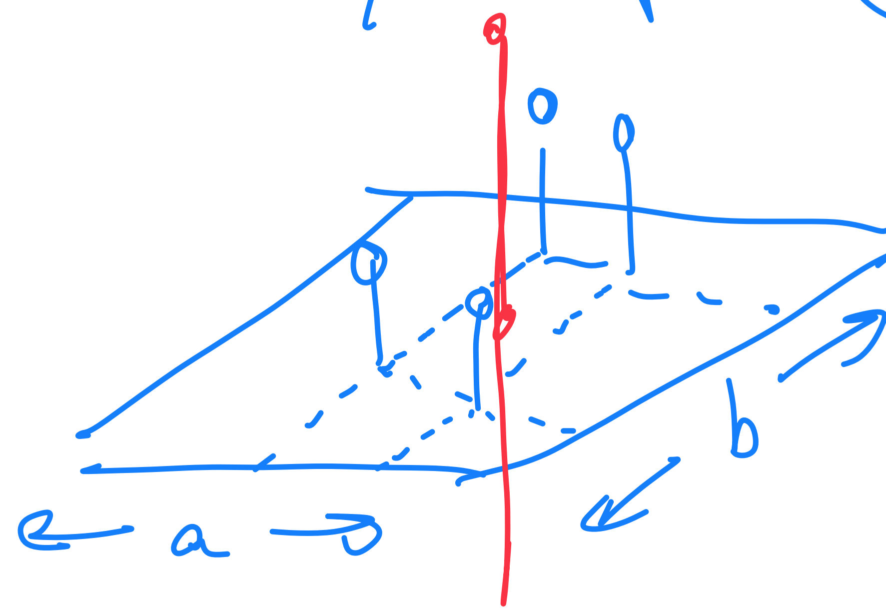
$$f_{XY}(a,b) \geq 0 \quad \forall \; a,b$$

$$\sum_i \sum_j f_{XY}(a_i,b_j) = 1$$

where a_i, b_j are all possible values for X & Y respectively

Single RV





$Pr(X=a)$ irrespective of Y

total probability $\quad f_{XY}(a,b) =$

$$f_X(a) = \sum_j f_{XY}(a, b_j) = \sum_j Pr(X=a, Y=b_j)$$

disjoint events

"marginalizing"

$$f_Y(b) = \sum_i f_{XY}(a_i, b)$$

marginal distributions for X & Y respectively

Ex:

Send bit over a network - error

S

R	0	1	
0	0.45	0.08	0.53
1	0.06	0.41	0.47
	.51	.49	1

$P(S=0)$

X Roll of 2 dice

Y \ X	1	2	3	4	5	6	
1	1/36	1/36	-	-	-	1/36	1/6
2							1/6
3							
4							
5							
6	1/36					1/36	1/6
	1/6	-	-	-	-	1/6	1

Flip 2 coins $H=1, T=0$

	0	1	
0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

Cumulative distribution function

$$F_{xy}(a,b) \equiv P_r(X \le a \,\&\, Y \le b)$$

$$= \sum_i \sum_j f_{xy}(a_i, b_j) \quad \forall\ a_i \le a,\ b_j \le b$$

Conditional Probability

$$\Pr(X=a \mid Y=b) = \frac{\Pr(\{X=a\} \cap \{Y=b\})}{\Pr(Y=b)} \quad *$$

$$\Pr(X=a \,\&\, Y=b) = \underline{p(X=a \mid Y=b)}\, \Pr(Y=b)$$

$$\Pr(X=a \mid Y=b) = \frac{f_{XY}(a,b)}{f_Y(b)}$$

Bits

R \ S	0	1	
0	.45	.08	.53
1	.06	.41	.47
	.51	.49	1

Compute

$$\Pr(R=1 \mid S=1)$$

$$= \frac{.41}{.49} = .8$$

$$\Pr(R=0 \mid S=0)$$

$$=$$

Independence

$$\Pr(X \mid Y) = P(X)$$

$$P(X \cap Y) = P(X)P(Y)$$

Events

Independence of discrete RVs.

$$f_{XY}(a,b) = f_X(a)\, f_Y(b) \quad \forall\, a, b.$$