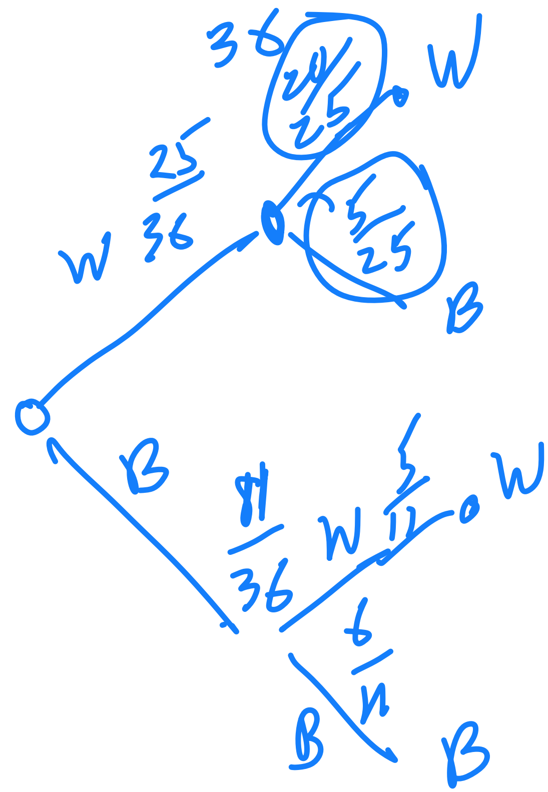
Two-sided Cards:

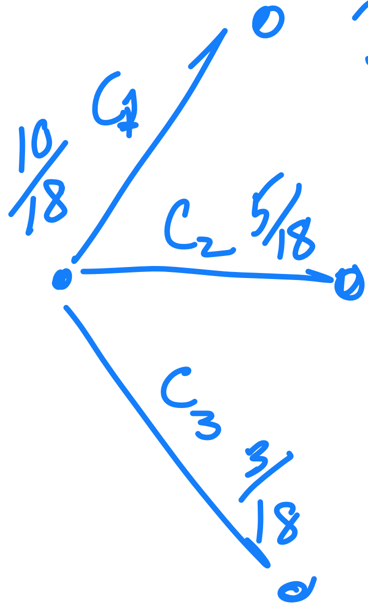
10 W/W

5 W/B

3 B/B



$$\Pr(B_2 \mid W_1).$$



$P(W|C_1) = 1$. $P(B|C_1) = 0$

$P(W|C_2) = \frac{1}{2}$ $P(B|C_2) = \frac{1}{2}$

$P(W|C_3) = 0$ $P(B|C_3) = 1$

$P(B_2 | W_1) = P(C_2 | W_1)$

$$P(C_2|W) = \frac{\overset{1/2}{P(W|C_2)}\,\overset{5/18}{p(C_2)}}{P(W)}$$

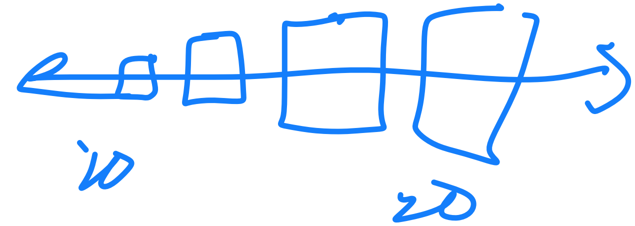
$$\overset{1}{P(W|C_1)}\,\overset{10/18}{p(C_1)} + \overset{1/2}{p(W|C_2)}\,\overset{5/18}{p(C_2)}$$

$$+ \cancel{\overset{0}{P(W|C_3)}\,p(C_3)} = P(W)$$

$$= \frac{25}{36}$$

$$\boxed{\frac{5/36}{25/36} = \frac{5}{25}}$$

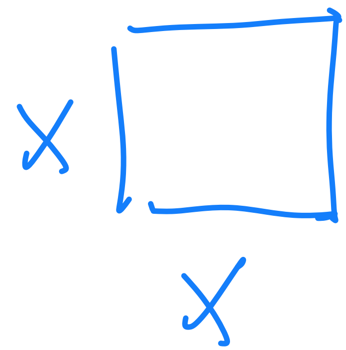
Expectation of Area.



House - area. square

size of side of house $X \sim U(10, 20)$ meters

Expected area (floor) house



$$E[X^2] = \int_{10}^{20} a^2 \overbrace{\frac{1}{10}}^{p(a)} \, da.$$

$$\int_{10}^{20} a^2 \frac{1}{10}\, da = \frac{1}{10} \frac{a^3}{3} \Big|_{10}^{20}$$

$$= \frac{1}{30} \left[8000 - 1000 \right] = \frac{7000}{30} = \frac{700}{3}$$

$$= 233$$

$$E[X]^2 = 15^2 = 225$$

Normal Distribution

pdf $f(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$

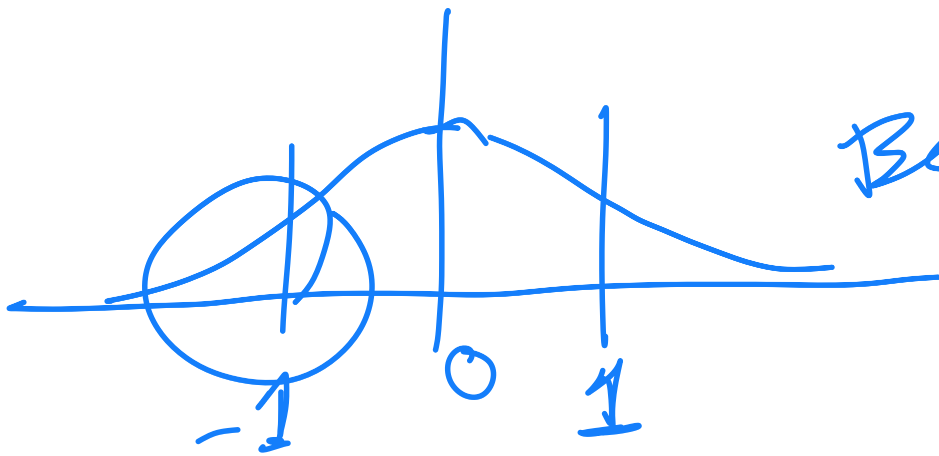
mean $= E[X] = \mu$

$var[X] = \sigma^2$

cdf $= F(x) = \int_{-\infty}^{x} f(a)\,da =$ "erf" no closed form

Notation $X \sim N(\mu, \sigma)$.

"standard normal" $N(0,1)$



Bell curve

"Gaussian"

"Mother" of all distributions

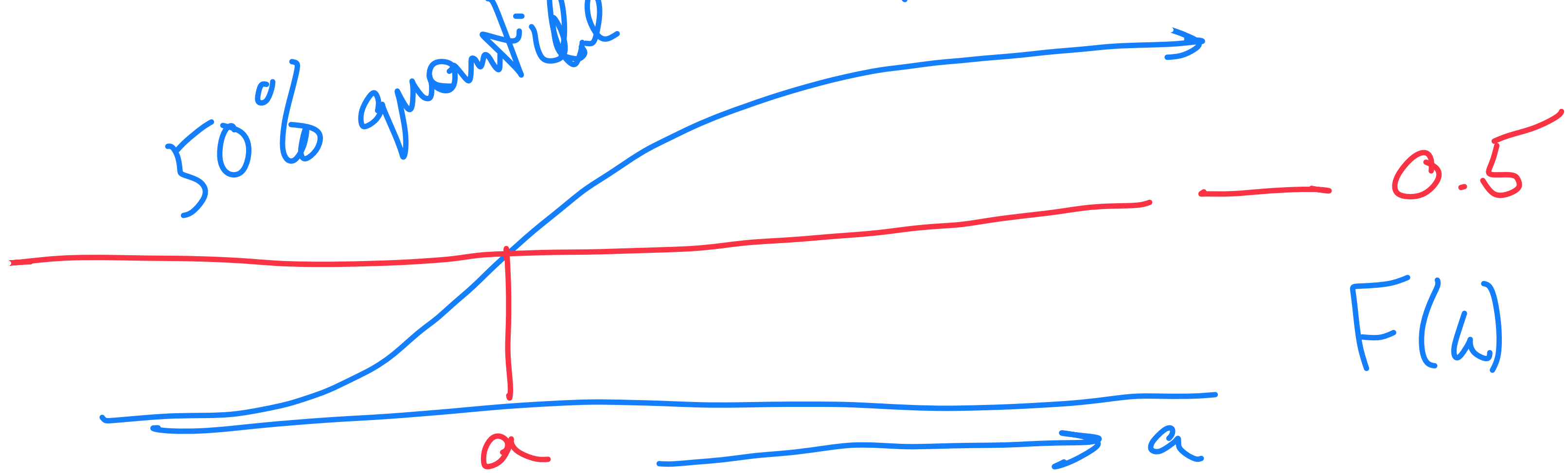
Sum of many random variables $\longrightarrow N$

"Quantiles"

50% quantile ≡ median

Property of a R.V.

Q quantile is the value of a RV for which $F(a) = Q$



Quantiles

milestones – interpretable
standardized tests.
90% quantile

baby weight
25% quantile weight

Understanding
25th, 50th,
75th
quantiles

Normal

50% quantile:

$f(x) = \frac{1}{\sqrt{2\pi}\,2} e^{-\frac{(x-3)^2}{2\cdot 4}}$ ex: μ, σ

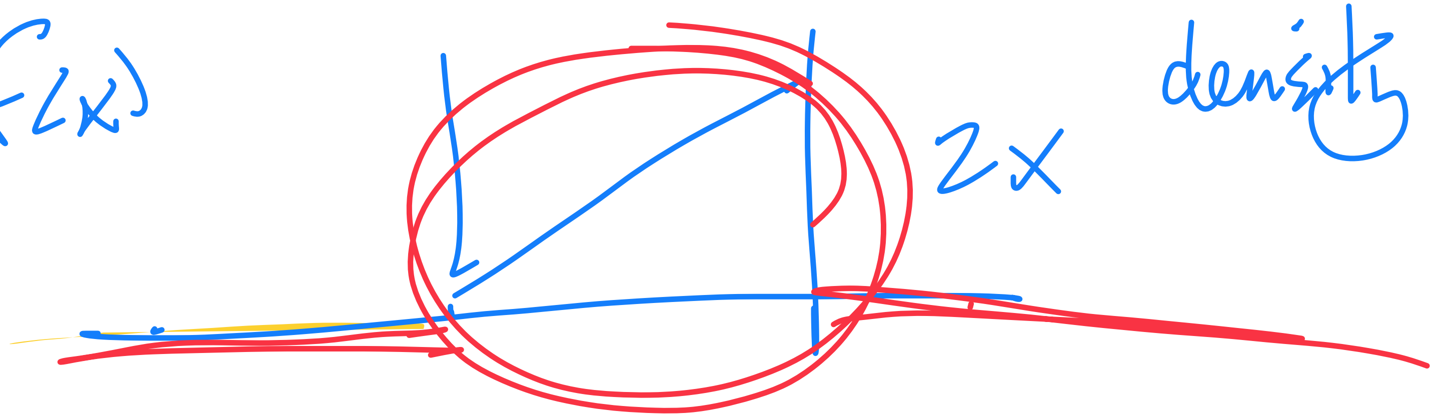
$F(x) = ? = .5$

New Example

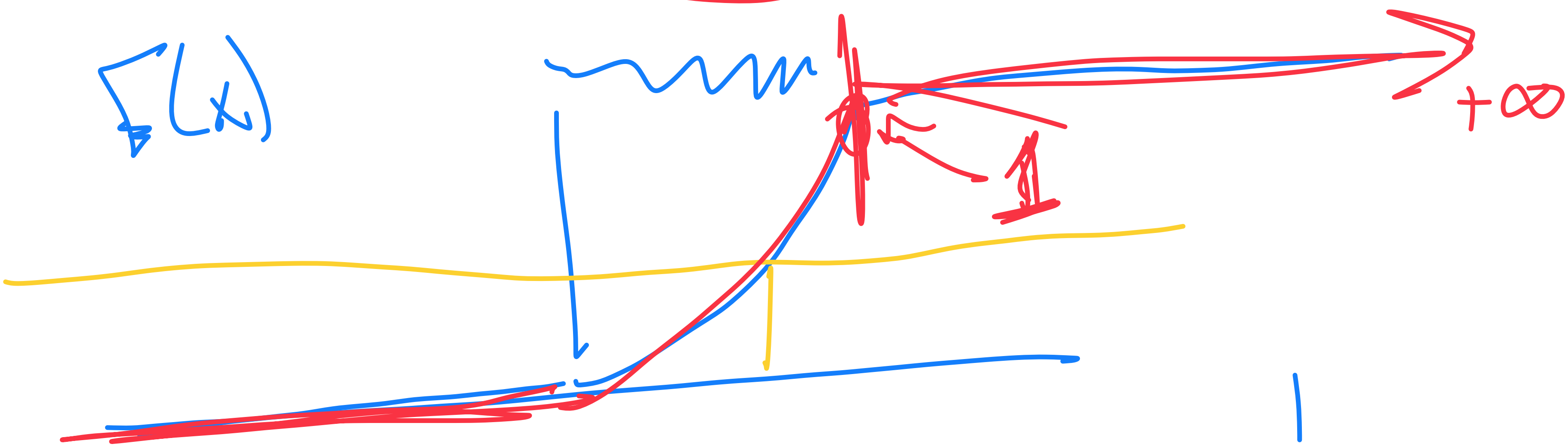
$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} x^2 & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases} \quad 0 \quad x \le 0$$

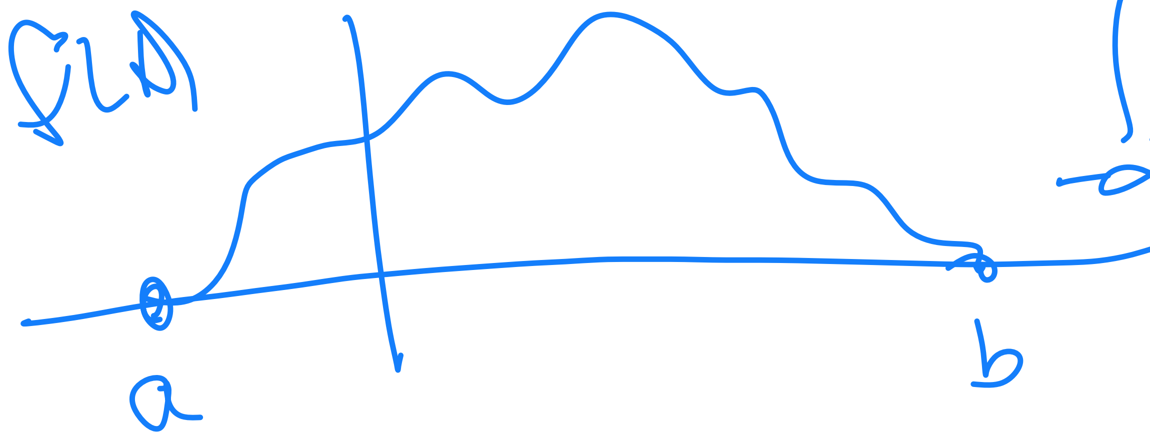
$f(x)$ density



$F(x)$

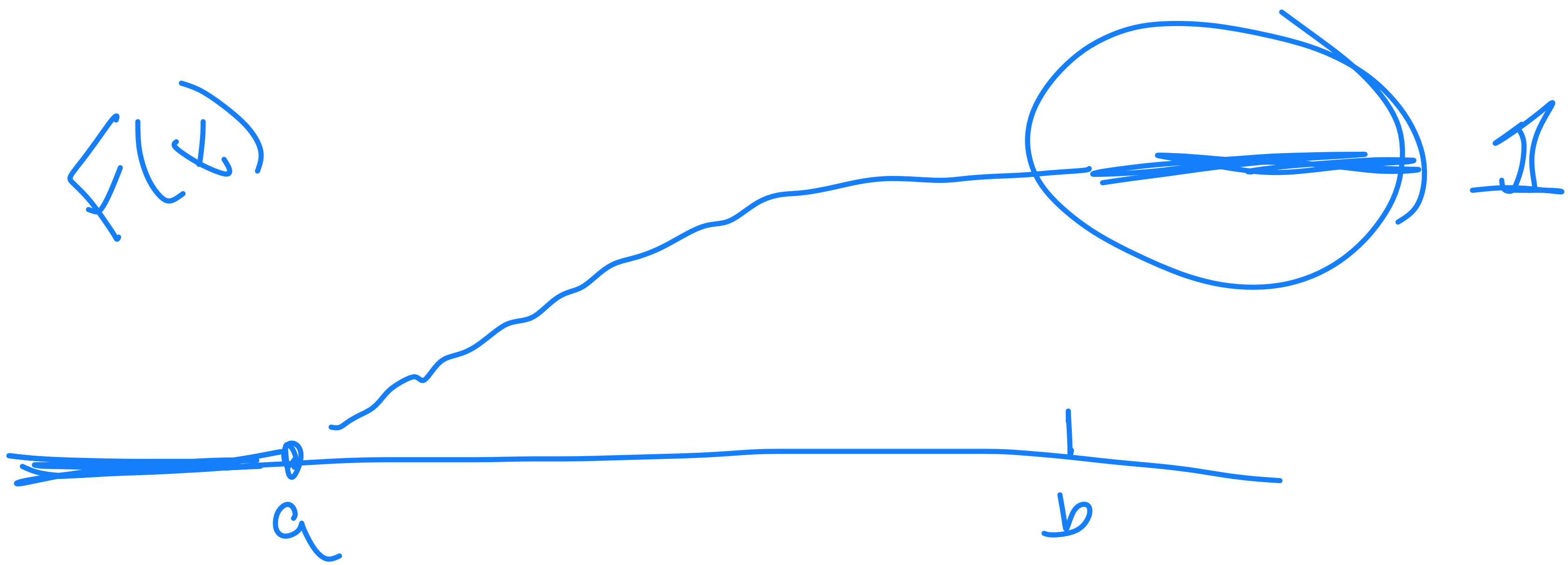


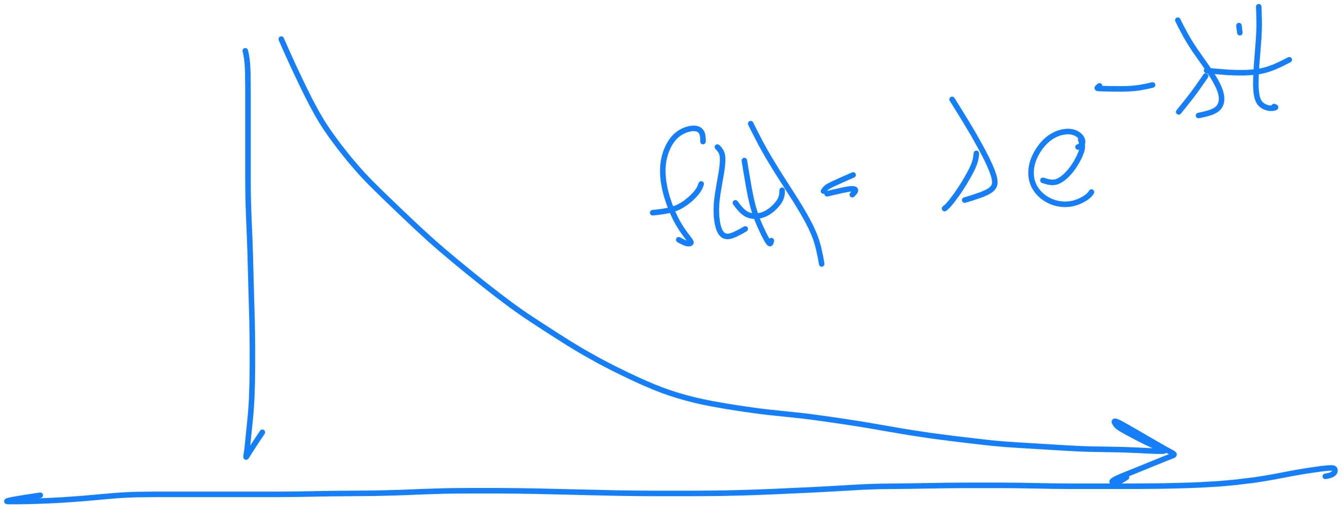
$$F(x) = x^2 = .5 \Rightarrow x = \frac{1}{\sqrt{2}}$$

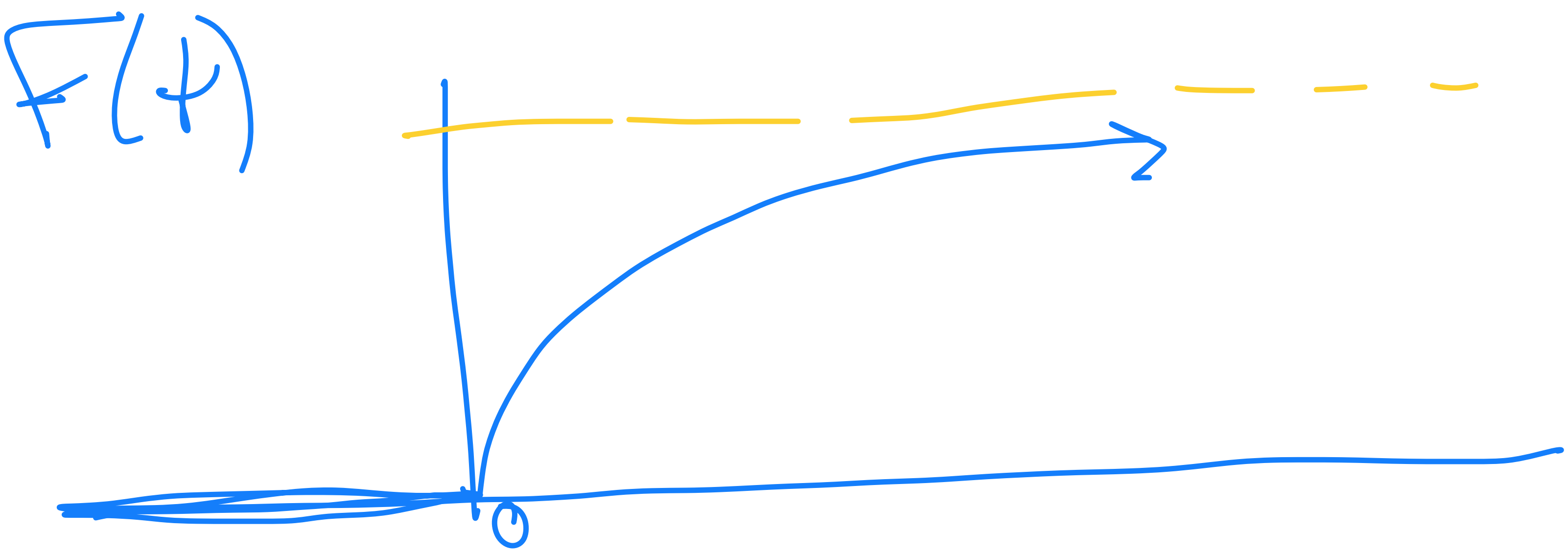


$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$= \int_a^b f(x)\,dx$$




$f(t) = \lambda e^{-\lambda t}$


$F(t)$
0