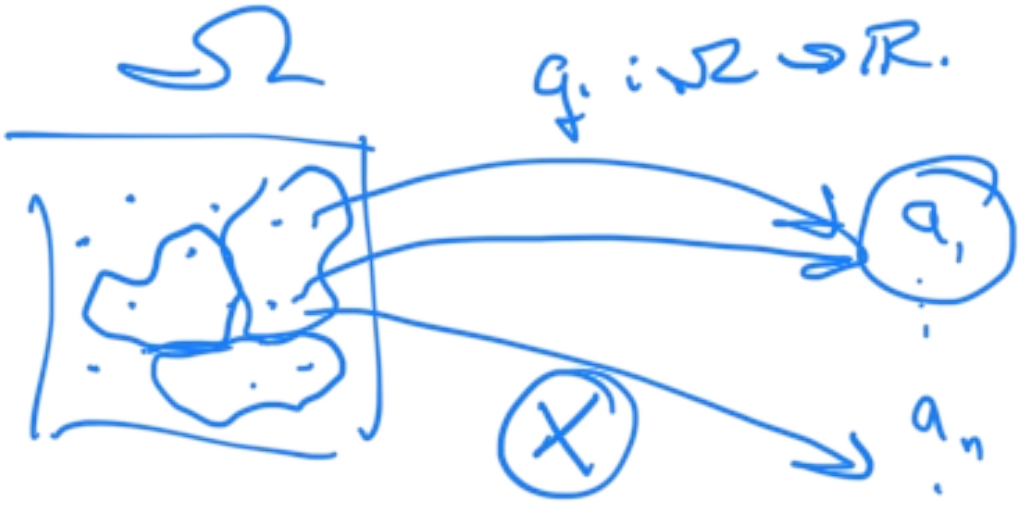
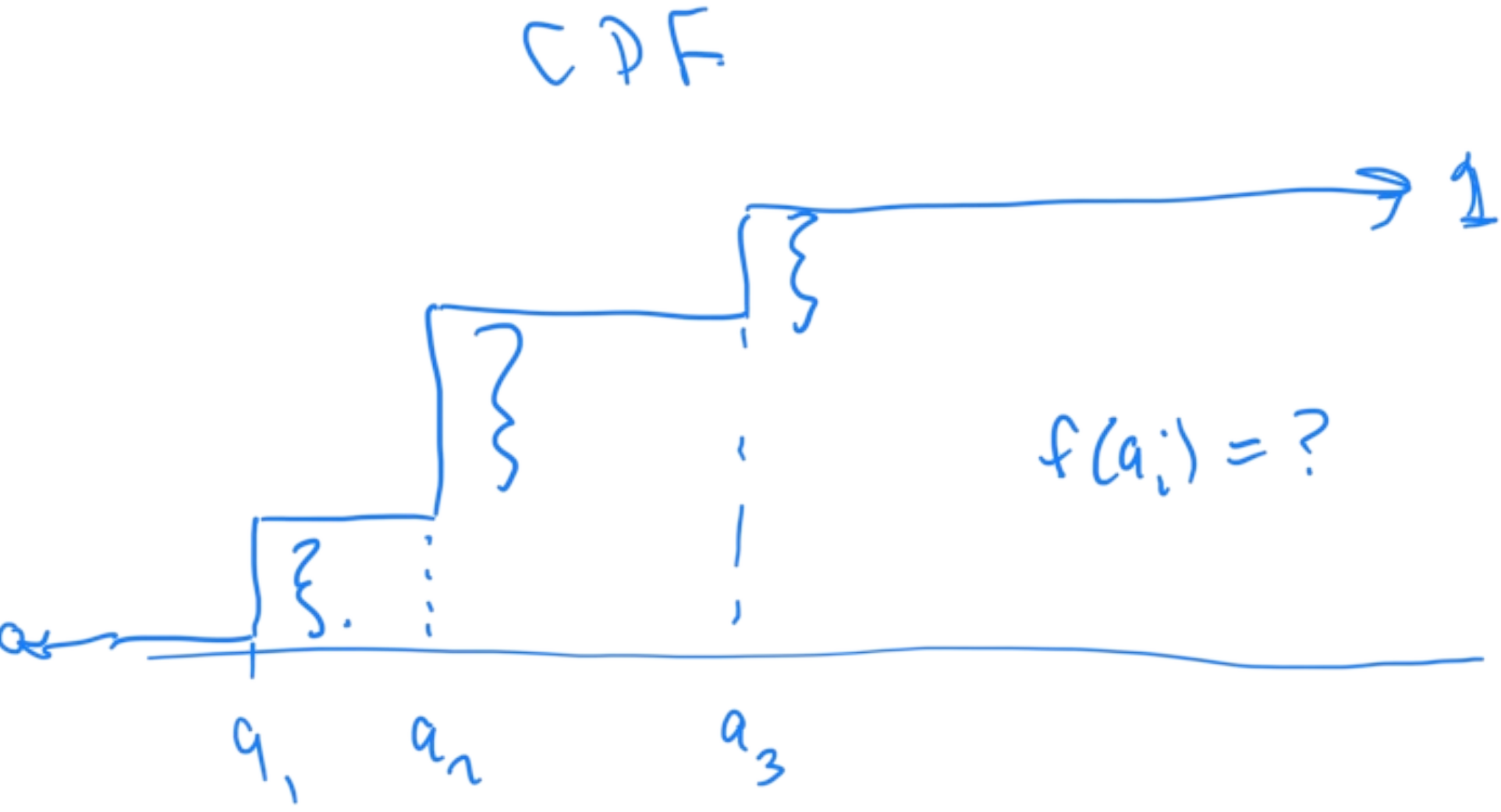
Random Variables



$$F(a) = \sum_{q_i \le a} f(q_i)$$

$f(a) = Pr(\{X = a\})$ ←

$F(a) = Pr(\{X \le a\})$.


CDF
1
$f(a_i) = ?$
a_1
a_2
a_3

Binomial

Bernoulli distribution example

Pr(H) = p

$$\{(TTTT),$$
$$(HTTT),(THTT),(TTHT),(TTTH),$$
$$(HHTT),(HTHT),(HTTH),(THHT),(THTH),(TTHH),$$
$$(THHH),(HTHH),(HHTH),(HHHT),$$
$$(HHHH)\}$$

Pr(T) = 1 − p

Pr	#
$(1-p)^4$	1
$(1-p)^3 p$	4
$(1-p)^2 p^2$	6
$(1-p) p^3$	4
p^4	1

$\binom{4}{2} = 6$

$\binom{4}{1} = 4$

Prob of k successes of n trials with B-variable of ... p

$$= \binom{n}{k} p^k (1-p)^{n-k}$$

$P(H) = p$

Geometric Distribution

Probability that it requires k trials to observe the first success.

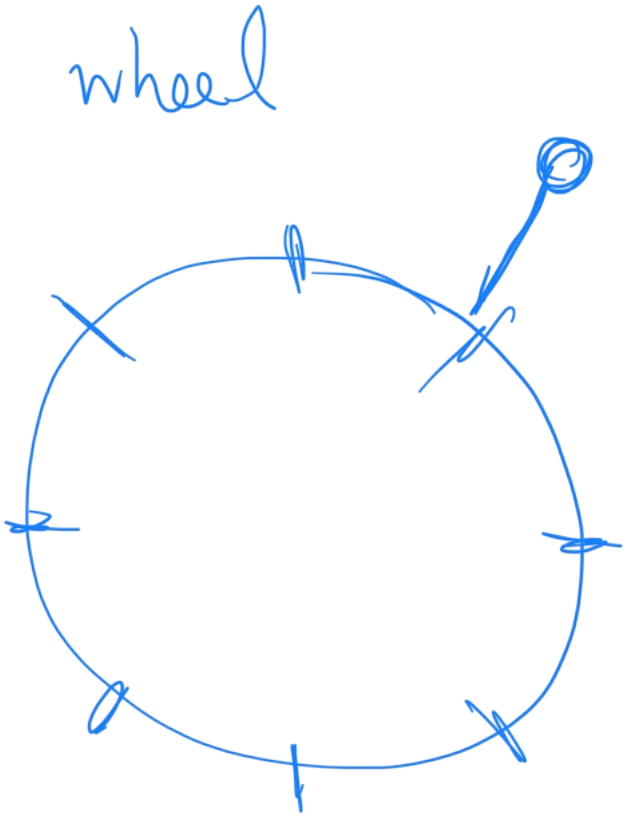
$\{(H), (TH), (TTH), (TTTH) \ldots\}$

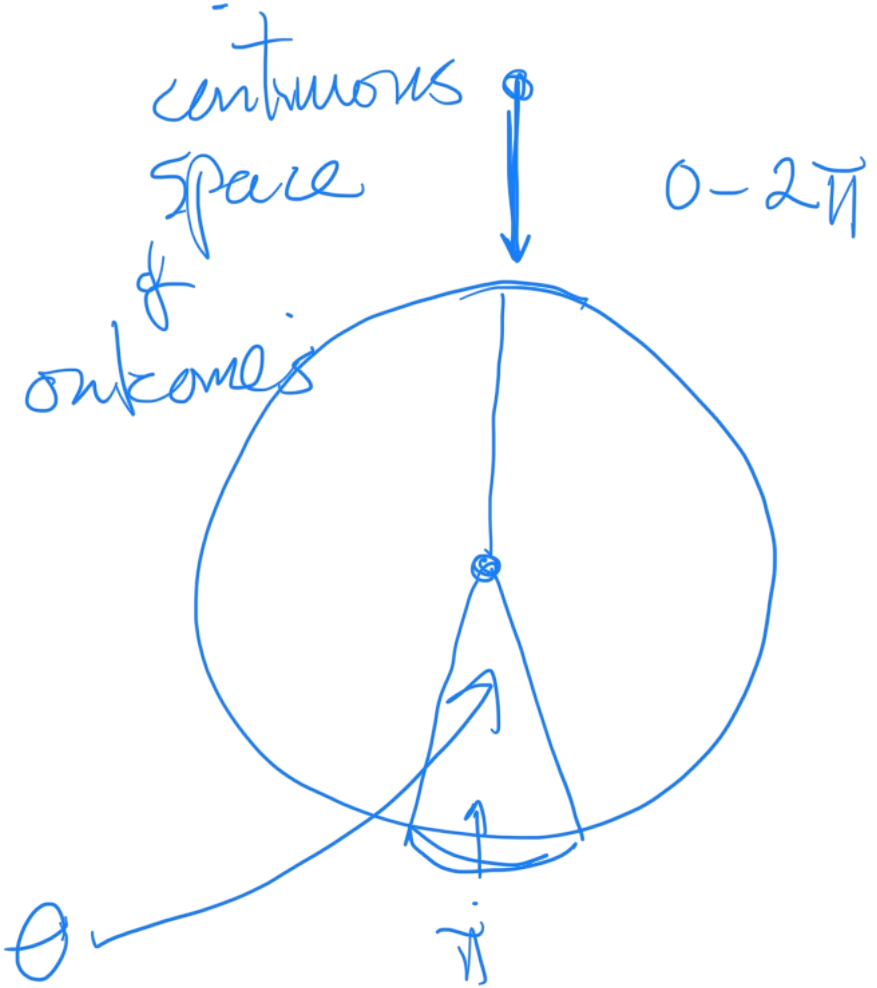
1 2 3 4 ....

p $(1-p)p$

$$Pr(k \text{ trials}) = (1-p)^{k-1} p$$

Continuous Random Variables





Cont RVs.

$$Pr(a \le x \le b) = \int_a^b f(x)\,dx$$

$$Pr(a-\varepsilon \le x \le a+\varepsilon) = \int_{a-\varepsilon}^{a+\varepsilon} f(x)\,dx$$

$$= 2\varepsilon f(a)$$

Cont. RVs.

$$\Pr(a \le x \le b) = \int_a^b f(x)\,dx$$

$f(a)$ ← probability density function
pdf "density function"

$$f(a) \ge 0 \quad \forall\ -\infty \le a \le \infty$$

$$\int_{-\infty}^{\infty} f(a) = 1$$

Cumulative Density Functions (CDF)

$$F_X(a) = P_r(X \le a)$$

$$F(a) = 0 \quad \lim a \to -\infty$$

$$F(a) = 1 \quad \lim a \to \infty$$

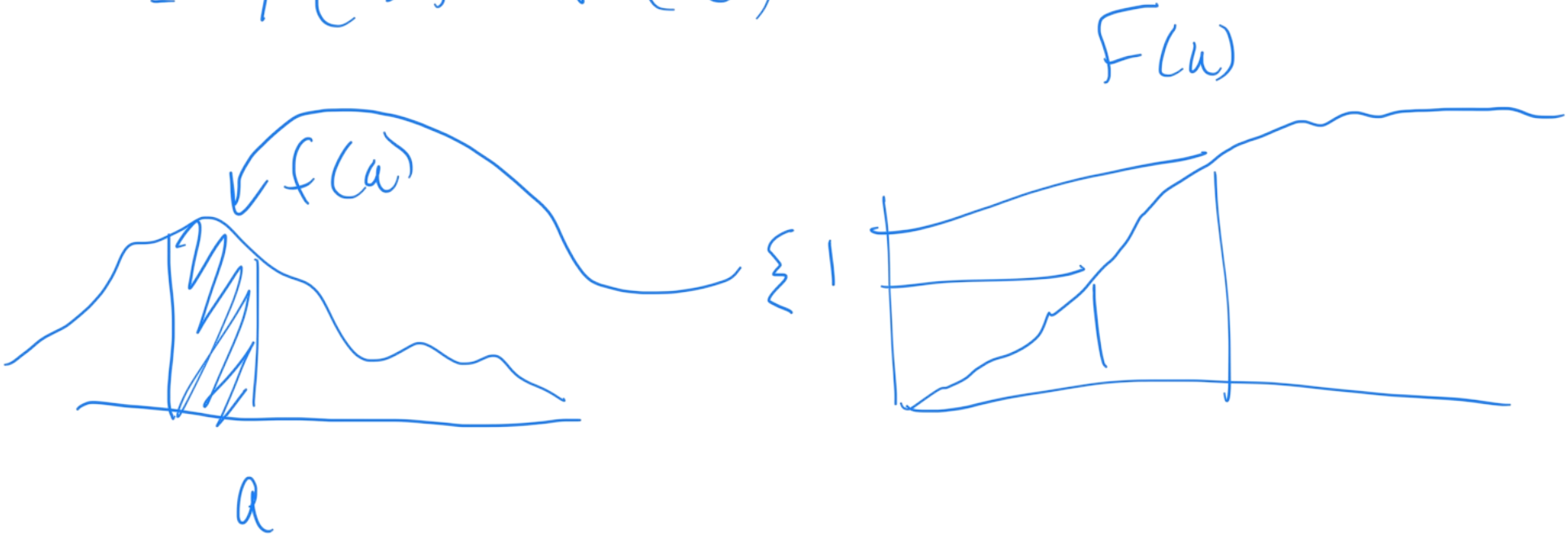
$$F(a) = \int_{-\infty}^{a} f(\alpha)\, d\alpha$$

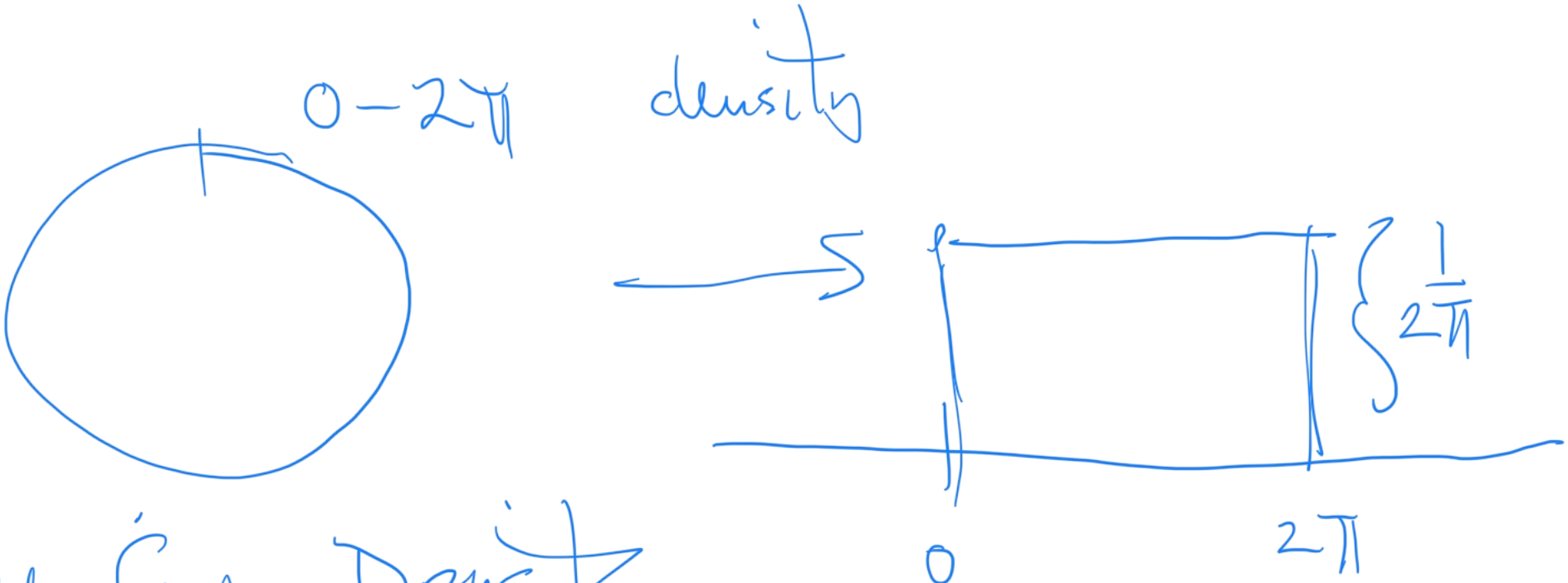
$$f(a) = \frac{d}{da} F(a)$$

CDF

$$Pr(a \le x \le b) = \int_a^b f(\alpha)\, d\alpha$$

$$= F(b) - F(a)$$





Uniform Density