Binomial Variance

$X \sim B(n, p)$ k successes in n trials with Ber param p.

mean

$E[X] = E[X_1 + X_2 \cdots + X_n] = np$.

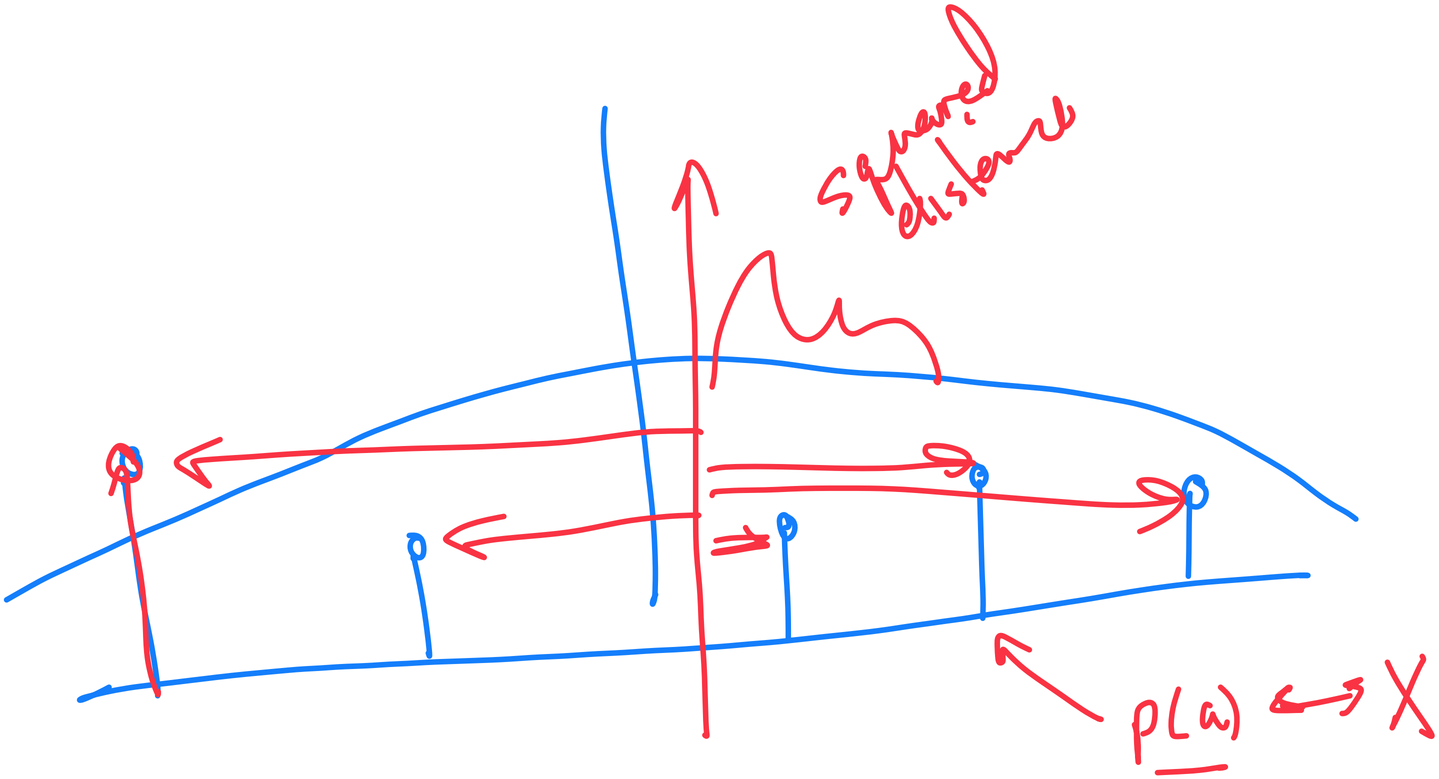
Ber.   ?   Ber.

$Var[Y] = p(1-p)$.

$Var = E[(X - E[X])^2]. = np(1-p)$.

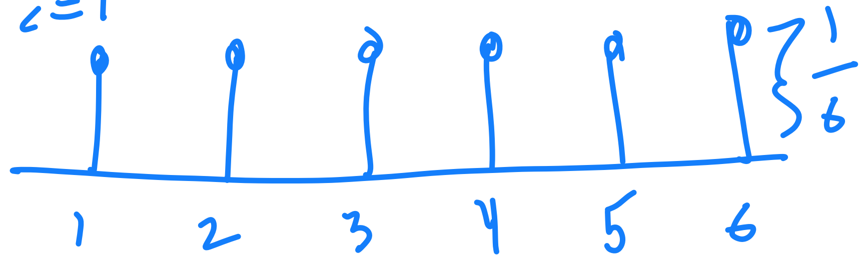
Sum of Independent RV's. $\longrightarrow$ Variances add.

Variaince



Roll of Die

$$E[X] = \sum_{i=1}^{6} i\,P(i) = \sum_{i=1}^{6} i\,\frac{1}{6} = \frac{21}{6} = 3.5$$



$$\mathrm{Var}[X] = \sum_{i=1}^{6} (i-3.5)^2 \frac{1}{6}$$

IDENTITY

$$\mathrm{Var}[X] \equiv E[(X - E[X])^2] = E[X^2] - E[X]^2$$

$$= E[X^2 - 2X E[X] + E[X]^2]$$

$$= E[X^2] - 2E[X E[X]] + \cancel{E[X]^2} = E[X^2] - E[X]^2$$

$$E[X^2] = \sum_{i=1}^{6} i^2 \frac{1}{6}$$

$$= \frac{1}{6}[1+4+9+16+25+36] = \frac{91}{6}$$

Var(X)

$$= \underbrace{\frac{91}{6}}_{E[X^2]} - \underbrace{(3.5)^2}_{E[X]^2} = 2.92.$$

Standard deviation = $\sqrt{\text{variance}}$

Properties of Expectation & Variance

- Expectation
  Linear $\quad E[aX+b] = aE[X]+b$ — not random.
- Exp & Var are themselves not random var.
- $Var[aX+b] = a^2 Var[X]$.
- $Var[X] = E[(X-E[X])^2] = E[X^2] - E[X]^2$

Uniform Variance

$X \sim U(a,b)$

$Var[X]$

$$E[X^2] = \int_a^b t^2 \frac{1}{b-a}\, dt.$$

$$= \frac{1}{b-a} \left.\frac{t^3}{3}\right|_a^b = \frac{1}{3} \frac{b^3-a^3}{b-a}$$

$$Var = \frac{1}{3}\frac{b^3-a^3}{b-a} - \left(\frac{a+b}{2}\right)^2.$$

$$= \frac{1}{3}\frac{\cancel{(b-a)}(b^2+ab+a^2)}{\cancel{(b-a)}} - \frac{a^2+2ab+b^2}{4}$$

$$\frac{(b-a)^2}{12} = Var[X] \qquad X \sim U(a,b).$$

Exponential Variance

$f(t) = \lambda e^{-\lambda t}$ mean $= \frac{1}{\lambda}$

$$E[X^2] = \int_0^\infty \text{☺}^2 \lambda e^{-\lambda \text{☺}} \, d\text{☺}$$

integration by parts.

$$= \frac{2}{\lambda^2}$$

$$\Rightarrow \mathrm{Var}[X] = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Normal Distribution

Barista question on HW3