Discrete Random Variables

"Random Variables" RV

Sample space Ω.

RV function $X : \Omega \to \mathbb{R}$.

meaning

finite RVs. $a_1, a_2, \ldots, a_n$. $|\Omega| = n$

infinite RVs $a_1, a_2, \ldots\ldots$ $|\Omega| = \text{inf}$

countably

Examples:

Sum of two dice

$\{(1,1),(1,2)\ldots\ldots(6,6)\}$.

$X = i+j \;\; \forall \; (i,j)$ in Ω.

$Pr(X = a) \;\; = Pr(\{(1,2)(2,1)\})$.

$a = 3$

Difference of two dice

$X = i-j \; \forall \, (i,j)$ in Ω.

negative

$P(T|F)$ ← True positive rate

$P(T|B)$ ← false positive rate

out of Yes

True result:

system result	T	F
T	TP 50	FP 25
F	FN 25	TN 50

TPR

$$= \frac{TP}{TP + FN} = \frac{TP}{P} = \frac{Pr(T \cap P)}{Pr(P)} = Pr(T|P)$$

Random Variable (ch 4)

$X : \Omega \to \mathbb{R}.$

Roll 2 dice $\{(1,1),(1,2), \ldots (6,6)\}.$

$X = i + j$ ← # dots on each die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$Pr(\{X = a\})$

$Pr(\{X = 4\}) = \frac{3}{36}$

$Pr(\{X = 5\}) = \frac{4}{36}$

$X = j \times i$

Event $\{X = 4\} = \{(3,2), (4,1), (1,4)\}$.

Event $\{X = 12\} = \{(3,4), (4,3), (6,2), (2,6)\}$.

$X = i + j$

$a_1 \ldots a_n$ ← possible RV values.

$2, 3, \ldots, 11, 12$

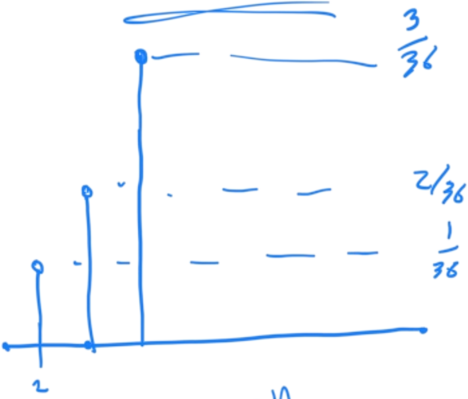
Probability mass function

$f(a) = \Pr(\{X = a\})$

$f(\omega)$	value	Prob.
a_1	2	1/36
a_2	3	2/36
a_3	4	3/36
a_4	5	4/36
a_5	6	5/36
⋮	7	6/36
⋮	8	⋮
a_{11}		1/36

$A = \{a_1, \dots, a_{11}\}$

$f(a) = 0 \ \forall \ a \notin A$



GRAPH

$f(a) \longleftrightarrow f_X(a)$

Book $p(a)$

Probability Distribution function:

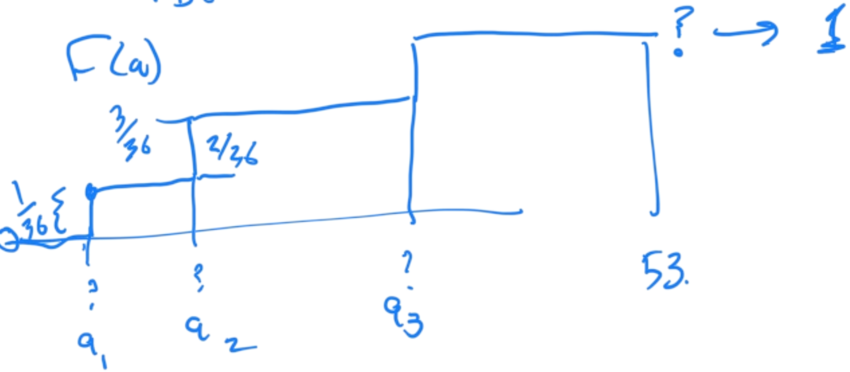
Cumulative Distribution Function

$$F_X(a) = Pr(\{X \leq a\}).$$

Ex: 2 dice, X-sum $\{X \leq 4\} = \{(1,1),(1,2),(2,1)$
$(2,2)(1,3)(3,1)\}$

$$F(4) = \frac{6}{36}.$$

PDF



Properties

$f(a) \geq 0$, $-\infty \leq a \leq \infty$ $\quad p(a_1)+p(a_2)+\dots = 1$

?

$a \leq b \iff F(a) \leq F(b)$

$$\lim_{a \to -\infty} F(a) = 0$$

$$\lim_{a \to \infty} F(a) = 1$$

Bernoulli distribution

pmf

$f(1) = p, \quad f(0) = 1 - p$

Binomial distribution

· Probabilities on repeated Bernoulli trials

- independent
- counts of "1"'s.
- H/T - coin ← how many tails?

n - number of trials

k - number of 1's (successes).

$n = 10$

$k = 4$

$p = .6$

$.6 \times .6 \times .6 \times .6 \times .4 \times .4 \times .4 \times .4 \times .4 \times .4$

$= (.6)^k \times (.4)^{n-k}$

$\binom{n}{k}$

Pr(# heads = 4)

Binomial distribution

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Pr of k successes in n trials.

n, p

$$\left\{ f(k) = \binom{n}{k} p^k (1-p)^{n-k} \right.$$