Prob Stats L08b

Variance

(& Geometric + Exponential Dist)

Feb 23, 2023

Review Expectation

R.V. $X \sim Dist(\theta)$

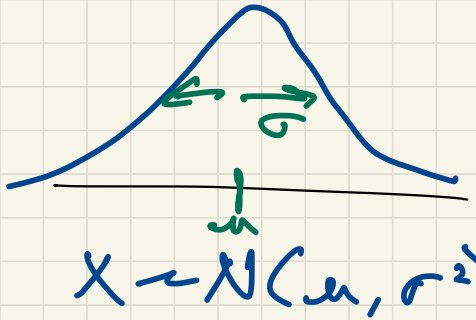
discrete

$$E[X] = \sum_{a_i} P_r(X = a_i)\, a_i$$

w_i

continuous

$$E[X] = \int_{a=-\infty}^{\infty} f_X(a)\, a\, da$$



$X \sim N(\mu, \sigma^2)$

$E[X] = \mu$

$X \sim Ber(p)$

1 w.p. p

0 w.p. $(1-p)$

$E[X] = p$

Variance

property of distribution or R.V. from distribution

↳ explains how spread out dist is.

R.V. $X \sim Dist(\theta)$

$$Var[X] = E\left[(X - E[X])^2\right]$$

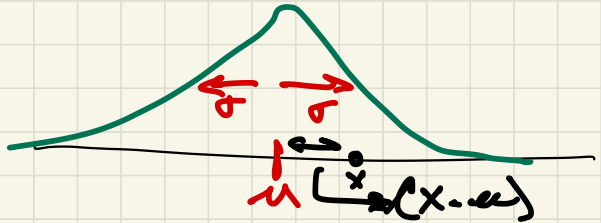
$N(\mu, \sigma^2)$

X: R.V.

$E[X]$: ↳ expected value of X constant.

standard deviation

$$\sigma_X = \sqrt{Var[X]}$$



Variance

Bernoulli R.V.

$X \sim Ber(p)$

$E[X] = p$

$Var[X] = E\left[(X - E[X])^2\right] = E\left[\underbrace{(X-p)^2}_{Y}\right]$

$E[Y] = \sum_i P_i(Y = a_i)\, a_i$

$Y = (X-p)^2$ $\quad X \in \{0,1\}$

$a_i \in \{(0-p)^2, (1-p)^2\}$

$= (1-p)(0-p)^2 + p(1-p)^2$

$(1-p)p^2 + p(1-p)^2 = [p(1-p)](\underbrace{p + (1-p)}_{1})$

$= p \cdot (1-p)$

$$\mathrm{Var}[X] = E\left[(X - E[X])^2\right]$$

($E[X]$: const.)

$$= E\left[X^2 + (E[X])^2 - 2X \cdot E[X]\right]$$

$$= E[X^2] + (E[X])^2 - 2E[X]E[X]$$

$$= E[X^2] + (E[X])^2 - 2(E[X])^2$$

$$\mathrm{Var}[X] = E[X^2] - (E[X])^2$$

$X = \{1, 2, \ldots 6\}$ Roll die $E[X] = 3.5$

$$Var[X] = E[(X - E[X])^2] = E[(X - 3.5)^2]$$

$$= \sum_{i=1}^{6} \frac{1}{6} (i - 3.5)^2$$

$$= \frac{1}{6}\Big(\underset{x=1}{(-2.5)^2} + \underset{x=2}{(-1.5)^2} + \underset{x=3}{(-0.5)^2} + \underset{x=4}{(0.5)^2} + \underset{x=5}{(1.5)^2} + \underset{x=6}{(2.5)^2} \Big)$$

$$= \frac{1}{6}\left(\frac{25}{4} + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} + \frac{25}{4} \right)$$

$$= \frac{1}{6}\left(\frac{70}{4} \right) = \frac{70}{24} = \frac{35}{12} \approx 2.92$$

$E[x] = 3.5$

$$Var[x] = E[x^2] - (E[x])^2$$

$$= E[x^2] - \frac{49}{4}$$

$$E[x^2] = \sum_{i=1}^{6} \frac{1}{6} i^2 = \frac{1}{6}\left(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2\right)$$

$$= \frac{1}{6}\left(1 + 4 + 9 + 16 + 25 + 36\right)$$

$$= \frac{91}{6}$$

$$Var[x] = \frac{91}{6} - \frac{49}{4} \approx 2.92$$

Geometric

$X \sim Geo(p)$

$$f_X(k) = (1-p)^{k-1} p$$

k events

$h = k - 1$

$$f_X(h) = (1-p)^{h} p$$

h failures

cdf

$$F_X(a) = P_r(X \leq a)$$

Exp continuous version Geo