What is Probability and Statistics and Why Should You Care?

CS 3130: Probability and Statistics for Engineers

January 9, 2023
What is Probability?

Definition

Probability theory is the study of the mathematical rules that govern random events. But what is randomness? Informally, a random event is an event in which we do not know the outcome without observing it. Probability tells us what we can say about such events, given our assumptions about the possible outcomes.
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Probability tells us what we can say about such events, given our assumptions about the possible outcomes.
Reasoning About Unknown Events (e.g. Future)
What is Statistics?

Definition

Statistics is the application of probability to the collection, analysis, and description of random data.

Statistics is used to:
▶ Design experiments
▶ Summarize data
▶ Draw conclusions about the world
▶ Explore complex data
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Applications of Probability and Statistics

Computer Science:
- Machine Learning
- Data Mining
- Artificial Intelligence
- Simulation
- Image Processing
- Data Management
- Visualization
- Software Testing
- Algorithms

Electrical Engineering:
- Signal Processing
- Telecommunications
- Information Theory
- Control Theory
- Instrumentation, Sensors
- Hardware/Electronics Testing
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General:

▶ Gambling
Applications of Probability and Statistics

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- Gambling (not recommended)
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- Stock Market Analysis
- Politics
- Sports
- Demographics
- Medicine
- Economics

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Applications of Probability and Statistics

General:

▸ Gambling (not recommended)
▸ Stock Market Analysis
▸ Politics
▸ Sports
▸ Demographics
▸ Medicine
▸ Economics
▸ All (Data) Sciences!!
Alan Turing: Connecting CS and Probability

▶ “Father of Computer Science”
▶ Most famous for:
  ▶ Computability, Turing machine
  ▶ Stored-program computer
  ▶ Turing test
  ▶ WWII cryptanalysis

Wrote a dissertation on probability theory!

Turing used probability and statistics to crack Enigma.
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Application: Machine Learning

**Machine Learning** builds statistical models of data in order to recognize complex patterns and to make decisions based on these observations.

Core tasks:

- Classification (recognition of street signs or cancer)
- Prediction (elections, movie preferences)
Application: Randomized Algorithms

Some algorithms benefit from using random steps rather than deterministic ones.

Example: QuickSort

One of the simplest & fastest sorting algorithms

Divide and Conquer: splits data based on random pivot

Takes $O(n \log n)$ time in expectation.

Example: stochastic optimization methods

Gradient descent optimizes cost functions: workhorse of machine learning

On large data sets (100s millions data points), just computing gradient is infeasible

Stochastic GD computes gradient on random sample: faster & more robust
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Application: Medical Image Analysis

- Must deal with noisy image data
- Example: finding an anatomical structure in a 3D image
- Often includes statistical analysis of resulting data

Fletcher et al, NeuroImage, 2010

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Big Data & Analytics

▶ The amount of digital data is exploding!
▶ Big data analysis is statistics + scalable CS.
▶ coresets and sketches (often randomized)

Volume of data created, captured, copied, and consumed worldwide

The volume of data generated, consumed, copied, and stored is projected to exceed 180 zettabytes by 2025

Source: statista.com
How Much is an Exabyte?

1 Exabyte = 1000 Petabytes = could hold approximately 500,000,000,000,000 pages of standard printed text.

It takes one tree to produce 94,200 pages of a book.

Thus it will take 530,785,562,327 trees to store an Exabyte of data.

In 2005, there were 400,246,300,201 trees on Earth.

We can store .75 Exabytes of data using all the trees on the entire planet.

Sources: http://www.whatsabyte.com/ and http://wiki.answers.com
(slide by Chris Johnson)

Note: 1 Zettabyte is 1000 exabytes
The Scientific Method

1. Define the question
2. Background research, observation
3. Formulate a hypothesis
4. Design and run an experiment
5. Analyze the results

Experimental measurements are noisy (randomness).
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2. Background research, observation
3. Formulate a hypothesis
4. **Design and run an experiment**
5. **Analyze the results**

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**Statistics is critical in the last two steps!**
Data Science

1. Process/squash enormous available data
2. Mine working data (calculate many statistics)
3. Analyze the results / Draw conclusions
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Every step is subject to noise and involves statistics.

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Prob/Stat — A Way of Thinking

1. Making decisions in uncertainty

   1.1 “If I do A then X will happen but if I do B then Y will happen”

   1.2 “If I do A then X is more likely to happen but if I do B then X is less likely to happen”

2. Understanding scientific results when they are presented

   2.1 “Taking this test could save lives”

   2.2 “There is a correlation between eating X and getting disease Y”
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About This Class — Learning Objectives

- Sample spaces and events, compute probabilities
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▶ Sample spaces and events, compute probabilities
▶ Random variables, reason about random processes
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- Aggregate properties of random events from large sample sizes — sampling and estimation
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▶ Construct and analyze estimators of distributions
▶ Reason about hypothesis testing in context of experiments.
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- Homeworks 40%
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- Quizzes last 15 minutes of each Thursday class. No exceptions. Lowest 3 grades will be dropped. You will need to have a computer/device in class.

Midterm — in class on Feb 29. You must attend and take in person.

Final exam — Time/location in University calendar. April 30, 3:30-5:30

Homeworks — due 10min before midnight, lose 10pts every 24 hours up to 20 points.
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- Tests and quizzes - on your own with only materials indicated.

- Homeworks — May discuss the questions and approaches with other students or look at examples from internet.

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Set aside several hours in the next week or so.
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What You Should Do Now

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   www.cs.utah.edu/~whitaker/cs3130
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4. Download and install R on your machine
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