Name:

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Homework 4: Continuous Random Variables, Joint Random Variables

Instructions: Write your answers directly on this pdf (via an editor, iPad, or pen/pencil). The answers should be in the specified place. Students will be responsible for loading their assignments to GradeScope, and identifying what page contains each answer.

The assignment should be uploaded by 11:50pm on the date it is due. There is some slack built into this deadline on GradeScope. Assignments will be marked late if GradeScope marks them late.

If the answers are too hard to read you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. A lecture hall is having wiring issues, and the projector occasionally powers down randomly following an exponential distribution with rate λ (power downs per minute).

   (a) A professor plans to give an 80 minute lecture. What is the power-down rate λ so that there is a probability of 0.05 that the lecture will **not** finish before the projector shuts down?

   (b) If the lecture is only 50 minutes, how does this change the rate needed to get a probability of 0.05 of **not** finishing the lecture without an outage?

   (c) Given the rates λ that you solved for in parts (a) and (b), what is **expected value** of how much time occurs between power-down events under the exponential distribution mode?

2. Suppose X is a binomial random variable where $n = 10$ is the number of trails and $p = 0.4$ is the probability of success of any trial.

   (a) Use the `plot` function in R to plot a histogram/stick plot using type="h" representing the distribution of X in RED. **Provide R code and plot in your response.**

(b) Use the `lines` function in R to superimpose a normal curve in BLUE on the plot from part (a) using the mean and standard deviation of X, i.e. use $\mu = np$ and $\sigma = \sqrt{np(1-p)}$. **Provide R code and plot in your response.**

(c) Use `pbinom` to calculate $\Pr(X \leq 4)$ and use `pnorm` to calculate $\Pr(X \leq 4)$. In this case, is the normal distribution a good approximation of the binomial distribution? **Provide R code in your response.**

(d) Repeat parts (a), (b) with $n = 100$ trials. Also, use `pbinom` to calculate $\Pr(X \leq 60)$ and use `pnorm` to calculate $\Pr(X \leq 60)$. In this case, is the normal distribution a good approximation of the binomial distribution? **Provide R code and plots in your response. You only need one plot (with two distributions) in your response.**

3. You are playing a game where you roll a die and win \$0 for rolling a 1, 2, or 3; \$1 for rolling a 4 or 5; and \$3 for rolling a 6. Each time you play the game, you must pay \$1.

   (a) How much money are you expected to win each round (including the one you pay)?

   (b) What is the variance of the amount of money that you win?

   (c) Assuming you like to make money, is this a game you want to play?

4. Below is a joint PMF for the following two random variables describing the state of the economy and the number of interviews it takes to find a job (assuming max of 4). These are:

- X = number job interviews to get a job: 1, 2, 3, 4 (or more)
- Y=state of the economy: Bad (0) , Average (1), Good (2)

X Y	Bad = 0	Avg. = 1	Good = 2
1	0.01	0.05	0.20
2	0.03	0.05	0.18
3	0.03	0.12	0.08
4	0.08	0.12	0.05

(a) What is the probability you will get a job within 2 interviews (2 or less)?

(b) What is the probability you will get a job within 2 interviews, given a good economy?

(c) If it takes you exactly 3 interviews to get a job, what is the probability that the economy is average or better?

(d) What is the probability that the economy is bad?

(e) Are these two variables independent? Prove your answer mathematically.

(f) What is the expected value(s) of this pair of variables?

(g) If it takes you only/exactly two interviews to get a job, what is the expected value of the economy?

(h) If the economy is bad, what is the expected number of interviews?

5. In January, let X represent the fraction of time it is snowing at Snowbird, and Y represent the fraction of time it is snowing at the SLC airport. Say they have the following joint density of X and Y

$$f(x, y) = \begin{cases} \frac{2}{11}(10x + y) & \text{for } x, y \in [0, 1] \times [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the marginal probability it is snowing at Snowbird $f_X(x)$?

(b) What is the probability that it is snowing at Snowbird less than half of the time (i.e., $\Pr(\{X \leq \frac{1}{2}\})$)?

(c) What is the conditional probability it is snowing at Snowbird less than half the time, if it is snowing at SLC exactly half the time?