CS3130/ECE3530: Probability and Statistics for Engineers **Due Tu 2/20**

Name:

uID:

Homework 3: Discrete and Continuous Random Variables

Instructions: Write your answers directly on this pdf (via an editor, iPad, or pen/pencil). The answers should be in the specified place. Students will be responsible for loading their assignments to GradeScope, and identifying what page contains each answer.

The assignment should be uploaded by 11:50pm on the date it is due. There is some slack built into this deadline on GradeScope. Assignments will be marked late if GradeScope marks them late.

If the answers are too hard to read you will lose points (entire questions may be given 0).

Please make sure your name appears at the top of the page.

You may discuss the concepts with your classmates, but write up the answers entirely on your own. **Be sure to show all the work involved in deriving your answers! If you just give a final answer without explanation, you may not receive credit for that question.**

1. Utah Jazz guard Jordan Clarkson makes 82% of his free throw attempts (and misses 18%). Let's say he will shoot 100 free throws next season. For each of the questions below, please give ALL of the following information: (1) the formula to compute the answer, and (2) the final number for the probability. (You will want to use a calculator or R to compute these.)

   (a) What is the probability that Clarkson will miss exactly 16 free throws next season?

   (b) What is the probability that Clarkson will make 85 or more free throws next season?

   (c) What is the probability that Clarkson will miss all of his free throws next season?

2. I bought a cheap toaster. Every time I use it there is a chance that it will catch on fire with probability $p = 0.25$, burn my toast, and destroy itself. But with probability $(1 - p)$ it makes perfect toast!

   (a) Suppose we toast slices of bread until a slice is burned. Let X be the number of slices until the first one is burned. What type of random variable is X?

   (b) Suppose we have 20 slices of bread. In R, plot the pdf of the random variable X. The points on your plot should be filled squares and colored red. **Include both your code and your plot in your answer.**

(c) In R, plot the cdf of X. The type of plot should be a stair step plot. Also, make the line color blue. **Include both your code and your plot in your answer.**

(d) What is the probability I put at least 7 pieces of bread into my cheap toaster before it destroys itself? **include R code used to get result.**

3. The score of a student on a certain exam is represented by a number between 0 and 1. Suppose that the student passes the exam if this number is at least 0.60. Suppose we model this experiment by a continuous random variable X, the score, whose probability density function is given by

$$f(x) = \begin{cases} 4x & \text{for } 0 \leq x \leq \frac{1}{2} \\ 4 - 4x & \text{for } \frac{1}{2} \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that the student fails the exam? **Show all your work (i.e., what you are integrating and differentiating, etc.)**

4. Let X be a continuous random variable that takes values in [0,1], and whose cumulative distribution function F satisfies

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 2x^3 - x^4, & \text{for } 0 \le x \le 1 \\ 1, & \text{if } x > 1 \end{cases}$$

(a) Compute $\Pr(\frac{1}{4} \le X \le \frac{3}{4})$

(b) What is the probability density function of X?

5. You manage a coffee shop with one barista (person making coffee). It take 3 minutes for them to make a coffee. You anticipate that the time between customers' will follow an exponential distribution with rate λ requests per minute.

   (a) What is the maximum order rate you can handle if you want the probability of a customer waiting more than 3 minutes to be less than 20%?

   (b) For that same order-rate constant, λ, what is the probability that a customer waits more than 3 minutes if the barista works faster and completes orders in 2 minutes?

6. Student take an exam, and the scores on the exam are normalized from 0.0 to 1.0 (instead of 0–100). It is known from previous exams that the probability density for scores (pdf) is:

$$f(a) = \begin{cases} 6(x - x^2) & \text{for} \quad 0.0 \leq a \leq 1.0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Graph the pdf for the exam grades.

(b) Compute and graph the cumulative distribution function for the grades.

(c) If a student need a score of at least 0.45 to pass the exam, what percentage of students do we expect to pass?