CS 3130/ECE 35350: Probability and Statistics for Engineers **Due Tu 1/24**

Homework 1: Getting Started with Probability

Reminder: Give partial credit for answers that show work that is partially correct. Default is 1/2 credit, but you can make it more or less in drastic cases. Don't count off for arithmetic mistakes, and don't take off twice when an incorrect number is used in a later answer.

1. **(25 points total / 5 points each part / no work needed)**
   (a) B^c
   (b) $B \cap G^c = B - G$
   (c) $B \cup G$
   (d) $B^c \cap G^c = (B \cup G)^c$ (DeMorgan's Law)
   (e) $(B \cup G) - (B \cap G) = (B - G) \cup (G - B)$

2. **(25 points total / 5 points each part / 1 point each for showing work, which should include stating the rule)**
   (a)

$$\begin{aligned} \Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B) && \text{(Inclusion-Exclusion or Union Rule)} \\ &= 0.3 + 0.5 - 0.25 \\ &= 0.55 \end{aligned}$$

   (b)

$$\begin{aligned} \Pr(A^c) &= 1 - \Pr(A) && \text{(Complement Rule)} \\ &= 1 - 0.3 \\ &= 0.7 \end{aligned}$$

   (c)

$$\begin{aligned} \Pr(A^c \cup B^c) &= \Pr((A \cap B)^c) && \text{(DeMorgan's Law)} \\ &= 1 - \Pr(A \cap B) && \text{(Complement Rule)} \\ &= 1 - 0.25 \\ &= 0.75 \end{aligned}$$

   (d)

$$\begin{aligned} \Pr(A \cap B^c) &= \Pr(A) - \Pr(A \cap B) && \text{(Difference Rule)} \\ &= 0.3 - 0.25 \\ &= 0.05 \end{aligned}$$

(e)

$$\begin{aligned}
\Pr(A^c \cap B^c) &= \Pr((A \cup B)^c) && \text{(DeMorgan's Law)} \\
&= 1 - \Pr(A \cup B) && \text{(Complement Rule)} \\
&= 1 - 0.55 && \text{(from part (a))} \\
&= 0.45
\end{aligned}$$

3. **(25 points total)**

   (a) **(5 points total / no work needed)**

$$\Omega = \{(1,2),(1,3),(2,3),(2,1),(3,1),(3,2)\}$$

   (b) **(8 points total / 2 points each / no work needed)**

$$\begin{aligned}
A &= \{(1,2),(2,3),(2,1),(3,2)\} \\
B &= \{(1,2),(2,1)\} \\
C &= \{(1,3),(2,3),(3,1),(3,2)\} \\
D &= \{(1,3),(3,1)\}
\end{aligned}$$

   (c) **(12 points total / 3 points for the event + 1 point for the probability)**

$$A^c = \{(1,3),(3,1)\}; \qquad \Pr(A^c) = \frac{2}{6} = \frac{1}{3} = 0.3\overline{3}$$

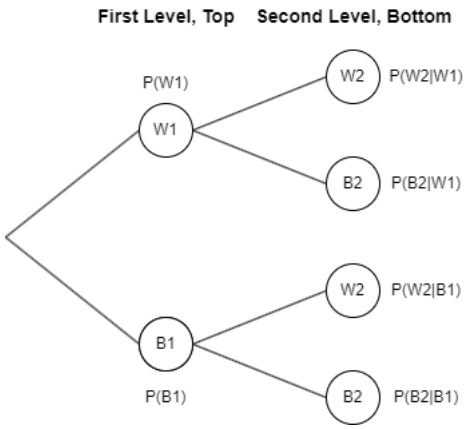
$$A \cup (C \cap D) = \{(1,2),(1,3),(2,3),(2,1),(3,1),(3,2)\}; \qquad \Pr(A \cup (C \cap D)) = \frac{6}{6} = 1$$

$$A \cup D^c = \{(1,2),(2,3),(2,1),(3,2)\}; \qquad \Pr(A \cup D^c) = \frac{4}{6} = \frac{2}{3} = 0.6\overline{6}$$

   $(D^c \subset A \rightarrow A \cup D^c = A)$

4. **(25 pts total)**

   (a) **(10 pts total / 1 pt for each number in the tree)**
   $B1, W1$ is "black top" or "white top", $B2, W2$ is "black bottom" or "white bottom"



First-level	Second-level	Joint Probabilities
$\Pr(B1) = \frac{11}{36}$	$\Pr(B2 \mid B1) = \frac{6}{11}$	$\Pr(B1 \cap B2) = \frac{6}{36} = \frac{1}{6}$
	$\Pr(W2 \mid B1) = \frac{5}{11}$	$\Pr(B1 \cap W2) = \frac{5}{36}$
$\Pr(W1) = \frac{25}{36}$	$\Pr(B2 \mid W1) = \frac{5}{25}$	$\Pr(W1 \cap B2) = \frac{5}{36}$
	$\Pr(W2 \mid W1) = \frac{20}{25} = \frac{4}{5}$	$\Pr(W1 \cap W2) = \frac{20}{36} = \frac{5}{9}$

(b) **(5 pts / no work needed)** $\Pr(W1) = \frac{25}{36}$

(c) **(5 pts / no work needed)** $\Pr(W2 \mid B1) = \frac{5}{11}$

(d) **(5 pts / 1 pt for showing work)**

$\Pr(B2) = \Pr(B2 \cap B1) + \Pr(B2 \cap W1) = \frac{1}{6} + \frac{5}{36} = \frac{11}{36}$ (Total Probability)

OR

$\Pr(B2) = \Pr(B1) = \frac{11}{36}$ (symmetry argument - top and bottom are same)