TEST 2 — Take Home

COMPUTATIONAL GEOMETRY
Spring Term 2000 — Number: CS 5963

Due Thursday, May 3 at 5:00pm

Name: _____
Student ID Number: _____

Ground rules:

- Open book (use only the text book), open notes.
- You should work *by yourself*. You may *not* use any medium which provides interaction with other people (e.g. posting to news groups, chat rooms, conversations with other students, staff, or faculty, are forbidden).
- You should spend no more than 8 hours (total) working the exam (this is more than enough time).
- Your answers (including equations, derivations, etc.) should be written on the pages given (including back sides as well).

Hints:

- The term "describe" does not mean complete sentences and paragraphs or essays. Pseudo code with short, accurate explanations or comments will suffice.
- All of the questions can be answered using the information from the course textbook and the lectures (combined with some thinking).

1. **[25 pts.]** Prove that the worst case for the incremental, randomized CONVEXHULL algorithm is $O(n^2)$. Describe the set of points and the input ordering (permutation) that produces this run time.

2. **[25 pts.]** This question relates to the robot-motion-planning problem from Chapter 13 in the text book. The road-map approach guarantees a correct solution (if one exists) but does not say anything about the quality of the solution, e.g. length of path.

   (a) Describe a modification to the breadth first search (as we discussed in class). Explain those conditions under which it would make a big improvement and those under which it would not.

   (b) As it stands, the entry point to the road map is always the center of the beginning trapezoid. Explain why this might not be the best approach, and give a method that fixes this problem. Hint: There is a correct way that does not involve complicated heuristics.

   (c) Describe the mechanism by which this algorithm can be modified to produce an optimal path.

3. **[25 pts.]** In the algorithm DELAUNAYTRIANGULATION there is a recursive procedure LEGALIZEEDGE. Give an iterative version of this procedure and discuss the advantages/disadvantages of your procedure relative to the recursive one.

4. **[25 pts.]** Consider a convex polytope with n vertices in $\Re^3$ whose interior is denoted as P. Give a data structure that allows one to test (with a $O(\log n)$ query time) whether or not P contains a point $p \in \Re^3$. How do you build such an data structure and how long does it take? Note: If you cannot find the $O(\log n)$ version, give any data structure that would allow such a query.