TEST 1 — Take Home

COMPUTATIONAL GEOMETRY
Spring Term 2000 — Number: CS 5963

Due Friday, March 23 at 5:00pm

Name: ____________________
Student ID Number: ____________________

Ground rules:

- Open book (use only the text book), open notes.
- You should work *by yourself*. You may *not* use any medium which provides interaction with other people (e.g. posting to news groups, chat rooms, conversations with other students, staff, or faculty, are forbidden).
- You should spend no more than 8 hours (total) working the exam (this is more than enough time).
- Your answers (including equations, derivations, etc.) should be written on the pages given (including back sides as well).

Hints:

- The term "describe" does not mean complete sentences and paragraphs or essays. Pseudo code with short, accurate explanations or comments will suffice.
- All of the questions can be answered using the information in the course textbook (and some thinking).

1. **[25 pts.]** The *convex inclusion* problems is as follows: Given a simple, closed, convex, planar, N-gon, P, and a point, q, determine whether or not q is in P.

   (a) Describe (psuedocode) an algorithm that has the following properties: "inclusion" is answered in $O(\log(N))$ time for any q. The data structures take $O(N)$ space, and depending on the algorithm will take $O(N)$ or $O(N\log(N))$ time to precompute.

   (b) The $O(N\log(N))$ is typically more general. How so? How would you extend this to non-convex polygons? Give (and justify) expressions for the precompute, storage, and run time.

2. **[25 pts.]** Consider the point-set, minimum distance problem: Given two sets of points A and B of points in the plane, each containing N elements, find the two closest points, one from A and the other from B.

   Give an algorithm that finds this pair of closest points. Show that the alogrithm is correct and that it requires $O(N\log(N))$ operations.

3. **[25 pts.]** The *medial axis*, $\mathscr{M}(P)$ of a polygon P is the set of internal points such that $p \in \mathscr{M}$ is equidistance from at least two points on the boundary of P, which has N sides. Describe an $O(N\log(N))$ algorithm that finds this set of points for convex polygons (Show that this algorithm is correct).

4. **[25 pts.]** The *Inverse of the Vornoi Diagram* problem is as follows: Given a planar subdivision with valence 3 (i.e. each vertex has degree 3) develop an algorithm to test if it is the Vornoi diagram of some (finite) set of points, S. If "yes", the algorithm should produce as output the set S.

   Describe this algorithm, show that it's correct, and analyze its run time.