

# **3-D Model Building Using CAGD Techniques<sup>1</sup>**

B. Bhanu, T. Henderson and S. Thomas

Department of Computer Science  
The University of Utah  
Salt Lake City, Utah 84112

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#### Abstract

This paper presents the ongoing work using of Computer Aided Geometric Design (CAGD) representations and models as a basis for the visual recognition of 3-D objects for robotic applications. We describe some techniques and algorithms which allow the generation of computer representations and geometric models of complicated realizable 3-D objects in a *systematic manner*. These representations and models are obtained using (a) available CAGD techniques, and (b) data acquired from various range finding techniques. As compared to previous work in machine vision, multiple hierarchical representations of an object obtained from geometric models can be used for finding orientation and position information.

## 1. Introduction

In the past a number of different techniques have been used for the representation and modeling of three-dimensional objects for computer vision applications [4, 14, 16]. However, there has been an absence of a *systematic approach* for building such models, for a large class of objects used in industrial environments, which can be used for matching with 2-D images or arbitrary 3-D views of objects. The models and matching strategies have been limited in scope because of the lack of generalization to other objects or even objects of similar types with minor variations in their descriptions. The emergence of Computer Aided Design/Computer Aided Manufacturing (CAD/CAM) technology as a driving force in manufacturing engineering has provided new opportunities for the use of geometric models of real world 3-D objects and to build systems to carry out the tasks of visual recognition and identification. In this paper we are concerned with the use of Computer Aided Geometric Design (CAGD) representations and models as a basis for the visual recognition of objects for robotic applications. Current CAGD systems offer an interactive design environment by providing facilities to create images of the designed parts, perform analysis functions on them (e.g., finite element analysis), and produce numerically-controlled machining information for manufacturing. Figure 1 shows our approach to CAGD based vision analysis.

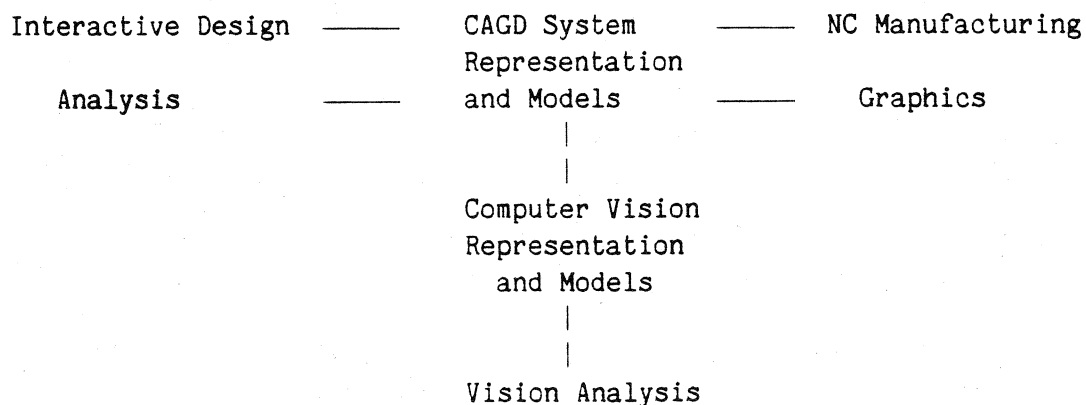


Figure 1. CAGD-Based Vision Analysis System

In the following we describe the work that has been done using these representations in the area of computer vision and CAGD. Here we present our representation scheme which allows multiple hierarchical representations of an object. This is followed by a discussion on modeling. Finally we describe a number of methods which can be used to generate data for building a vision model and present our initial results.

## **2. Representation in Vision**

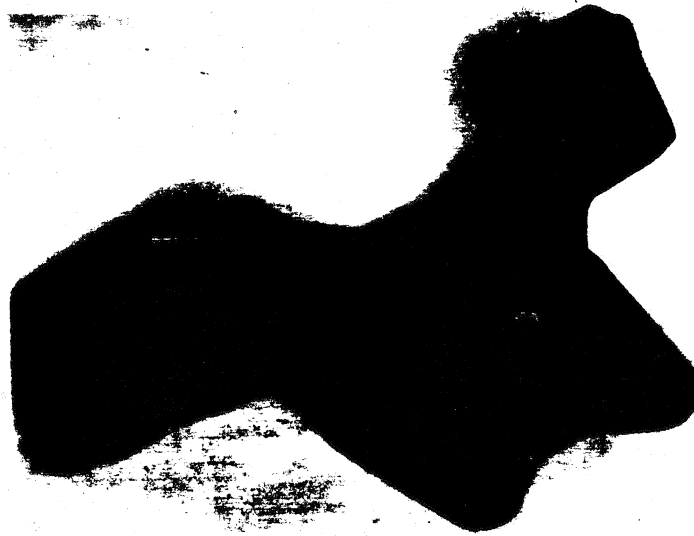
Geometric modeling is one of the key components of a domain-independent model-based 3-D industrial machine vision system. Here our interest is in the representation, modeling and recognition of rigid, opaque 3-D solid objects. Three general classes for the representation of 3-D solid objects are (a) surface or boundary, (b) volume, and (c) sweep [22]. In the boundary representation schemes, a 3-D solid object is represented by segmenting its boundary into a finite number of bounded "faces" and describing the structural relationships between the faces. Another approach to surface representation is to express the surfaces as functions on the "Gaussian Sphere" [25]. Volumetric representations include spatial occupancy, cell decomposition and constructive solid geometry [23]. Sweep representations consist of translational sweep, rotational sweep, 3-D sweep and general sweep.

Since a direct model of a 3-D object in terms of its volume (e.g., as a 3-D array) may easily exhaust the memory capacity of a system, representation by oct-trees has been considered [15]. These may make space array operations more economical in terms of memory space. A simple approach to analyzing 3-D objects is to model them as polyhedra. This requires a description of the objects in terms of vertices, edges and faces. Baumgart [3] developed a 3-D geometric modeling system ("Geomed") for application to computer vision. He used a face-based representation for planar polyhedral objects, called the "winged-edge" representation. The Euler primitives are

used for polyhedron construction and shape operators include union, intersection and difference. "Geomed", a system of 3-D modeling routines for manipulating winged edge polyhedra, provides many capabilities; for example, arbitrary polyhedra may be constructed, altered or viewed in perspective with hidden lines eliminated. Bolles et al. [6] have used a CAD model that contains a standard volume-surface, edge-vertex description as well as pointers linking topologically connected features. Their preliminary model uses a pointer structure similar to Baumgart's "winged-edge" representation.

Wesley et al. [28] have used polyhedral models for automated mechanical assembly in their geometrical modeling system GDP. Their Automated Parts Assembly System (AUTOPASS) [17] language has never been implemented. Before automated assembly can be successful, it is essential to have robust representations, models, and general purpose techniques for determining the orientation and position of 3-D objects for a large class of industrial parts.

For curved and more complex objects, other representations and models have been used, such as generalized cylinders (or cones). Generalized cylinders or cones are a quite popular representation in computer vision [1, 8, 20]. However, there are some problems with this representation. There are infinitely many possible generalized cones representing a single object. More constraints are needed to get a unique description. Although it is possible to represent arbitrary shapes with generalized cones by making them arbitrarily complex, their computation is difficult. They are also not well suited to descriptions of non-elongated objects and objects of arbitrarily deformed surfaces enclosing little volume. The generalized cone primitives used by Nevatia and Binford [20] and Brooks [8] are not sufficient to represent the automobile casting shown in Figure 2. Note that this casting does not contain any major horizontal or vertical surface and is quite complicated in shape.



**Figure 2.** Automobile Part

Although sweep representations, such as generalized cylinders, and volume representations, such as constructive solid geometry, imply surface description, they fail to describe the junction or surface peculiarities. In the recognition of 3-D objects from partial views, we detect surfaces first, and only after seeing several different views of the object do we have enough data to obtain volume properties. For objects constructed from thin sheet-like material, surfaces are natural candidates for representation. Further, surfaces are seen first. As such, they are important for computer vision. Hence the need for surface or boundary based representations.

York et al. [29] have used a structured collection of Coons surface patches for representing 3-D objects whose boundary curves are approximated by cubic B-splines. Their design of Coons patches is cumbersome, since it requires that a simple surface patch be designed on paper before it may be entered into the data base. Brady [7] proposes a symbolic representation of visible surface based on "curvature patches." They are computed locally by determining the tangent vectors that indicate directions in which

the surface changes. Example directions include the principal curvature directions and the directions in which the normal curvature is zero. Smooth changes in curvature patch descriptions are obtained to determine the larger scale structure of a surface. It is not clear that curvature patch surfaces are perceptually "fairer" than surfaces developed in CAD.

A region growing technique has been used by Bhanu and Henderson [4, 14] to obtain a planar surface representation of 3-D objects. They have used only geometrical information, i.e., position of a point in 3-D space. They have not used any topological information. However, such information is of great use; it can be derived or made available by the CAGD representation of 3-D objects and can be effectively used. Other techniques which have been explored by us include the split and merge technique, surface normals and clustering techniques for the segmentation of range data. Faugeras [13] has also used a region growing algorithm working on a 3-D graph to approximate object surfaces. For a critique and details of various representation and modeling techniques for 3-D objects in computer vision, see [4, 14].

### **3. Representation in CAGD**

Constructive solid geometry (CSG) and boundary representations are the best understood and currently most important representation schemes in computer aided design. Present day 3-D wireframe models used in CAD and model-based vision have many deficiencies including ambiguity - it is easy to build a wireframe model that can be surfaced in several ways [23]. In CSG, the basic idea is that complicated solids can be represented as various ordered "additions" and "subtractions" of simpler solids by means of modified versions of Boolean set operators-union, difference and intersection [22]. For inherent boundary representations a number of different approaches are used. These include Coons patches, bicubic surface patches, Bezier methods and B-splines [2].

Current Geometrical Modeling Systems (GMS) use a limited class of primitives such as rectilinear blocks and conic surfaces (cylinders, cones and spheres). Although these are sufficient to cover a large number of conventional unsculptured parts, a GMS which includes sculptured solids is highly desirable. Also since the sculptured design is surface oriented, it is easier to incorporate it in a boundary based system. In general, boundary modelers tend to support stepwise construction of the models more easily than CSG modelers but require greater data storage. CSG modelers are inadequate for modeling sculptured parts: they have no capability at all for constructing and using sculptured surfaces as part of the boundary of the solid model. Some advantages of boundary representation are: there are many known surface models available from which to choose [2]; the mathematics of surface representation is well developed and complex shapes can often be represented with a single primitive [10, 26]; and it results in an intuitive model. A minor disadvantage is that it may be difficult to ensure the validity of a boundary representation of a set. On the other hand, CSG representations are not unique in general, since a solid may be constructed in many ways; the final result may not be easily visualized by looking at the primitives. However, the CSG representation is concise, validity is guaranteed and such a representation can be easily converted to a boundary representation. The comparison of CSG and boundary representation methods can be found in [23].

Recently there have been attempts to use a set of manipulative operations for boundary models for solid objects to construct a solid modeling system [18, 26]. These are designed for CAD/CAM environments, rather than for computer vision applications. In [18], a set of Euler operators is used on the topology of a boundary model, that is on the relative arrangement of its faces, edges and vertices. The operations allow the system to perform arbitrary modifications necessary for boundary representation models, the faces of which are planar polygons.



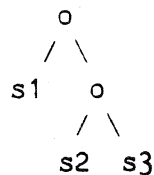
Until recently it was not possible to carry out Boolean operations on sculptured surfaces. Recent work by Thomas [26] attempts to combine the best attributes of CSG and surface-based representation systems by using subdivision techniques developed by Cohen et al. [12]. He uses a uniform boundary representation. The "primitives" are solids bounded by B-spline surfaces. As compared to the other work in solid modeling, his method does not require that the objects being combined have closed boundaries; they must only satisfy a weak completion criterion. Thus this method results in a powerful shape description system which allows the combination of primitives using set operations into arbitrarily-complex objects bounded by curved surfaces and the production of a model which represents such objects. Adjacency information about surface points and the intersection curve between two surfaces as a polyline can be obtained. Although he has used B-spline surfaces, his techniques are applicable to any surface representation scheme. As an example, Cobb [10] describes a number of powerful shape description and modification facilities. All this work has been incorporated in the Alpha\_1 system [11]. (More details about Alpha\_1 are presented below.) Thus, the advantages of both CSG and sculptured surface representation can be obtained in the shape representation of objects and the combination of objects via set operations. As a result of these significant advances in CAGD, it is of special interest that we use the Alpha\_1 system for exploring the computer vision application.

Alpha\_1, an experimental CAGD based solid modeler system incorporating sculptured surfaces [11]. It allows in a single system both high quality computer graphics and freeform surface representation and design. It uses a rational polynomial spline representation of arbitrary degree to represent the basic shapes of the models. The rational spline includes all spline polynomial representations for which the denominator is trivial. Nontrivial denominators lead to all conic curves. Alpha\_1 uses the Oslo algorithm [12] for computing discrete B-splines. Subdivision, effected by the Oslo

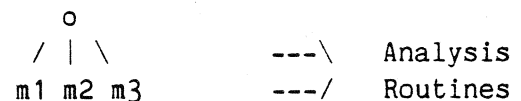
algorithm, supports various capabilities including the computation associated with Boolean operations, such as the intersection of two arbitrary surfaces [26]. B-splines are an ideal design tool, they are simple yet powerful; many common shapes can be represented exactly using rational B-splines. For example, all of the common primitive shapes used in CSG systems fall into this category. Other advantages include good computational and representational properties of the spline approximation: the variation diminishing property, the convex hull property and the local interpolation property. There are techniques for matching a spline-represented boundary curve against raw data. Although the final result may be an approximation, it can be computed to any desired precision (which permits nonuniform sampling). At present, tolerancing information is not included in the object specification in Alpha\_1 system. It is planned to be incorporated in the future. Once it is available, we can make our models in terms of classes of objects (rather than a single object) which are functionally equivalent and interchangeable in assembly operations.

Figure 3 shows the relation between the CAGD model and the generalized vision model.

CAGD Model



Generalized Vision Model



**Figure 3.** Relation of CAGD Model to Vision Analysis

Given the CAGD model (perhaps by combining several modeling paradigms), a corresponding set of vision models (with some control structure) is generated. Once these models are available, they provide the basis for standard 2-D and 3-D scene

analysis. An early example of such an interactive system is the ACRONYM system [8] designed for applications in computer vision and manipulation. The world is described to ACRONYM as volume elements and their spatial relationships and as classes of objects and their subclass relationships. It uses a hybrid CSG and general sweep scheme for the representation of rigid solids. The representations are CSG-like trees whose leaves are generalized cylinders. Like PADL (a geometric modeling system [9]) it allows variation in size, limited variation in structure and variation in structural relationships of the modeled objects. However, in ACRONYM, it may be difficult to design algorithms for computing properties of objects (e.g., centroid).

### **3.1. Modeling**

One of the limitations of the earlier work in 3-D scene analysis has been the restriction to a single range image of the object in building a 3-D model [1, 20]. This results in a model of only part of the object, not the complete object. Nevatia and Binford [20] used tree structured generalized cone models of objects detected using a laser range finder. In [4], the 3-D model was automatically built by combining object points (obtained by using a laser ranging system) from a sequence of range data images corresponding to various views of the object, applying the necessary transformations and then approximating the surface by polygons. Another approach for automatic generation of a 3-D model which combines the information from several views is the work reported by Potmesil [21]. He uses a bicubic parametric patch as the basic surface element. A heuristic search algorithm is used to register partially overlapping surface segments into a common space and then the overlapping sections are merged. The match and merge process is iteratively repeated for all 3-D surface segments until a complete model of the object in a single 3-D space is created. It is to be noted that, as compared to these 3-D model building approaches in vision, CAD systems for industrial parts normally build models which are viewpoint independent and provide a 3-D description of the volume

occupied by the part. Thus they can be used in vision and manipulation applications. Bolles et al. [6] have used a preliminary CAD model based on Baumgart's "winged-edge" representation. Several other 3-D models used in computer vision are discussed in [4].

#### 4. 3-D Vision Model Building

As we have discussed there is at present a lack of convenient 3-D solid modelers which are capable of generating object descriptions suited to the problem of 3-D object recognition and localization. Here our objective is to provide and use such a description. In our system, parts are designed and modeled using the Alpha\_1 CAGD system [24]. It models the geometry of solid objects by representing their boundaries as discrete B-splines. It allows the combination of primitive objects into more complex objects using set operations. It supports several modeling paradigms, including direct manipulation of the B-spline surface, creation and combination of primitive shapes, and high-level shape operators such as bend, twist, and warp. The single underlying mathematical formulation of Alpha\_1 simplifies implementation, but it is sufficiently powerful to represent a very broad class of shapes. It is able to create images of the designed objects, to perform certain analysis functions on them, and to produce numerically-controlled machining information for manufacturing [24, 26].

By using the Alpha\_1 system, several methods are available to generate data from which a vision model can be built. (a) *Ray tracing* is a common technique to produce realistic images of a solid model. Light rays are traced from the eye to the object and then to the light source to determine the illumination at each point in the scene. Evenly spaced rays can be traced to the model, and the intersection points between the rays and the model gives the range information [27]. This simple technique can be used to *simulate a laser scanner* and a set of 3-D points on the surface of the object can be obtained [5]. (b) A procedure which is simple to program and produces an irregular set of points

(nonuniform) is to subdivide the B-spline surface which comprises the boundary of the object [12]. The bounding surface can be subdivided until all the resulting pieces are smaller than some resolution. Then the center points of all the small surface pieces can be obtained as the surface representation of the object by 3-D points. (c) Noting that (a) and (b) discard information which is available in the original model, during the subdivision process in (b) while generating surface points, a spatial adjacency graph may also be generated. It could be very useful for surface approximation, since it contains topological information. (d) Methods as described in (a) to (c) produce the surface description in terms of the points on the surface of the object. These points have to be processed further to get the higher level surface representation. It is possible to obtain data in a much more intelligent fashion. As an example, during the subdivision stage, we can detect surface pieces which are planar (or quadric). Now the data available will be in terms of single polygons (or quadric or mixed types). Several of these polygons can be merged using the spatial adjacency graph to obtain a structured collection of large faces. Here the characteristic of the bounding surfaces (such as flat, cylindrical, etc.) retained in the model can also be used as an aid in the segmentation of actual range data and in the recognition task [5]. We are currently investigating the above methods in detail in order to obtain intermediate level descriptions for the vision model.

*The higher level multiple hierarchical representation model-* An efficient representation depends upon the type of objects and the intended application. Classes of objects based on shape can be grouped as elongated, polyhedral, curved surfaces and complex objects which incorporate the components of other shape types. For each of these types a particular representation may be more suitable than others. Three important features of 3-D model representation are [19] (a) that its coordinate system is object-centered (b) that it includes volume primitives, that may exploit the space occupied by an object and not just its visible surfaces, and (c) primitives of various sizes

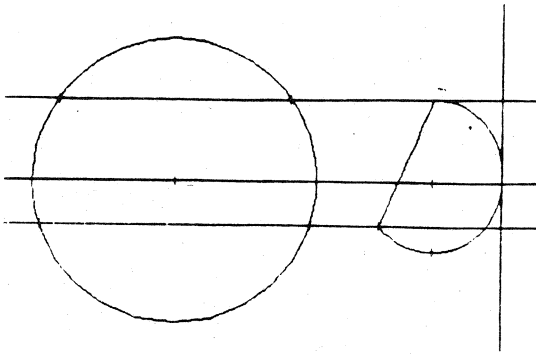
are included, arranged in a modular hierarchical manner. In our CAGD based vision system, we claim to have all three features. Different parts of an object are designed such that they can have their own coordinate system. Once we have designed the complete surface, these parts are combined using set operations and we have the volume bounded by the surface. Properties such as the center-of-mass of the volume can be computed and all the points on the surface of the object can be referred to with respect to the body center of the object. Thus we can define canonically the coordinate scheme for an arbitrary shape and have the advantages of both a single coordinate frame that embraces the entire object and the distributed coordinate system for each articulated component or individual shape characteristics. By using set operations on volumetric primitives, we have a hierarchical representation of the object shape.

Thus CAGD systems may support several 3-D object modeling paradigms. Likewise, computer vision systems can be based on one of several 3-D representation schemes ([14]). Most CAGD systems and computer vision systems rely on only one representation, not several (e.g., ACRONYM is based on generalized cylinders). If an object is represented under a single modeling paradigm, we have a *homogeneous representation*. We believe, for the reasons mentioned above, that it is possible and necessary to support several modeling types in both the design phase and the analysis phase. For example the complicated casting shown in Figure 2 may comprise a combination of subcomponents, each modeled with a different paradigm. The paradigms are surface, volume and generalized cylinder or (cone). The subcomponents of the object using these paradigms are represented hierarchically. Such a representation is called a *heterogeneous representation* and it can be derived from the CAGD representation. Thus the model of the object consists of multiple hierarchical representations. From the considerations of matching we allow a certain amount of redundancy in the model in the form that, in addition to whatever natural representation is defined, we also have surface

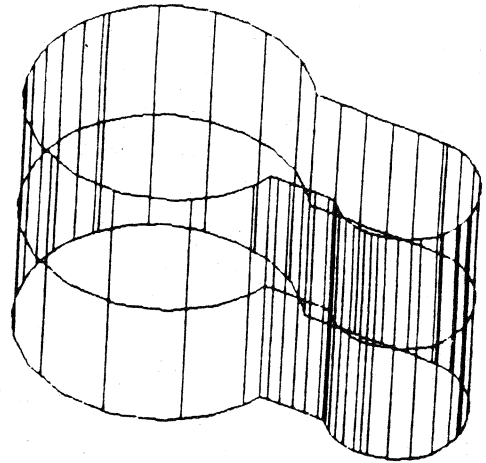
representation in terms of planar faces for all objects; this is used for indexing purposes. Of course, as pointed out above, from the CAGD model we derive the information about the overall size of the objects, local features (such as the number of holes, their location and corners, etc.) and properties of the bounding surfaces.

#### 4.1. An Example

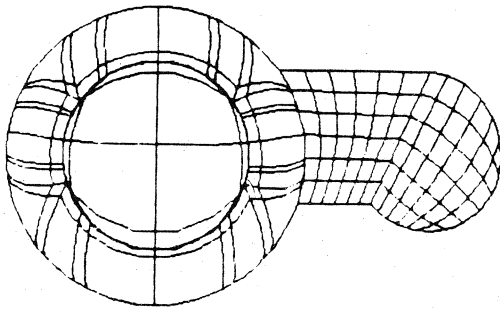
We will now consider the entire design and analysis process for part of the object shown in Figure 2. As can be seen, the piece has a highly irregular surface which poses significant representational problems. Figures 4 to 7 show the "left head" of the automobile part (Figure 2), during several stages in the design. The stage 1 (Figure 4) in the design is to lay out the outline of the face of the part. Points and construction lines are placed to define the centers and endpoints of the circular arcs, and straight line segments are inserted between the arcs. In stage 2 (Figure 5) the side surface is produced by extruding the face outline. In stage 3 (Figure 6) the surface of the face is produced in several steps. It is divided into three sections corresponding to the large cylinder, the small cylinder and the connecting neck. The outline of each of these is broken into four segments and a Boolean sum surface computed. The division into parts is done so that the resulting surfaces have identical parametrization along their matching edges, and can therefore be rejoined into a single surface. The circular depression is produced by embedding its circular outline in the face surface and extruding it to the proper depth. It remains represented by a single B-spline. In stage 4 the bottom face is formed by reflecting the top surface through the center plane of the object. The separate surfaces are then assembled into a model of the "left head." Figure 7 shows a hidden line view of the left head. An example of adjacency graph obtained from CAGD modeling scheme as discussed above is shown in Figure 8.



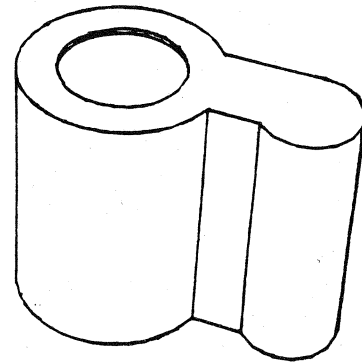
**Figure 4.** Stage 1 in the Design



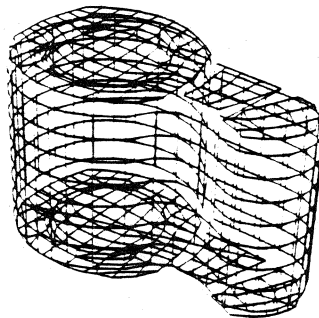
**Figure 5.** Stage 2 in the Design



**Figure 6.** Stage 3 in the Design



**Figure 7.** Stage 4 in the Design

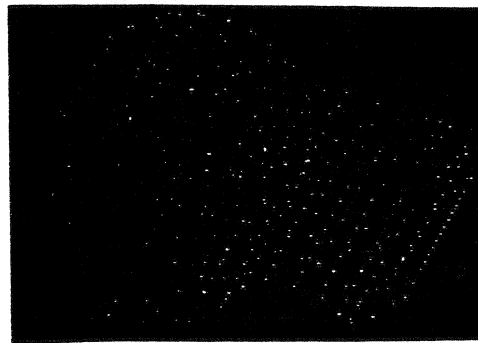


**Figure 8.** Adjacency Graph of Model

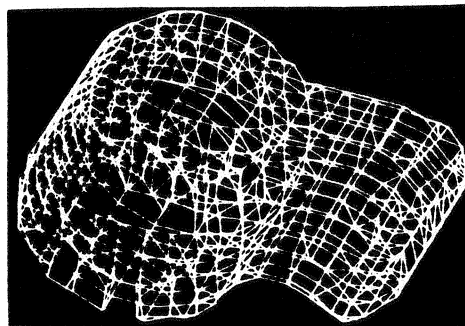
Once the part has been designed (or a subcomponent in this case), some method must be selected to generate the required data for building computer vision models. As discussed earlier, several methods can be used. Here we have chosen to use ray tracing



to simulate the action of a laser range finder. Evenly spaced rays are traced to the model and the intersection points between the rays and the model are recorded. Such a set is shown in Figure 9(a). Once the set of points has been obtained, each point can be connected to its nearest neighbors by building the *spatial proximity graph* (see Figure 9b). From this graph it is easy to generate polygonal approximations to the data and then to perform matching based on those models [4, 14]. Given such a polyhedral model, scene analysis is performed by deriving descriptions of objects in the scene and matching those descriptions to the model. Final representation is a semantic network whose nodes can have multiple hierarchical representations and arcs describe geometric constraints and relationships. Recognition algorithms are based on such representations and make use of arc descriptions.



(a) Sampled Points



(b) Spatial Proximity Graph

**Figure 9.**

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