From Sensorimotor Data to Concepts: The Role of Symmetry

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UUCS-12-005

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5 October 2012

Abstract

We have proposed the use of symmetry theories as the basis for the interpretation of sensorimotor data and the creation of more abstract representations. Here we outline a cognitive architecture to implement such an approach and provide a set of specific mechanisms for 1-D and 2-D sensorimotor processing. The overall goal is to integrate low-level sensorimotor data analysis and behavior with more abstract affordance representations.

1 Introduction

Here we provide a description of the basic mechanisms that permit the parsing of signals into symbolic representations of the symmetries present in the data. The representation permits the synthesis of motor commands which are coordinated with expected sensory data streams. In addition, this representation provides a basis for robots to share concepts, even if that requires some form of sensorimotor calibration. In particular, this paper describes:

- an architecture for cognitive processing,
- 1-D sensorimotor stream symmetry operators, and
- 2-D visual stream symmetry operators.

The ultimate goal is the integration of these into an active autonomous embedded agent.

In previous work [7, 10, 15, 16], we have described the role of symmetry in cognition, as well as more specifically the *Symmetry Engine* (see Figure 1). In this approach, signal and

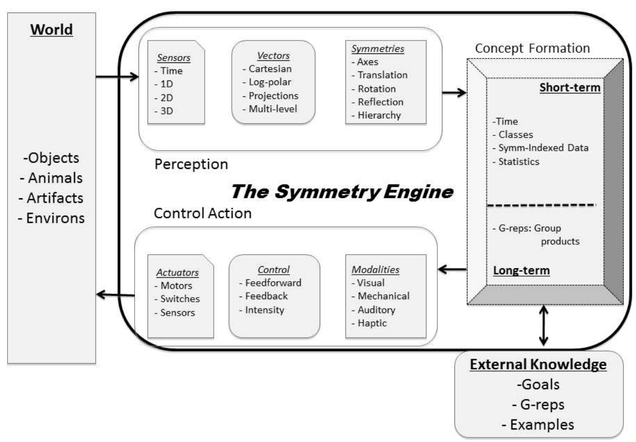


Figure 1: The Symmetry Engine. *Perception* requires an appropriate set of operators to construct group-theoretic representation (*G-reps*); this includes vector constructors, symmetry detectors, and symmetry-based data indexing and variance operators. *Control action* requires the ability to map *G-reps* onto action sequences to achieve desired results in the world. *Concept Formation* operators allow the exchange of *G-reps* with other agents.

motor command streams are characterized by their symmetries, and then more abstract representations (parameterized strings) are produced. Several advantages accrue from this: (1) compression into a more compact representation, (2) computational efficiency in matching, and (3) ease of sharing the representations. E.g., communicating ”a red $O(3)$ symmetry” is better than sending all the 2D images, 3D surface data, and motor commands that were used to obtain the data. Moreover, long-term knowledge maintenance requires efficient storage and retrieval, especially as the body of knowledge grows over time.

Hypothesis: We propose that robot affordance knowledge acquisition and perceptual fusion can be enabled by means of a common sensorimotor semantics which is provided by a set of group symmetry theories embedded a priori in each robot. These theories inform the production of structural representations of sensorimotor processes, and these representations, in turn, permit perceptual fusion to broaden categories of activity.

Symmetry here means an invariant, and involves finding data sets with associated operators that map the set to itself (see Weyl [43]). Viana's characterization fits well with our view [39]: “Symmetry provides a set of rules with which we may describe certain regularities among experimental objects.” Our goal is therefore to find operators which leave certain aspects of state invariant.

2 Related Work

A good deal of recent research activity has focused on the acquisition and representation of robot behavioral knowledge, ranging from multi-media databases ([8, 11, 13]) and ontologies ([1, 3, 36, 37, 38, 41]), to web-based robot knowledge repositories (e.g., RoboEarth [40]). Elsewhere ([15]) we have discussed how the cognitivist and dynamical systems approaches impact this problem, as well as the role of innate knowledge in robotics (see [12, 14]). While this is a grand enterprise that may indeed lead to affordance construction and sharing at the knowledge level, it is unclear that there is an adequate semantic basis for robots to exploit or share such knowledge. There is evidence that even sharing standard object representation models across robot platforms is difficult [18]. Moreover, great reliance is placed on human programmers to provide ontologies, as well as the frameworks for any form of sharing (e.g., sensor models, maps, coordinate frames, etc.).

Note that there has been some recent move to include low-level continuous representations in the more standard symbolic cognitive architectures (see Laird [20, 44] who add a continuous representation to Soar, and also Choi [6] who propose ICARUS, a symbolic cognitive architecture for their humanoid robot; however, the high-level concepts of these systems do not arise through the sensorimotor process). One of our goals is to promote a broader dialog between the AI, cognitive systems, and robotics communities.

We explore the use of symmetry analysis as a basis for the semantic grounding of sensorimotor affordance knowledge; this includes symmetry detection in signals, symmetry parsing in knowledge representation, and symmetry exploitation in structural bootstrapping and knowledge sharing. We are working in close cooperation with our colleagues, Profs. R. Dillmann and T. Asfour at the Karlsruhe Institute of Technology, involved in the European Union Xperience project (http://www.xperience.org/). The overview they give of the Xperience project is:

> Current research in enactive, embodied cognition is built on two central ideas: 1) Physical interaction with and exploration of the world allows an agent to acquire and extend intrinsically grounded, cognitive representations and, 2) representations built from such interactions are much better adapted to guiding behavior than human crafted rules or control logic. The Xperience project will address this problem by *structural bootstrapping*, an idea taken from child language acquisition research. Structural bootstrapping is a method of building generative models, leveraging existing experience to predict unexplored action effects and to focus the hypothesis space for learning novel concepts. This developmental approach enables rapid generalization and acquisition of new knowledge and skills from little additional training data.

This gives us a larger context within which to test our hypothesis that symmetry-based structuring of knowledge provides a more robust semantic basis than current methods, and in particular symmetry as applied to: the acquisition of affordances from signals, representation as Object-Action Complexes (OACs provide a framework for modeling actions and their effects), and exploitation in generative action discovery (structural bootstrapping). The Xperience team is developing a set of cognitive benchmarks for robotic systems.

Symmetry plays a deep role in our understanding of the world in that it addresses key issues of invariance. By determining operators which leave certain aspects of state invariant, it is possible to either identify similar objects or to maintain specific constraints while performing other operations. We have shown how to use symmetry in range data analysis for grasping [9]. Popplestone and Liu showed the value of this approach in assembly planning [25], while Selig has provided a geometric basis for many aspects of advanced robotics using Lie algebras [34, 35]. Recently, Popplestone and Grupen [31] gave a formal description of general transfer functions (GTF's) and their symmetries. In computer vision, Michael Leyton has described the exploitation of symmetry [22] and the use of group theory as a basis for cognition [23], and we expand on his approach here. Leyton [23] argues that the *wreath group product* is a basic representation for cognition as stated in the *Grouping Principle*: "Any perceptual organization is structured as an n-fold wreath product $G_1 \wr \ldots \wr G_n$" and argues that "human perceptual and motor systems are both structured as wreath products." For a more biologically motivated cognitive architecture which learns features for hierarchical models to recognize invariant objects, see [42] as well as other papers from the Honda research group on their approach to cognitive architecture [4, 5, 32]. Of course, many researchers in computer vision and graphics have proposed symmetry detection and analysis methods [17, 21, 24, 26, 27, 28, 29, 30]. We integrate several of these methods and describe improvements on them in the 2-D image analysis given method below.

3 Cognitive Architecture

Figure 2 provides a more detailed view of the general cognitive architectural framework proposed for the *Symmetry Engine*. A particularly important feature is the Behavior Unit

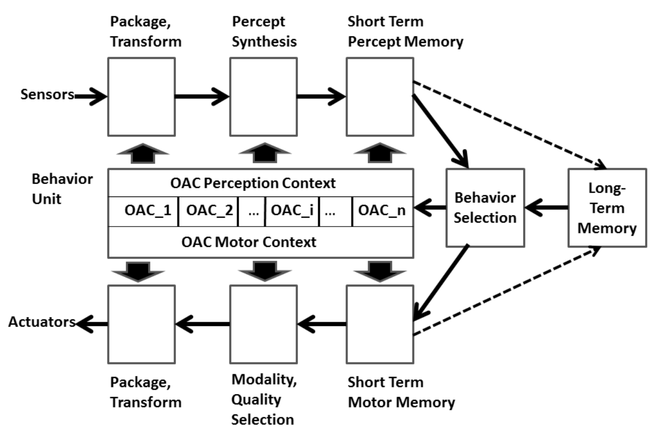


Figure 2: General Cognitive Framework Architecture.

(in the middle of the figure). Behavior is encoded as *Object-Action Complexes*. The *Behavior Selection* function chooses the next OAC sequence based on the current OAC, the current states of the *Short-Term Perception Memory* and the *Short-Term Motor Memory*, as well as the available behaviors in the *Long-Term Memory*. As an OAC executes, it provides context to both the perception and motor pipelines. Data arrives continuously from the sensors and first undergoes specific packaging and transformation procedures, then is formed into percepts (symmetry characterizations), and finally, results are stored in the *Short-Term Memory*. Similarly, motor commands are moved to the *Short-Term Motor memory* where they are then interpreted according to modality and the specific qualities desired, and finally, these more symbolic representations are decoded into specific motor commands for the actuators. In the work reported here, we deal mainly with the perception side of the architecture and describe the mechanisms for symmetry detection, classification and encoding.

As a simple example of the perceptual-motor duality of the architecture, consider a square shape. As described in more detail below, the boundary of the shape can be represented as a point which undergoes a specific sequence of symmetry transforms: translation (half the side length), reflection (about the line perpendicular to the endpoint), and rotation (4-fold about the z-axis). This same representation can be used to issue motor commands to trace out the shape or to circumnavigate it (e.g., go around a table).

4 1-D Sensorimotor Data Stream Symmetry Operators

We have shown that symmetries can significantly increase the efficiency of solving the sensorimotor reconstruction problem [16]. This is done by means of building models for theories (see Figure 3) in terms of sets and operators that are discovered in the world, and then fit to a specific theory. Moreover, relations over these objects must be determined.

We have developed the following specific mechanisms to discover, characterize and exploit symmetries in sensorimotor data:

- **Similarity Characterization**: Generally speaking, this includes correlation and distance functions, but more particularly in our case, is mainly a check as to whether the same symmetry is present. The latter is done hierarchically, first assuring that the symmetry type is the same (e.g., periodic), and then that the symmetry parameters match (e.g., period and basic shape).
- **Sets**: The basic sets are the streams of sensor and motor signal data. Other examples of sets include things like data sequences from similar sensors (e.g., range finders, cameras, etc.), or data sequences that satisfy some symmetry (e.g., pixel data which is all the same color). A basic operator here is the discovery of equivalence classes.
- **Operators**: These are typically the result of a motor command sequence which effect some change in the world (e.g., rotate by θ).

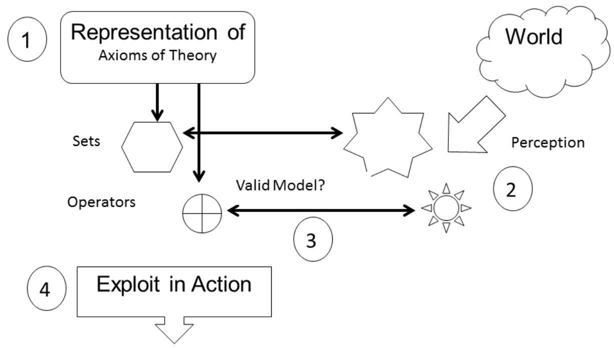


Figure 3: Building Models by Mapping Perceptions onto Theories. (1) Axioms are defined for the theory; (2) Perceptions are formed into sets and operator hypotheses; (3) A determination is made as to whether the specific sets and operators model the theory; (4) The theory may be applied to these sets and operators to achieve action.

- **Symmetry Classification**: Symmetries must be discovered in the sensor and motor data streams, starting at the 1D level, and working on up through 2D and 3D, as well at the various levels of representation. For example, at the 1D level this includes finding constant functions, lines, periodic, reflective sequences; we also seek to characterize data that is really just noise (Gaussian). The output here is a token such as S_n, the symmetric group of order n which can represent a constant signal (any sample can be mapped to any other sample).

- **Morphisms**: Determine for a given set, X, and function, f, whether $\forall x_1, x_2 \in X, x_1 \sim x_2 \rightarrow f(x_1) \sim f(x_2)$, where $x \sim y$ means that x is similar to y.

- **Groups**: Determine if a given set, X, and operator, +, satisfy the group axioms:

  1. Closure: $\forall x, y \in X, x + y \in X$
  2. Identity: $\exists e \in X \ni \forall x \in X, x + e = x$
  3. Inverse: $\forall x \in X, \exists x^{-1} \in X \ni x + x^{-1} = e$
  4. Assoc.: $\forall x, y, z \in X, (x + y) + z = x + (y + z)$

Details of these mechanisms may be found elsewhere [10].

5 2-D Imagery Symmetry Operators

We have begun developing symmetry operators for 2-D imagery as well. The goal is to produce collections of abstractions culled from a scene; Figure 4 shows an example of the group representations of segments in an office scene. Although wreath products were

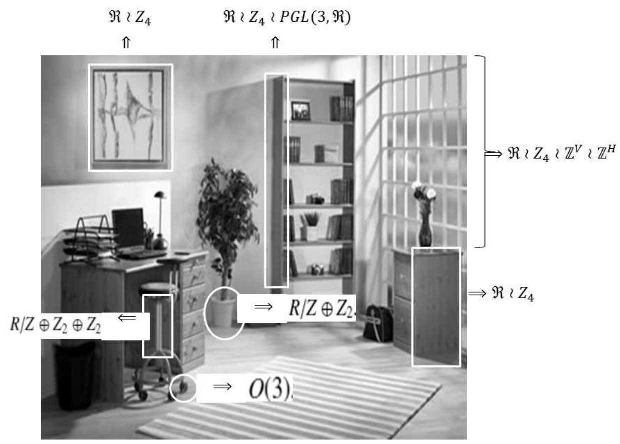


Figure 4: Group Representations Discovered in an Office Scene.

proposed for this by Leyton as it accounts for the combinatorics of the group action, we believe that a lower cost representation is possible and necessary; we call this the *G-rep* (Group Representation), and is a sequence of groups which describe the shape creation process augmented by detailed shape basis, color, scale, etc. information for the specific object. Note that these representations may also be created for temporal objects.

Given a set of symmetry elements and axes, it is necessary to determine how they are best combined into *G-reps*. Little detail on this process has been given in the literature. Consider, for example, Leyton's favorite example, the square. While it is true that $\Re \wr \mathcal{Z}_4$ captures the symmetries of the square, it also characterizes all the diverse shapes shown in Figure 5.

We associate a set of properties with symmetry elements and the *G-reps* constructed from them as described above. As for the shape itself, the essential characterization can be given in terms of what we call the *shape basis*; this is the smallest part of the shape that informs the reflection and/or rotation symmetries. Figure 6 shows the shape basis circled in the shapes given in Figure 5.

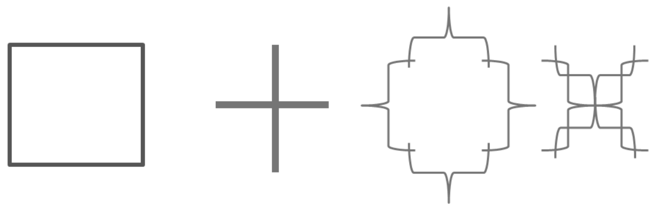


Figure 5: Four Shapes with the Same Wreath Product Representation.

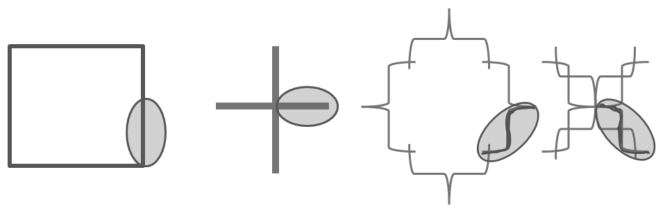


Figure 6: *Shape Basis* for Each of Four Shapes.

Several symmetry detectors reported in the literature have been implemented (e.g., the planar reflective symmetry transform [30], rotation symmetry group detectors [21], etc.). For example, given the image in Figure 7, the following symmetries are found:

```
type: 'continuous'
num_lobes: 0

type: 'dihedral'
num_lobes: 4

type: 'continuous'
num_lobes: 0

type: 'cyclic'
num_lobes: 5
```

Figure 8 shows the 4 symmetry sets found in the image.

Several symmetry detectors use the Frieze Expansion Pattern (FEP) to recover symmetries, where the FEP is the image formed by taking a specific pixel, p, as the origin, then taking the line at orientation θ degrees ($\theta = [1, 2, \ldots, 180]$) through p and forming column θ in the FEP image such that p is located at the center row of the image and the part of the object from p out in direction θ goes in the upper half of the image, while the opposite direction

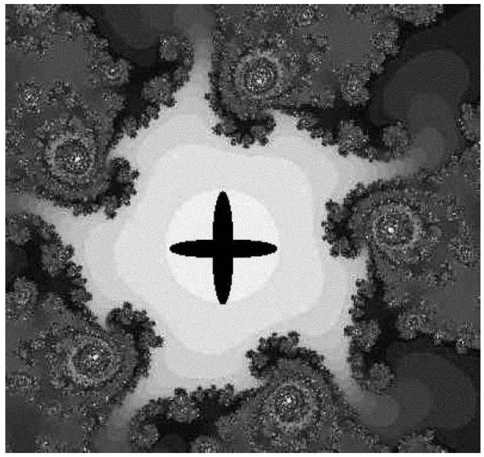


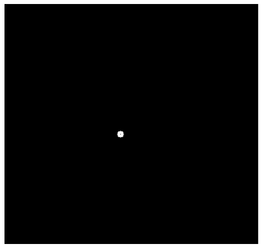
Figure 7: Example Image for Symmetry Detection.

from p goes in the bottom half of the image (see Figure 9 (upper right) for the FEP of a square). In studying this more closely, we have found some additional useful properties of the FEP:

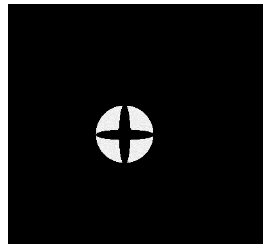
1. Reflective 2-D Symmetries can be found as 1-D translational symmetries.
2. Certain features in the 1-D curves found in an FEP can be used to identify the shape basis for a *G-rep*.

5.1 Reflective 2-D Symmetries in the FEP

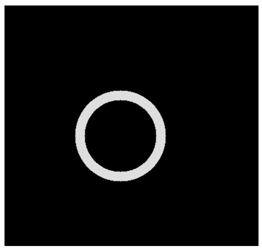
A 2-D reflective symmetry is a set of points in the plane that are invariant under reflection across a symmetry axis line through the set. Podolak's method considers every orientation at every pixel. However, reflective axes can be found as follows: For every segmented object with center of mass CM and FEP F at cm, then if F_1 is the top half of F and F_2 is the bottom half of F, then let $F\prime$ be F_2 flipped left right and then flipped up-down; next check for translational similarity between F_1 and $F\prime$, and where the similarity is high, there is a reflective axis. Figure 9(lower left) shows the 1-D similarity measure for the square (as a probability of a reflective axis versus angle).



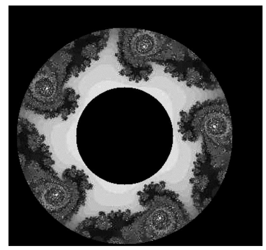
O(2) Continuous Rotation Group



Dihedral 4 Group



O(2) Continuous Rotation Group



Cyclic 5 Group

Figure 8: Symmetries Found in Chaos Image.

5.2 Identifying the *G-rep* Shape Basis in the FEP

Given an FEP, if there are reflective axes, then the shape basis for the full figure must be found between two successive reflective axes. This is shown in Figure 9 (lower right). In this case for the square, this is any of the half side segments.

6 Conclusions and Future Work

We have described our vision of a cognitive architecture based on symmetry detection and classification. 1-D and 2-D symmetries have been described, and results on sensorimotor data given. In future work we must address many issues, including:

***G-rep* Grammar** Two levels of representation are required for *G-reps*, one for group representations and one for associated properties. A standard notational grammar and parser will be developed for *G-reps*. For properties, one way to proceed is to declare them in terms of standard sensor and actuator categories (e.g., camera, range device, etc.). It is preferred, however, to express properties in terms of the robot's own sensors and actuators,

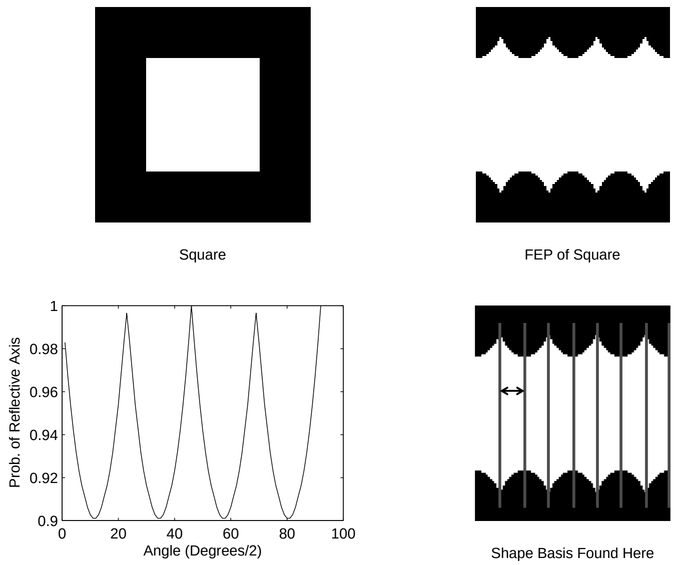


Figure 9: FEP of Square.

and then provide mechanisms by which a robot can provide the appropriate information so that another robot can determine the relation, if any, to its own sensor and actuators (i.e., it may need to solve the sensorimotor problem for another robot).

***G-rep* Indexing and Storage** Once a repertoire of *G-reps* is available, suitable storage and indexing schemes must be developed. This is crucial for the effcient working of the Short and Long Term Memories. One approach is to achieve some form of prime factorization in the group representation. Although the work of Rhodes [33] indicates that this is possible for semi-group representations of automata, the details remain to be worked out for shapes and sensorimotor processes.

Control in *G-reps* We also seek to produce structural descriptions of complete behaviors, including control aspects, and we intend to pursue two strategies. First is to use *G-reps* to specify perceptual entities and embed these in some form of affordance representation. The second approach is to incorporate control directly into the *G-rep*. We propose to pursue both, and with respect to the first, will use the Object-Action Complexes representation which has been developed by the Xperience team [2, 19]. As for the second option, one approach is to encode control signals into basic behavior units related to trajectories or configurations in $SE(3)$ using standard matrix representations. Selig [34, 35] has provided an in-depth description of group theoretical representations in robot control. In addition, shared control will be studied as suggested by [35] in which movement is limited by (group) constraints. Scleronomic constraints are represented by functions defined on the groups.

Structural Bootstrapping Once *G-reps* can be synthesized for affordances, then bootstrapping can be accomplished as follows. Given a *G-rep* with group sequence $G_1 \wr G_2 \wr \ldots \wr G_i \wr \ldots \wr G_n$, then it is abstractly the case that any group equivalent entity to G_e may be substituted in its place as a hypothesized new *G-rep*: $G_1 \wr G_2 \wr \ldots \wr G_e \wr \ldots \wr G_n$. Of course, this will need to be checked in the real world. E.g., a young child knows that it can get into a full-sized car; when presented with a toy car, the child may try to get into it, not realizing that there is a problem with scale.

Symmetry Engine Architecture Another major goal is to work out the details of the cognitive architecture and to implement a prototype, and test these ideas on physical robots. We plan to have working versions running on a variety of robots in the near future.

Acknowledgement: This work was support in part by NSF grants 10-21038 and 06-54414, and the rotational symmetry detection code was implemented by Evan Kroske during his NSF IGERT internship at the University of Utah.

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