Passive Localization Methods based on Distributed Phenomena

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Abstract This paper is devoted to methods for localizing individual sensor nodes connected in a network. The novelty of the proposed method is the model-based approach (i.e., rigorous exploitation of physical background knowledge) using local observations of a distributed phenomenon. By unobtrusively exploiting background phenomena, the individual sensor nodes can be localized by only locally measuring its surrounding without the necessity of heavy infrastructure. Two approaches are introduced: (a) the polynomial system localization method and (b) the simultaneous reconstruction and localization method. The first approach (PSL-method) is based on restating the mathematical model of the distributed phenomenon in terms of a polynomial system. Solving the system of polynomials for each individual sensor node directly leads to the desired locations. The second approach (SRL-method) basically regards the localization problem as a simultaneous state and parameter estimation problem in terms of a Bayesian approach. By this means, the distributed phenomenon is reconstructed and the individual nodes are localized in a simultaneous fashion, while considering remaining stochastic uncertainties.

1 Introduction

The research work presented here is a modified version of [15], however explanations about the novel localization process are given in an considerably extended way, with the focus on illustratively describing the actual process with its different stages, i.e., identification/calibration stage and acutal usage stage. For more details

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Fig. 1 Classification of localization methods: **(a)** Active localization, such as methods based on artificial signals between nodes and global positioning systems, and **(b)** passive localization, such as methods based on locally measuring a naturally existing distributed phenomenon.

about the used Bayesian estimator and its prospective applications, we refer to our previous research work [8, 15, 16, 17, 21].

Recent developments in various areas dealing with sensor networks and the further miniaturization of individual nodes make it possible to apply wireless sensor networks for observing natural large-area physical phenomena [2]. Examples for such physical quantities are temperature distribution [17], chemical concentration [22], fluid flow, structural deflection or vibration in buildings, or the surface motion of a beating heart in minimally invasive surgery [1].

For the reconstruction of such distributed phenomena, the individual sensor nodes are densely deployed either inside the phenomenon or close to it. Then, by distributing local information to a global processing node, the phenomenon can be coöperatively reconstructed in an intelligent and autonomous manner [6, 13, 16]. In such scenarios, the sensor network can be exploited as a huge information field collecting data from its surrounding and then providing useful information both to mobile agents and to humans. Hence, respective tasks are accomplished more efficiently, thanks to the extended perception provided by the sensor network. By this means, sensor networks can forecast or prevent dangerous situations, such as forest fires, seismic sea waves, or avalanches [4].

For most sensor network applications, the sensory data has only limited utility without location information. In particular for the accurate reconstruction of distributed phenomena, the locations of the individual sensor nodes are necessary. Manually measuring the location of every node in the network becomes infeasible, especially when the number of sensor nodes is large, the nodes are inaccessible or in the case of mobile sensor deployments. That makes the localization problem one of the most important issues to be considered in the area of sensor networks.

Classification of Localization Methods In general, the main goal of a localization system is to provide an estimate about the location of the individual nodes in the sensor network in the area of interest. There are several ways to classify the huge diversity of localization methods. In this work, they are classified into *active* localization methods and *passive* localization methods; depicted in Fig. 1.

- ***Active localization methods:*** The active localization methods obtain an estimate of the sensor node location based on signals that are artificially stimulated and measured by the network itself or by a global positioning system. The stimuli usually used in such scenarios consist of artificially generated acoustic events. It is obvious that the active localization process is performed in fairly controlled and well accessible environments. As it stands, these circumstances incur significant installation and maintenance costs. A comprehensive survey on active localization methods can be found in [5].
- ***Passive localization methods:*** In the case of passive localization methods, which in contrary occur in a non-controlled and a possibly inaccessible environment, the stimuli necessary for the localization process are generated in a natural manner. In Fig. 1 **(b)** prospective examples of natural physical systems which can be used as stimuli for localizations, are stated. The clear advantage of using passive methods for the localization is that they do not need additional infrastructure. This certainly keeps the installation and maintenance costs at a very low level. In addition, these methods become particularly important for applications where global positioning systems are simply not available. This is for example the case of sensor networks for monitoring the snow cover [4], applications in deep sea, indoor localization [11, 19, 21], or robotic-based localization [9].

There are various techniques and methods that can be considered for localization systems using different kind of infrastructures in different scenarios. In general, for the estimation of a distributed phenomenon by a sensor network, the existing infrastructure could consist of both a number of sensor nodes with *known locations* and nodes with *unknown or uncertain locations*. For the minimization of the installation and maintenance costs, it is benefical to develop a method that requires no additional hardware such as a global positioning system or other heavy infrastructures. Moreover, there are various application scenarios without the possibility to access a global positioning system for the localization, such as the indoor localization of mobile phones [20, 21] or sensor networks deployed deep inside the snowpack for predicting snow avalanche risks, to name just a few. For that reason, a novel *passive process* is proposed which does not require such a global positioning system or the localization based on landmarks. It is important to emphasize that the passive localization technique proposed in this work can be employed in combination with other localization methods for further improving the location accuracy.

Key Idea of the Proposed Localization Method For the *passive localization* of sensor nodes, we present *model-based approaches* based on local observations. The novelty of the methods introduced in this work is the rigorous exploitation of a strong *mathematical model* of the distributed phenomenon for localizing individual sensor nodes. Furthermore, within this framework, the often remaining uncertainties in the sensor node locations can be considered during the reconstruction process of the distributed phenomenon [17]. The use of such a mathematical model for node localization was proposed in [4]. However, there was no consideration of uncertainties naturally occuring in the measurements and in the used model. The key idea of the proposed localization approach is depicted in Fig. 2. Roughly speaking, for lo-

Fig. 2 Visualization of the key idea of the proposed novel localization method based on locally measuring a distributed phenomenon. **(a)** Possible distribution $p_k(x,y)$ of a physical system characterized by a strong mathematical model. **(b)** Sectional drawing of the system at a specifi c location in x-direction. Depicted are the possible locations (deterministic case) and the respective density function $f^e(r_k^{sy})$ (stochastic case).

calizing sensor nodes, the mathematical model and the resulting distribution of the spatially distributed phenomenon is exploited in an *inverse manner*. That means, locally measured physical quantities are used to obtain possible locations where the measured values could have been generated.

In this research work, we introduce two different methods for the model–based passive localization of sensor nodes based on local observations: **(a)** the polynomial system localization method, and **(b)** the simultaneous reconstruction and localization method. The *first approach* (***PSL-method***) is purely deterministic, meaning that neither uncertainties in the model description nor in the measurements are considered. This direct method is based on restating the model of the distributed phenomenon in terms of a polynomial system including the state of the physical system and the location to be identified. Then, solving a system of polynomial equations leads directly to the desired location of the sensor node. The *second approach* (***SRL-method***) considers uncertainties both in the mathematical model and the measurements during the localization process. It is shown that the localization problem can be regarded as a simultaneous state and parameter estimation problem, with node locations as the parameters to be identified. This leads to a high-dimensional nonlinear estimation problem, making the employment of special kinds of estimators necessary. By this means, the sensor nodes are localized and the distributed phenomenon is reconstructed in a simultaneous fashion. The improved knowledge can be exploited for other nodes to localize themselves.

2 Problem Formulation

The main goal is to design a novel localization method for sensor network applications, where individual nodes are able to locally measure a distributed phenomenon

only. We assume to have a strong mathematical model of the phenomenon, i.e., with known model structure and model parameters. This model could possibly result from an earlier identification task; visualized in Fig. 3 **(a)**. Based on this mathematical model and local measurements, newly deployed or movable sensor nodes can be efficiently localized without using a global positioning system; see Fig. 3 **(b)**.

Considered Distributed Phenomenon Throughout this paper, we consider the localization based on the observation of a distributed phenomenon described by the one-dimensional diffusion equation

$$L\left(p(r,t)\right) = \frac{\partial p(r,t)}{\partial t} - \alpha \frac{\partial^2 p(r,t)}{\partial r^2} - s(r,t) = 0 \ , \tag{1}$$

where $p(r,t)$ denotes the distributed state of the phenomenon at the spatial coordinate r and at the time t. The diffusion coefficient α can be varying in both time and space. Given an estimated solution $p(\cdot)$, the aim is the estimation of the location r_k^{si} of the individual sensor nodes based on local measurements $\hat{\underline{y}}_k$ of a realization of the distributed phenomenon $p(\cdot)$. In this work, we consider the worst-case scenario where the node location is *completely unknown* and the phenomenon $p(\cdot)$ still contains some uncertainties. The same methods can be utilized for the purpose of simply considering uncertainties in the locations during the reconstruction of distributed phenomena.

3 Overview of the Passive Localization Method

The *model-based passive localization* method proposed in this research work can be considered as a ***two-stage technique***: The first stage is the so-called *identification/calibration stage*, which is responsible for building a sufficiently accurate probabilistic model of the considered physical phenomenon and its environment. This can be regarded as a system identification and training phase. Then, during the *localization stage*, the previously created and identified model is exploited to estimate the location of individual sensor nodes by local measurements of the distributed phenomenon. This stage can be seen as the usage stage performing the actual localization task based on locally measuring the distributed phenomenon.

3.1 Identification/Calibration Stage

For the derivation of a sophisticated model describing the underlying distributed phenomenon exploited for the localization, a series of calibration measurements is required. This can be performed by using a certain number of sensor nodes sens-

ing the physical quantity at known locations. Here, these nodes are assumed to be responsible only for identifying the underlying phenomenon, however, not necessarily for the actual localization process. At each sensor node with the precisely known position $r_k^{\mathrm{BS}i}$, a realization of the distributed phenomenon $p_k(\cdot)$ is locally measured.

For physical phenomena distributed over a wide area, gathering the measurements can become tedious. However, the automation of this process can be achieved using mobile devices (with an accurate independent navigation system) moving in the area of interest in an autonomous and self-organized manner. Such system, for example, was proposed in [14], where a mobile robot autonomously collects information about the signal strength for indoor localization purposes [11, 12].

The identification or calibration stage strongly differs in the way they actually make use of the measurements obtained. In this research work, the localization based on *static* as well as *dynamic* phenomena is of interest. In particular, depending on the type of the system, the description to be obtained during the identification stage is different. For static systems a mathematical model only in terms of a probability density function is required, whereas for dynamic systems additional parameters describing the dynamic and distributed behavior need to be identified and calibrated.

Static Phenomena In the case of localizing sensor nodes based on a static distributed phenomenon, the identification stage consists only of finding an appropriate model description in terms of the conditional density function $f^e(p|r)$. This description characterizes the distribution of the considered physical quantity and its uncertainty in the area of interest. In this sense, for each position r a density function about the distributed phenomenon is obtained. There are several ways for the actual derivation of the model describing the distribution of the physical quantity. For example, this can be achieved by *data-driven approaches* [3] which use the calibration measurements to directly estimate the underlying density function $f^e(p|r)$ of the static distributed phenomenon. Another possibility is to use *probabilistic learning techniques*, such as the *simultaneous probabilistic localization and learning* method (*SPLL-method*) proposed in [10], which additionaly allows the simultaneous localization during the identification and calibration stage.

Dynamic Phenomena For dynamic distributed phenomena, it is not sufficient to derive a description only about the current spatial distribution of the physical quantity, rather additional parameters characterizing the dynamic behavior are necessary. The main advantage of exploiting dynamic phenomena for the localization is that additional information about the dynamics allows excluding specific values of the otherwise possibly ambiguous location estimates. However, this advantage is opposed by the more sophisticated and costly identification/calibration stage that must be accomplished before or simultaneous to the actual localization stage. That means, the precise identification of the structure and the parameters of the model description for the distributed phenomenon is required. This can be achieved by the ***S***imultaneous ***R***econstruction and ***I***dentification method (*SRI-method*), see [16].

Fig. 3 Visualization of *two tasks* for the estimation of a distributed phenomenon. The individual tasks are managed by a planning and scheduling process (not considered in this research work).

3.2 Localization Stage

In the localization stage, the individual sensor nodes with unknown location r_k^{si} measure the underlying distributed phenomenon locally, e.g., temperature distribution or signal strength distribution. The locations of the N sensor nodes to be identified are collected in the parameter vector $\underline{\eta}_k^M$, according to

$$\underline{\eta}_k^M := \left[r_k^{s1}, r_k^{s2}, \ldots, r_k^{sN}\right]^T \in \mathbb{R}^N \quad .$$

In the following, two different approaches for the passive localization are introduced: **(a)** the polynomial system localization method (PSL-method) and **(b)** the simultaneous reconstruction and localization method (SRL-method).

4 Polynomial System Localization Method

This section is devoted to a deterministic approach for the localization of individual nodes in a sensor network based on local measurements of a distributed phenomenon. The key idea of the proposed direct method is to solve the partial differential equation (1) in terms of the unknown node locations. This leads to a straightforward solution as long as the resulting nonlinear equations can be readily solved. Solving these equations for all sensor locations is called the *Polynomial System Localization Method* (PSL-method). The PSL-method basically consists of two steps: **(1)** spatial and temporal discretization of the mathematical model, and **(2)** reformulating and finally solving the resulting system of polynomial equations in terms of the desired locations.

1) Spatial and Temporal Discretization The simplest method for the spatial and temporal discretization of distributed phenomena is the finite-difference method [13, 4]. In order to solve the partial differential equation (1), the derivatives

need to be approximated with finite differences according to

$$\frac{\partial p(r,t)}{\partial t} = \frac{p_{k+1}^i - p_k^i}{\Delta t} \quad , \quad \frac{\partial^2 p(r,t)}{\partial r^2} = \frac{\frac{p_k^{i+1}-p_k^i}{r_k^{i+1}-r_k^i} - \frac{p_k^i-p_k^{i-1}}{r_k^i-r_k^{i-1}}}{\frac{1}{2}(r_k^{i+1}-r_k^{i-1})} \tag{2}$$

where Δt is the sampling time. The superscript i and subscript k in p_k^i denote the value of the distributed phenomenon at the discretization node i and at the time step k. Plugging the finite differences (2) into the mathematical model of the distributed phenomenon (1), in general, leads to a system of polynomial equations of degree three. However, for the case of one unknown sensor node location, this reduces to a single quadratic equation, as shown in the next subsection.

2) Solving Polynomial System Equations Based on the spatial and temporal discretization, the partial differential equation (1) may be expressed as a finite difference equation and put in the following form at each discretization point, p_k^i, in the interval in question

$$\begin{aligned} 0 =& A_k^i (r_k^{i+1} - r_k^i)(r_k^i - r_k^{i-1})(r_k^{i+1} - r_k^{i-1}) \\ & - B_k^i (r_k^i - r_k^{i-1}) + C_k^i (r_k^{i+1} - r_k^i) \quad , \end{aligned} \tag{3}$$

where

$$A_k^i = \frac{p_{k+1}^i - p_k^i}{2\alpha\Delta t} \quad , \quad B_k^i = p_k^{i+1} - p_k^i \quad , \quad C_k^i = p_k^i - p_k^{i-1} \quad .$$

At this point, it is important to mention that r_k^i represents the unknown location of the sensor node to be localized and r_k^{i+1} and r_k^{i-1} are the *known locations* of neighboring nodes. The derived system equation (3) can be simply regarded as an explicit relation between three positions on a line (two known endpoints and one unknown location between them), and four values of the measured phenomenon (all known and one at each location at time t and one at the unknown location at time $t+1$). In order to derive the unknown location r_k^i of sensor node i, the polynomial system of equations (3) needs to be solved and the root selected, which best fits the conditions (e.g., must be between the two known locations r_k^{i-1} and r_k^{i+1}).

The PSL-method assumes a densely deployed sensor network in which every node i communicates with its neighboring nodes $i-1$ and $i+1$. This means that measurements of the distributed phenomenon p_k^{i-1} and p_k^{i+1} need to be transmitted between adjacent nodes. It can be stated that the denser the sensor nodes are deployed, the more accurate the individual nodes in the network can be localized. The proposed localization approach involves neither errors in the mathematical model nor uncertainties in the measurements. However, it can be easily implemented and has low computational complexity.

5 Simultaneous Reconstruction and Localization Method

For the state reconstruction of distributed phenomena, the precise knowledge about the node locations is essential for deriving precise estimation results. However, using any kind of positioning system, some uncertainties in the location estimate remain. In order to obtain consistent and accurate reconstruction results, these uncertainties in the node location need to be systematically considered during the reconstruction process. Hence, the simultaneous method proposed in this section does not only allow **(a)** the *localization of sensor nodes*, but especially **(b)** the systematic consideration of uncertainties in the node locations during the *state reconstruction process*.

After the derivation of a finite-dimensional model for the node localization based on a system conversion, a method for the ***S***imultaneous ***R***econstruction of distributed phenomena and node ***L***ocalization (*SRL*-method) is introduced. There are four key features characterizing the novelties of the proposed method: **(a)** approach is based on local measurements only, **(b)** systematic consideration of uncertainties in the model description and the measurements, **(c)** derivation of an uncertainty measure for the estimated node location in terms of a density function, and **(d)** the simultaneous approach allows improving the estimation of the distributed phenomenon.

5.1 Conversion of Distributed Phenomena

The model–based state reconstruction of distributed systems based on a distributed–parameter description (1) is quite complex. The reason is that a Bayesian estimation method usually exploits a lumped–parameter system description. In order to cope with this problem, the system description has to be converted from a distributed–parameter form into a lumped–parameter form. In general, the conversion of the system description (1) can be achieved by methods for solving partial differential equations, such as modal analysis [1], the finite-difference method [13, 4], the finite-element method [17], and the finite-spectral method [7]. Basically, these methods consist of two steps, namely spatial decomposition and temporal decomposition.

1) Spatial decomposition By means of the spatial decomposition, the partial differential equation (1) is converted into a set of ordinary differential equations [17]. First, the solution domain $\Omega = [0, L]$ needs to be decomposed into N_x subdomains Ω^e. Then, the solution $p(r,t)$ in the entire domain Ω is represented by a piecewise approximation according to

$$p(r,t) = \sum_{i=1}^{N_x} \Psi^i(r)\, x^i(t) \quad , \tag{4}$$

where $\Psi^i(r)$ are analytic functions called *shape functions* and $x^i(t)$ their respective weighting coefficients. It is important to note that the individual shape functions $\Psi^i(r)$ are defined on the entire solution domain. The essence of all afore-

mentioned conversion methods lies in the choice of the shape functions $\Psi^i(r)$, e.g., piecewise linear functions, orthogonal functions, or trigonometric functions [17].

2) Temporal discretization In order to derive a discrete-time system model the system of ordinary differential equations (derived from the spatial decomposition) needs to be discretized in time. The temporal discretization produces a linear system of equations for the state vector $\underline{x}_k$ containing the temporal discretized weighting factors x_k^i of the finite expansion (4). The resulting discrete-time lumped-parameter system represents the distributed system (1).

In the case of *linear* partial differential equations (1), the aforementioned methods for the spatial and temporal decomposition always result in a *linear* system of equations according to

$$\underline{x}_{k+1} = \mathbf{A}_k \underline{x}_k + \mathbf{B}_k \left(\underline{\hat{u}}_k + \underline{w}_k^x\right) \quad . \tag{5}$$

The global state vector $\underline{x}_k$ characterizes the state of the distributed system and the vector $\underline{w}_k^x$ represents the system uncertainties. The structure of the system matrix $\mathbf{A}_k$ and the input matrix $\mathbf{B}_k$ merely depend on the applied conversion method [17].

5.2 Derivation of Measurement Model

In this section, we derive the *measurement model* for the purpose of *localizing* sensor nodes based on local observations of a physical phenomenon. The sensor nodes are assumed to measure directly a realization of the distributed phenomenon $p(r_k^{si}, t_k)$ at their individual locations r_k^{is}. Then, the measurement equation for the entire network is assembled from the individual shape functions $\Psi^j(\cdot)$ as follows

$$\underline{\hat{y}}_k = \underbrace{\begin{bmatrix} \Psi^1(r_k^{s1}) & \cdots & \Psi^{N_x}(r_k^{s1}) \\ \vdots & \ddots & \vdots \\ \Psi^1(r_k^{sN_s}) & \cdots & \Psi^{N_x}(\eta_k^{sN_s}) \end{bmatrix}}_{\mathbf{H}_k(\underline{\eta}_k^M)} \underline{x}_k + \underline{v}_k \quad , \tag{6}$$

where $\underline{v}_k$ denotes the measurement uncertainty and N_s represents the number of sensor nodes used in the network. The measurement model (6) directly relates the measurements $\hat{y}_k^i$ to the state vector $\underline{x}_k$ characterizing the distributed phenomenon and to the location vector $\underline{\eta}_k^M$. The structure of the measurement matrix $\mathbf{H}_k$ for localizing sensor nodes in a network is shown in the following example:

Example of Measurement Model In this example, we visualize the structure of the measurement matrix $\mathbf{H}_k$ subject to *piecewise linear shape functions*. The entire solution domain Ω is represented by $N_x = 4$ shape functions $\Psi^i(\cdot)$. In addition, there are two sensor nodes located at r_k^{s1} and r_k^{s2} in the

Fig. 4 Visualization of dynamic system and estimator for the node localization based on local observations. **(a)** The system description contains a high-dimensional linear substructure. The individual node locations r_k^{si} collected in the parameter vector $\underline{\eta}_k$ characterizes the measurement matrix $\mathbf{H}_k(\cdot)$, and thus, the individual measurements $\underline{\hat{y}}_k$ **(b)** The Bayesian estimator is based on *sliced Gaussian mixture densities* consisting of a Gaussian mixture and Dirac mixture.

subdomains Ω^1 and Ω^2. Then, the measurement model is given as follows

$$\begin{bmatrix} \hat{y}_k^1 \\ \hat{y}_k^2 \end{bmatrix} = \begin{bmatrix} \overbrace{c_1^1 + c_2^1 r_k^{s1}}^{\Psi^1(r_k^{s1})} & \overbrace{c_3^1 + c_4^1 r_k^{s1}}^{\Psi^2(r_k^{s1})} & 0 & 0 \\ 0 & \underbrace{c_1^2 + c_2^2 r_k^{s2}}_{\Psi^2(r_k^{s2})} & \underbrace{c_3^2 + c_4^2 r_k^{s2}}_{\Psi^3(r_k^{s2})} & 0 \end{bmatrix} \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \\ x_k^4 \end{bmatrix} + \begin{bmatrix} v_k^1 \\ v_k^2 \end{bmatrix} \quad ,$$

where the constants c_i^j arise from the definition of the piecewise linear shape functions in each subdomain and thus the geometry of the applied grid for the finite elements. The extension to orthogonal polynomials and trigonometric functions can be derived in a straightforward fashion [17, 16].

From the previous example, it is obvious that the structure of the measurement matrix $\mathbf{H}_k$ merely depends on the location collected in the parameter vector $\underline{\eta}_k^M$ of the individual sensor nodes. That means, for the accurate reconstruction of the distributed phenomenon (1) based on a sensor network, the exact node locations are necessary. Due to this dependency, deviations of true locations from the modeled node locations lead to poor estimation results, as shown in our previous research work [15]. On the other hand, thanks to the dependency of the measurement matrix $\mathbf{H}_k$ on the node locations, the localization problem can be stated as a *simultaneous state and parameter estimation* problem. By this means, the distributed phenomenon can be reconstructed and the nodes can be localized in a simultaneous fashion.

5.3 Augmented System Description for Node Localization

For the simultaneous node localization and reconstruction of distributed phenomena, the unknown locations of the sensor nodes $\underline{\eta}_k^M$ are treated as additional state variables. By this means, conventional estimation techniques can be used to *simultaneously* estimate the location and the state of the distributed phenomenon. Hence, an *augmented state vector* $\underline{z}_k$ containing the system state $\underline{x}_k$ and the additional unknown node locations $\underline{\eta}_k^M$ is defined by $\underline{z}_k := [\underline{x}_k^T, \underline{\eta}_k^T]^T$.

The augmentation of the state vector with additional unknown parameters leads to the so-called *augmented system model*. In the case of localizing sensor nodes, the augmentation leads to the following augmented system model

$$\begin{bmatrix} \underline{x}_{k+1} \\ \underline{\eta}_{k+1}^M \end{bmatrix} = \begin{bmatrix} \mathbf{A}_k \underline{x}_k + \mathbf{B}_k \hat{\underline{u}}_k \\ \underline{a}_k(\underline{\eta}_k^M) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_k \underline{w}_k^x \\ \underline{w}_k^\eta \end{bmatrix} \quad , \tag{7}$$

and measurement model

$$\hat{\underline{y}}_k = \underbrace{\mathbf{H}_k(\underline{\eta}_k^M)\underline{x}_k}_{\underline{h}_k(\underline{x}_k, \underline{\eta}_k^M)} + \underline{v}_k \quad , \tag{8}$$

where the nonlinear function $\underline{a}_k(\cdot)$ describes the dynamic behavior of the node locations contained in the vector $\underline{\eta}_k^M$ to be estimated.

The structure of the augmented system model (7) and (8) for the node localization is depicted in Fig. 4 **(a)**. In this case, it is obvious that the augmented measurement model is *nonlinear* in the augmented state vector $\underline{z}_k$ due to the multiplication of $\mathbf{H}_k(\underline{\eta}_k^M)$ and the system state $\underline{x}_k$. Since the parameter vector $\underline{\eta}_k^M$ characterizes the measurement matrix $\mathbf{H}_k$, it also has a direct influence on the actual measured values. It is important to emphasize that the measurement model (8) contains a high-dimensional linear substructure, which can be exploited by the application of a more efficient estimator. In the following section, we briefly describe a Bayesian estimator allowing the decomposition of the localization problem.

5.4 Estimation based on Sliced Gaussian Mixture Densities

There are several methods to exploit the linear substructure in the combined linear/nonlinear system equation (7) and measurement equation (8). The marginalized particle filter [18] integrates over the linear subspace in order to reduce the dimensionality of the state-space. Based on this marginalization, the standard particle filter is extended by applying the Kalman filter to find the optimal estimate for the linear subspace (which is associated with the respective individual particles). The marginalized filter certainly improves the performance in comparison to the standard particle filter. However, some drawbacks still remain. For instance, special measures have to be taken in order to avoid effects like sample degeneration and

impoverishment. More importantly, it does not provide a measure on how well the true joint density is represented by the estimated one.

For that reason, a more systematic Bayesian estimator is employed for the simultaneous reconstruction of distributed system and node localization. For the exploitation of linear substructures in general nonlinear systems, we introduced in our previous research work [8] a systematic estimator, the so-called ***S***liced ***G***aussian ***M***ixture ***F***ilter (SGMF). There are two key features leading to a significantly improved estimation result compared to other state of the art estimation approaches.

- **Novel density representation** The utilization of a special kind of density allows the decomposition of the general estimation problem into a linear and nonlinear problem. To be more specific, as a density representation the so-called *sliced Gaussian mixture density* is employed for the simultaneous reconstruction and localization of sensor nodes.
- **Systematic approximation** The systematic approximation of the density resulting from the estimation update leads to (close to) optimal approximation results. Thus, less parameters for the density representation are necessary and a measure for the approximation performance is provided.

Despite the high-dimensional nonlinear character, the systematic approach for the simultaneous reconstruction and localization for large-area distributed phenomena is feasible thanks to the decomposition based on the Sliced Gaussian Mixture Filter. Furthermore, the uncertainties occuring in the mathematical system description and arising from noisy measurements are considered by an integrated treatment. The systematic estimator exploiting linear substructures basically consists of three steps: the *decomposition* of the estimation problem, the utilizaton of an *efficient update*, and the *reapproximation* of the density representation [16, 8].

6 Simulation Results

In this section, the performance of the proposed localization methods is demonstrated by means of simulation results.

Assumption of Simulated Case Study In this simulation, we consider the localization based on the one-dimensional partial differential equation (1), with assumed initial condition and Dirichlet boundary conditions as considered in [15]. The nominal parameters for the system model (5) are given by

$$s(r,t) = 0 \ , \quad \alpha = 1 \quad \Delta t = 0.2 \ , \quad r^s_{\text{true}} = 16 \ ,$$

where r^s_{true} denotes the true node location. The aim is the localization of a sensor node with initially unknown location based on local observations only. The system noise term is $\mathbf{C}^w_l = \text{diag}\{20,\ldots,20\}$, the noise term for the node

Fig. 5 Results of the polynomial systems localization method (PSL-method) for various neighboring nodes with known locations. The true location to be identifi ed is $r^s_{\text{true}} = 16$.

location is given by $C_n^w = 0.02$, and for the local measurement of the node to be localized is assumed to be $C_v = 0.01$. Here, we compare different approaches for the passive localization based on local measurements: **(a)** PSL-method, **(b**) deterministic approach introduced in [4] (CSN-method), **(c)** SRL-method based on sliced Gaussian mixture filter (50 slices), **(d)** SRL-method based on marginalized particle filter (500 particles). These approaches are compared based on 100 independent simulation runs.

The simulation results for the PSL-method are depicted in Fig. 5. It is important to mention that this deterministic approach was simulated with perfect information, i.e., there is noise neither in the system nor in the measurements. Furthermore, we assume that the sensor node to be localized receives information about distributed phenomenon and locations from neighboring nodes. Since the diffusion equation has derivatives involving Δt and Δx, the PSL-method is sensitive to the distance between the two adjacent known locations. Evidence of this effect is shown in Fig. 5 which plots the values found by the PSL-method for known points of varying distance from the unknown. It is obvious that the denser the nodes are deployed the more accurate the location can be identified.

The simulation results for the SRL-method with considering all the aforementioned uncertainties is shown in Fig. 6. Here, we assume the sensor network consists only of a single sensor node locally measuring the phenomenon. Furthermore, the sensor node has only very uncertain knowledge about the initial distributed phenomenon, see Fig. 6 **(a)**.

Fig. 6 **(c)** depicts one specific simulation run for the estimation of the unknown node location η_k^S. It can be seen that after a certain transition time the SRL-method based on sliced Gaussian mixture filter (with 50 slices) offers a nearly exact location estimation, while the determinstic approach CSN-method strongly deviates (due to neglecting system and measurement noises). The root mean square error (rms) of all 100 simulation runs over time is depicted in Fig. 6 **(d)**. It is obvious that in this ex-

Fig. 6 Comparison of SRL-method based on SGMF, SRL-method based on MPF, and deterministic approach CSN-method. **(a)-(b)** Improvement of estimation of distributed phenomenon thanks to *simultaneous* approach. **(c)** Specifi c simulation run for the estimation of the node location r_k^s collected in the parameter vector $\underline{\eta}_k^M$. The true location is assumed to be $r_{\text{true}}^s = 16$. **(d)** Root mean square error (rmse) over time of 100 simulation runs.

ample the SRL-method based on the Sliced Gaussian Mixture Filter (with 50 slices) outperforms both the deterministic approach (CSN-method) and the approach based on marginalized particle filter (with 500 particles); mainly due to the consideration of uncertainties and the systematic and deterministic approximation of the density.

Comparing Fig. 6 **(a)** and **(b)**, it is obvious that thanks to the *simultaneous property* of the SRL-method, not only can the sensor node be accurately localized, but also the estimation about the distributed phenomenon can be further improved. This can be exploited by other sensor nodes to localize themselves.

7 Conlusions and Future Work

In this paper, we introduce the methodology of two novel localization approaches for sensor nodes measuring locally only their surrounding. The *PSL-method* is a deterministic approach and is mainly based on restating the mathematical model in terms of the location. In the case of *no noise* in the model description and the measurement, this method leads to sufficient results for a *dense sensor network*. The stochas-

tic SRL-method basically reformulates the localization problem into a simultaneous state and parameter estimation problem. This leads to a high-dimensional nonlinear estimation problem, which makes the employment of special types of estimators necessary. Here, the Sliced Gaussian Mixture Filter (SGMF) and the marginalized particle filter (MPF) are applied for the decomposition of this estimation problem. Thanks to the stochastic approach, the SRL-method leads to better estimation results for the location, even with noisy information. Furthermore, the simultaneous approach allows to improve the knowledge about the phenomenon, which then can be exploited by other nodes for the localization.

The application of the proposed localization methods (PSL-method and SRL-method) to sensor networks provides novel prospects. The network is able to localize the individual nodes without relying on a satelite positioning system (which is not always available) as long as a strong model of the surrounding is available.

For the PSL-method it is necessary to incorporate uncertainties into the mathematical model as well as the sensors, and to study the robustness of the method in the presence of noise. Another issue for future work is that if the locations of several nodes are unknown, they may be solved separately using the method described in this paper; however, we should compare it to the simultaneous solution of the system of degree three equations. So far, the model parameters and structure were assumed to be precisely known for the SRL-method. In many real world applications, the parameters contain uncertainties. The combination of the parameter identification of distributed phenomena and the node localization is left for future work. Finally, we intend to test the proposed localization methods on real sensor data.

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