Ridge and Ravine Detection in Digital Images

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Abstract

A novel and efficient ridge and ravine detection method is given.

1 Introduction

Ridges and ravines are important features in some image analysis tasks and represent a basic topographic type in digital terrain data. Several methods have been proposed to recover these features, but they have major shortcomings including (1) their sensitivity, and (2) their computational cost (usually as a result of fitting a polynomial). We describe here an approach based on the Laplacian operator that has a firm theoretical foundation and which is relatively inexpensive to compute.

Haralick[Haralick and Shapiro, 1992] describes how the facet model approach can be used to recover ridges and ravines. A bicubic polynomial is fit to a patch in the image; ridges are then characterized by a negative second derivative across the ridge line and a zero first derivative in the same direction. The only difference for a ravine is that the second derivative across the ravine is positive. (Haralick's book reviews several earlier techniques for ridge and ravine detection; note that Rosenfeld and Kak maintain that the Laplacian can be used to detect lines.) The computational cost is high due to the ten coefficients that are computed at each pixel.

A more recent technique related to our approach is that proposed by Gauch and Pizer[Gauch and Pizer, 1993]. In their approach, they find places where the "intensity falls off sharply in two opposite directions." They determine curvature extrema of the level curves of the image in order to achieve this. Unfortunately, their calculation requires the evaluation of a large polynomial in the first-, second- and third-order partial derivatives of the image, where cubic splines are used to calculate the partial derivatives.

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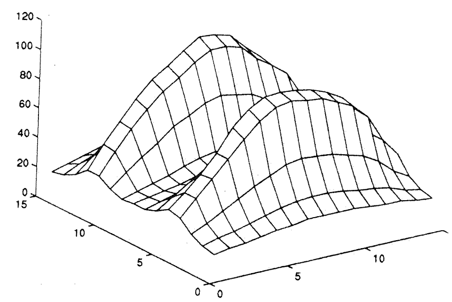

Figure 1: Ridges Viewed as Topography

2 Curl Method

Our method is based on the following sequence of observations concerning the behavior of the gradient in the neighborhood of a ridge or ravine. Figure 1 shows an image as a surface in 3D (this is a subimage of a medical image with intensity bands). The gradient (see Figure 2) produces vectors on the side of a ridge which point toward the ridge and which point away from a ravine. Although the gradient can be analyzed directly to determine the location of ridges and ravines, it is computationally more convenient to do the following:

- Rotate (locally) each gradient vector -90 de-

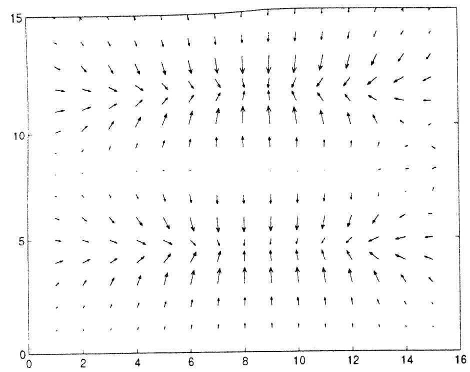

Figure 2: Gradient Vectors of Ridge Image

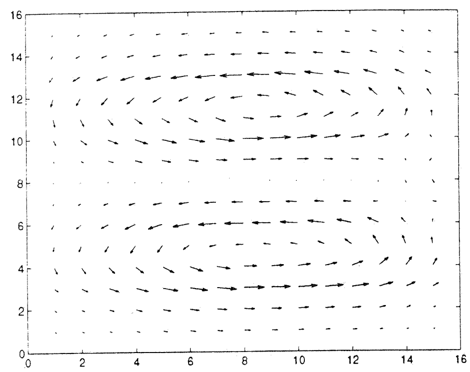

Figure 3: Rotated Gradient

grees about the out of image axis.

- Calculate the curl at each point to determine the opposed flow that exists at ridge lines.
- Calculate the extremum of this function across the ridge.

Figure 3 shows the rotated gradient for the image of Figure 1, while Figure 4 shows the extracted ridge pixels.

Now that the ideas should be clear, we give a formal development of this technique. Let the image function be $f(x,y)$. Then the gradient is:

$$\bigtriangledown f = f_x(x,y)\cdot\bar{i} + f_y(x,y)\cdot\bar{j} + 0\cdot\bar{k}$$

The rotation is:

$$rot(\bigtriangledown f) = f_y(x,y)\cdot\bar{i} - f_x(x,y)\cdot\bar{j} + 0\cdot\bar{k}$$

The curl of this is:

$$curl(rot(\bigtriangledown f)) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_y & -f_x & 0 \end{vmatrix}$$

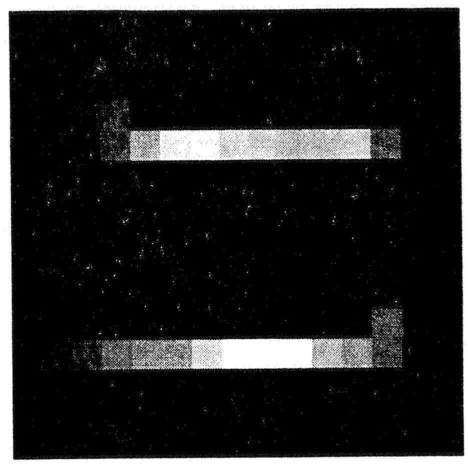

Figure 4: Extrema of Curl across Ridge

$$= 0\cdot\bar{i} + 0\cdot\bar{j} + (-f_{xx} - f_{yy})\cdot\bar{k}$$

which is just the negative of the Laplacian.

Finally, a principal direction of curvature for a ridge pixel is:

$$\alpha = \frac{atan2(f_{xy}, f_{yy} - f_{xx})}{2}$$

as well as $\alpha + \frac{\pi}{2}$. We search in these directions to determine that the pixel is a local maximum across the ridge. Note that ravines can be found in a similar way, as they are (negative) minima. (Also, it is possible that vectors of different strengths pointing the same direction give rise to a response; they can be easily filtered if necessary.)

3 Summary

We believe that the curl of the rotated gradient provides an excellent basis upon which to construct a robust ridge and ravine detector. It is comparable to existing operators, but much less costly. We have tried this technique on a number of types of images and found the results to be very good.

References

[Gauch and Pizer, 1993] John Gauch and Stephen Pizer. Multi-resolution analysis of ridges and valleys in grey-scale images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15(6):635–646, June 1993.

[Haralick and Shapiro, 1992] Robert Haralick and Linda Shapiro. *Computer and Robot Vision.* Addison-Wesley Pub Co, Reading, MA, 1992.