Flat Surface Reconstruction Using Minimal Sonar Readings

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Abstract

An optimal sensing strategy is given for recovering vertical walls with two spread-beam sonar readings. Experimental results demonstrate the effectiveness of this technique.

1 Introduction

Sonar sensors in common use today (e.g., the Polaroid sensor) produce with reasonable accuracy the range to the nearest surface, but the direction to that surface is not explicitly determined; rather the surface is known to lie within a certain angle spread centered about the line of direction of the sensor (e.g., 22.5° for the Polaroid sensor). (See Figure 1.) Multiple sonar

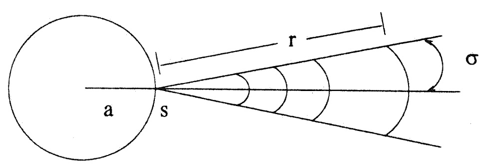

Figure 1: Beamspread of Sonar Sensor

readings are required to disambiguate the location (pose) of the reflecting surface. Several researchers have investigated the use of sonar in mobile robotics [Bozma and Kuc, 1991, Crowley, 1985, Elfes, 1987, Matthies and Elfes, 1988], and others have directly addressed the problem of wall detection [Bozma and Kuc, 1990, Kleeman and Kuc, 1995, Peremans *et al.*, 1993].

We have investigated optimal sensing strategies for the determination of the pose of vertical, flat walls in the robot's environment [Henderson *et al.*, 1996]. We start with the recovery of a single wall in the sensor's field of view. Given a single sonar sensor located on a circular ring at a distance a from the center of the ring, we show that two sonar readings, with one sensing position rotated with respect to the other (with certain conditions on the angle of rotation), suffice to recover the pose of a wall. This reduces to a plane geometry problem in which the wall is represented as a line in the plane, and its equation is determined.

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2 Pose Determination of Wall

Assume that the environment consists of a single wall whose pose is to be determined (i.e., a line in the plane). There are two key insights:

1. a single sonar reading determines a set, S, of possible lines, and
2. if correctly positioned, a second sonar reading can disambiguate which line in S gave rise to the two readings.

Under certain conditions there is a one-to-one relation between the second sonar reading and the lines in S.

Lemma 1.

Consider the circle C_1 shown in Figure 2. Given a point, P, in the circle not at the center, C, then the shortest distance from P to the tangent line at A is maximum, and to the tangent line at B is minimum. This distance monotonically decreases as the tangent line moves from A to B. (Note that only for the tangent lines at A and B are the lines through C and P perpendicular to the tangents perpendicular.)

Now, we will prove that the distance from the second sonar to the tangent lines along the arc AB decreases monotonically. Suppose not; then there exist two points D_1 and D_2 on the circle between A and B such that P is equidistant from the tangent lines to circle C_1 at D_1 and D_2 (i.e., $|PE_1| = |PE_2|$ where E_1 and E_2 are the points of intersection of the perpendiculars to the tangent lines at D_1 and D_2, respectively.) Consider the two triangles PE_1F and PE_2F, we have:

$$\overline{PF}^2 = \overline{PE_1}^2 + \overline{E_1F}^2 = \overline{PE_2}^2 + \overline{E_2F}^2 \quad (1)$$

From the assumption that $|\overline{PE_1}| = |\overline{PE_2}|$ and Equation 1, we have:

$$|\overline{E_1F}| = |\overline{E_2F}| \quad (2)$$

From the two triangles CD_1F and CD_2F, since $|\overline{CD_1}|$ and $|\overline{CD_2}|$ are equal (both equal to the radius of the circle C_1,) and since $\overline{CF}$ is a common side in both triangles, therefore,

$$|\overline{D_1F}| = |\overline{D_2F}| \quad (3)$$

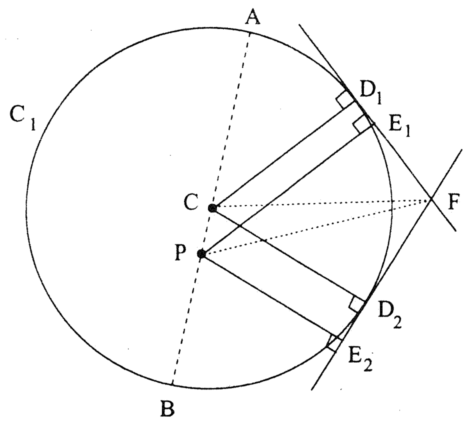

Figure 2: Distance from P to the tangent lines.

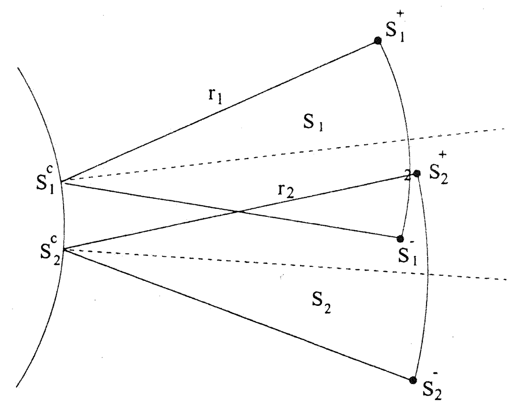

Figure 3: Notations used in the algorithm.

Also, from Figure 2, we can see that

$$|\overline{E_1F}| = |\overline{D_1F}| - |\overline{D_1E_1}| \quad (4)$$

and

$$|\overline{E_2F}| = |\overline{D_2F}| + |\overline{E_2D_2}| \quad (5)$$

Combining Equations 2,3,4,5 we get:

$$|\overline{D_1E_1}| + |\overline{E_2D_2}| = 0 \quad (6)$$

which means that $|\overline{D_1E_1}| = 0$ and $|\overline{D_2E_2}| = 0$, and so the two points E_1 and E_2 coincides with the two points D_1 and D_2, respectively. And since $\overline{PE_1}$ and $\overline{PE_2}$ are perpendiculars to the tangent lines of circle C_1, then P must coincide with the center of the circle; C, which contradicts the condition that P should be a point not at the center of the circle C_1.

To see that this solves the wall pose recovery problem, note that so long as the sonar sensor is rotated a non-zero amount about the center of the non-zero radius sonar ring, but less than the angle made by a tangent to the robot that goes through the sector corners, then the lemma applies and the pose of the wall can be found.

3 An Implementation

Given two sonar readings r_1 and r_2, we can determine the pose of the wall, assuming that the wall is flat and in the field of view of both sensors. First, let's define some points as shown in Figure 3. Two sensors are located on a circular arc at locations S_1^c and S_2^c with fields of view represented by the two sectors S_1 and S_2, respectively. The corners of each sector are defined by the points S_i^+ and S_i^- as shown in Figure 3.

There are five different cases for the orientation of the wall with respect to the two sensors. These cases are shown in the Figures 4 through 8 and can be summarized as follows:

Case I: the wall is tangent to neither sector arc and goes through S_1^+ and S_2^+.

Case II: the wall is tangent to S_1 but not to S_2 and goes through S_2^+.

Case III: the wall is tangent to both arcs of S_1 and S_2.

Case IV: the wall is tangent to S_2 but not to S_1 and goes through S_1^-.

Case V: the wall is tangent to neither sector arc and goes through S_1^- and S_2^-.

For each of these cases, by fixing r_1, the value of r_2 can determine which region the wall is in. The following algorithm can be used to determine the wall pose given the two sensor readings r_1 and r_2.

- if $r_1 \leq r_2$ then
  1. draw a tangent line from point S_1^+ to the arc of sector S_1 (see Figure 4).
  2. if the distance from point S_2^c to that tangent is less than or equal r_2, then the wall is in the first region, and it is represented by the line segment connecting S_1^+ and S_2^+.
  3. else, draw a tangent to the arc of sector S_2 from point S_2^+, as shown in Figure 5.
  4. if the distance from the point S_1^c to that tangent is greater than or equal r_1, then the wall is in the second region and that tangent represents the wall.
  5. else, the wall is in the third region (Figure 6), and is represented by the common tangent to the two arcs.
- else if $r_1 > r_2$ then
  1. draw a tangent line from point S_2^- to the arc of the sensor at S_2 (see Figure 8).
  2. if the distance from point S_1^c to that tangent is less than or equal r_1, then the wall is in the fifth region, and it is represented by the line segment connecting S_1^- and S_2^-.
  3. else, draw a tangent to the arc of sector S_1 from point S_1^-, as shown in Figure 7.
  4. if the distance from the point S_2^c to that tangent is greater than or equal r_2, then the wall is in the forth region and that tangent represents the wall.

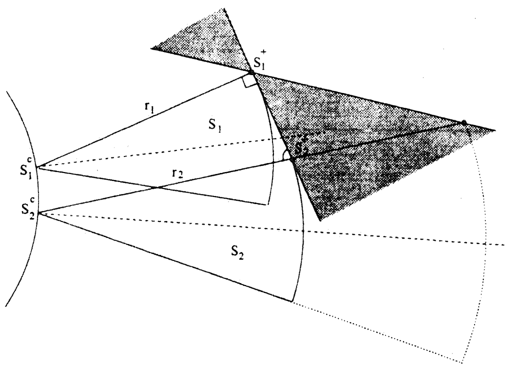

Figure 4: First region.

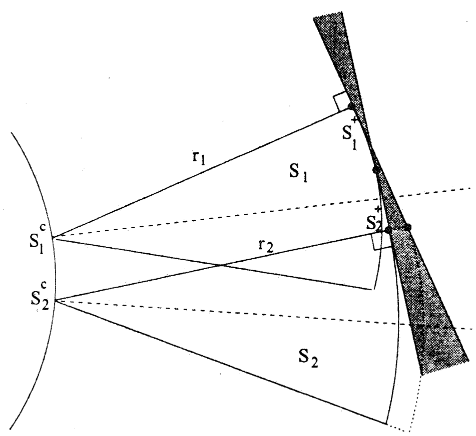

Figure 5: Second region.

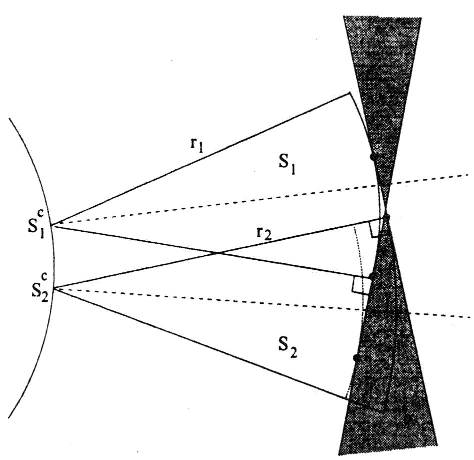

Figure 6: Third region.

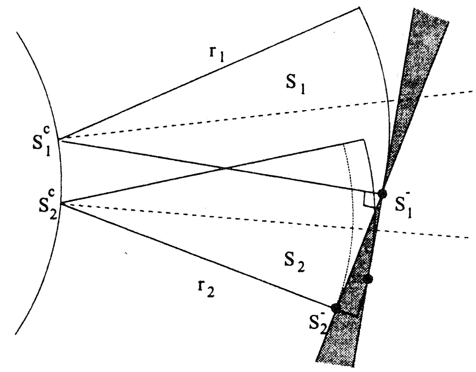

Figure 7: Forth region.

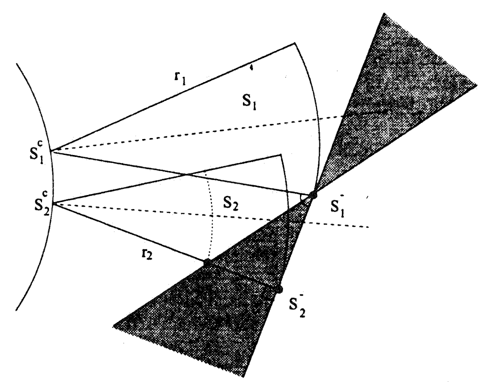

Figure 8: Fifth region.

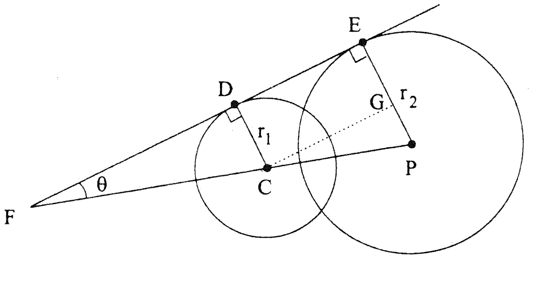

Figure 9: Finding the common tangent of two circles.

5. else, the wall is in the third region (Figure 6), and is represented by the common tangent to the two arcs.

Now, the only thing left is to find the common tangent to two circles. Figure 9 shows the basic idea of finding the common tangent. By connecting the two centers, C and P, and extending the line segment $\overline{CP}$ to F where the distance $|\overline{PF}|$ can be calculated from the equality:

$$\frac{r_1}{r_2} = \frac{|\overline{FC}|}{|\overline{FP}|} \tag{7}$$

where

$$|\overline{FC}| = |\overline{FP}| - |\overline{CP}| \tag{8}$$

From point F, we draw a line that makes an angle of θ with the line $\overline{FP}$ where

$$\sin\theta = \frac{|r_2 - r_1|}{|\overline{CP}|} \tag{9}$$

4 Experimental Results

In practice, sonar sensors located on a ring and with at most 18° difference in their directions can be used pairwise to recover hypotheses about walls present in the environment (this is due to the fact that a sonar/wall incident angle of greater than 60 degrees is necessary to get a return). We present here some experimental data taken with walls located in known positions with respect to the sonar ring and compare the calculated poses.

First, we consider the setup shown in Figure 10. The wall (a large modular office partition wall) was placed at a fixed distance, d, but at various tangents to the circle of radius d centered at the sonar sensor, S_1. A reading is then taken from the second sonar sensor, S_2, and the pose calculated. Figure 11 shows the range reading from a central sonar and two side sonars and clearly indicates the stability of the range of the central sonar and the monotonic nature of the two side sonars.

Second, we repeat this experiment, but with the robot interacting with actual walls in an office. The pose of the walls was determined by measurement with the center of the sonar ring the origin, and the forward facing sonar of the robot giving the x-axis. Figure 12 shows the the angle error between the computed wall orientation and the actual wall orientation.

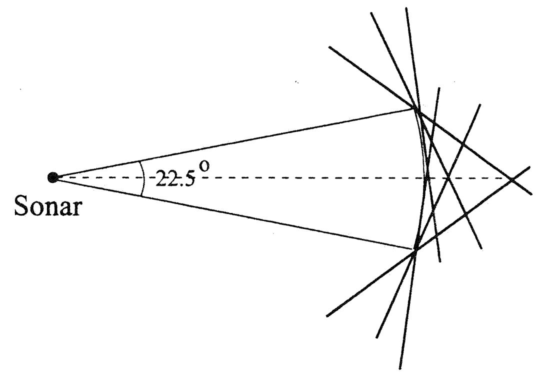

Figure 10: Reference Wall Experiment

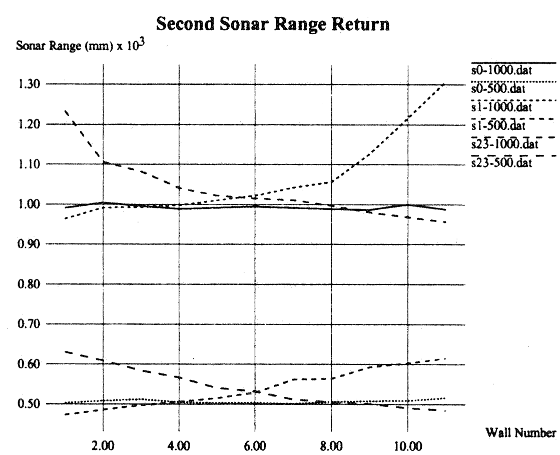

Figure 11: Controlled Line Recovery Data

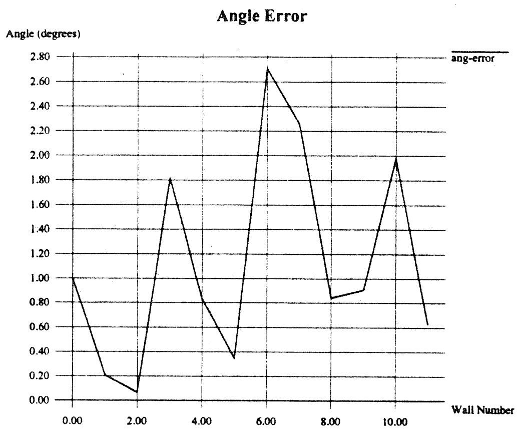

Figure 12: Environment Line Recovery Data

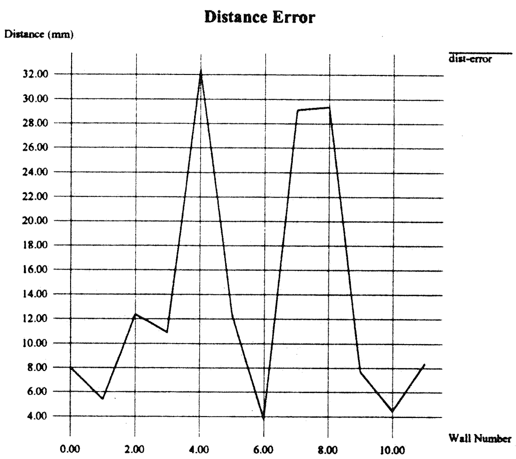

Figure 13: Environment Line Recovery Data

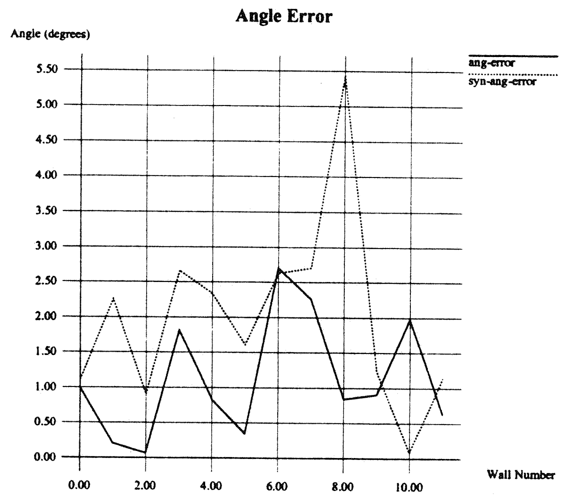

Figure 14: Method Comparison

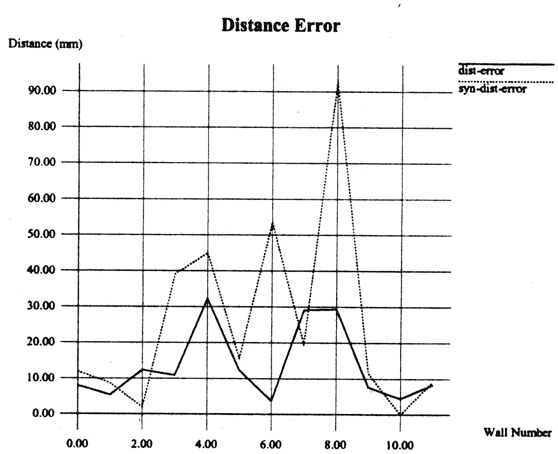

Figure 15: Method Comparison

Figure 13 shows the the distance error between the computed wall and the actual wall (where distance is the normal distance from the origin to the wall).

In addition, we compared our method with a more standard approximation used in the mobile robot community. A pose estimate can be made by assuming there is no beam spread on the sonar so that two distinct points on the wall are given by the two sonar readings. The line is then defined by these two points. A comparison of the error in this method and the error in our method is given in Figures 14 and 15.

5 Conclusions

An optimal sonar sensing strategy is given for the use of two sonar readings for the recovery of wall positions in the environment. This technique can be used to generate hypotheses of wall surfaces which helps define precise strategies to take more data which corroborate or disconfirm the hypotheses. The underlying geometrical arguments may also be relevant to other kinds of sensors with similar beam spread physics.

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