

# Formal Verification of Cyber-Physical Systems

## An Invitation

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# Pop Quiz

Solve this:

$$\frac{dx}{dt} = xe^{-t}$$

$$x(0) = 5$$

# Dynamical Systems

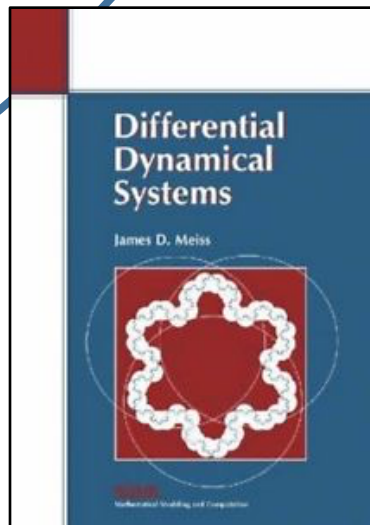
- State:  $x$
- Rule describing how to go from one state to next.

# Continuous vs. Discrete Time

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}, t)$$

$$t \in \mathbb{R}$$

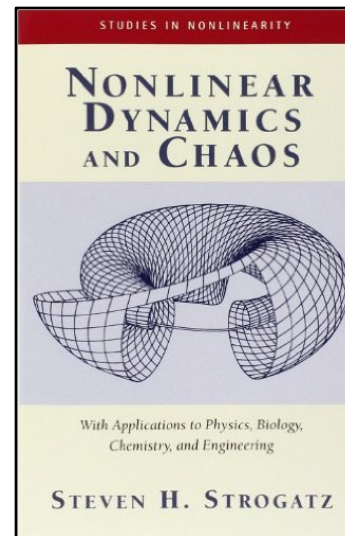
$$f(\mathbf{x}, t)$$



$$\mathbf{x}(t + 1) = f(\mathbf{x}(t), t)$$

$$t \in \mathbb{N}$$

$$\mathbf{x}(t + 1)$$

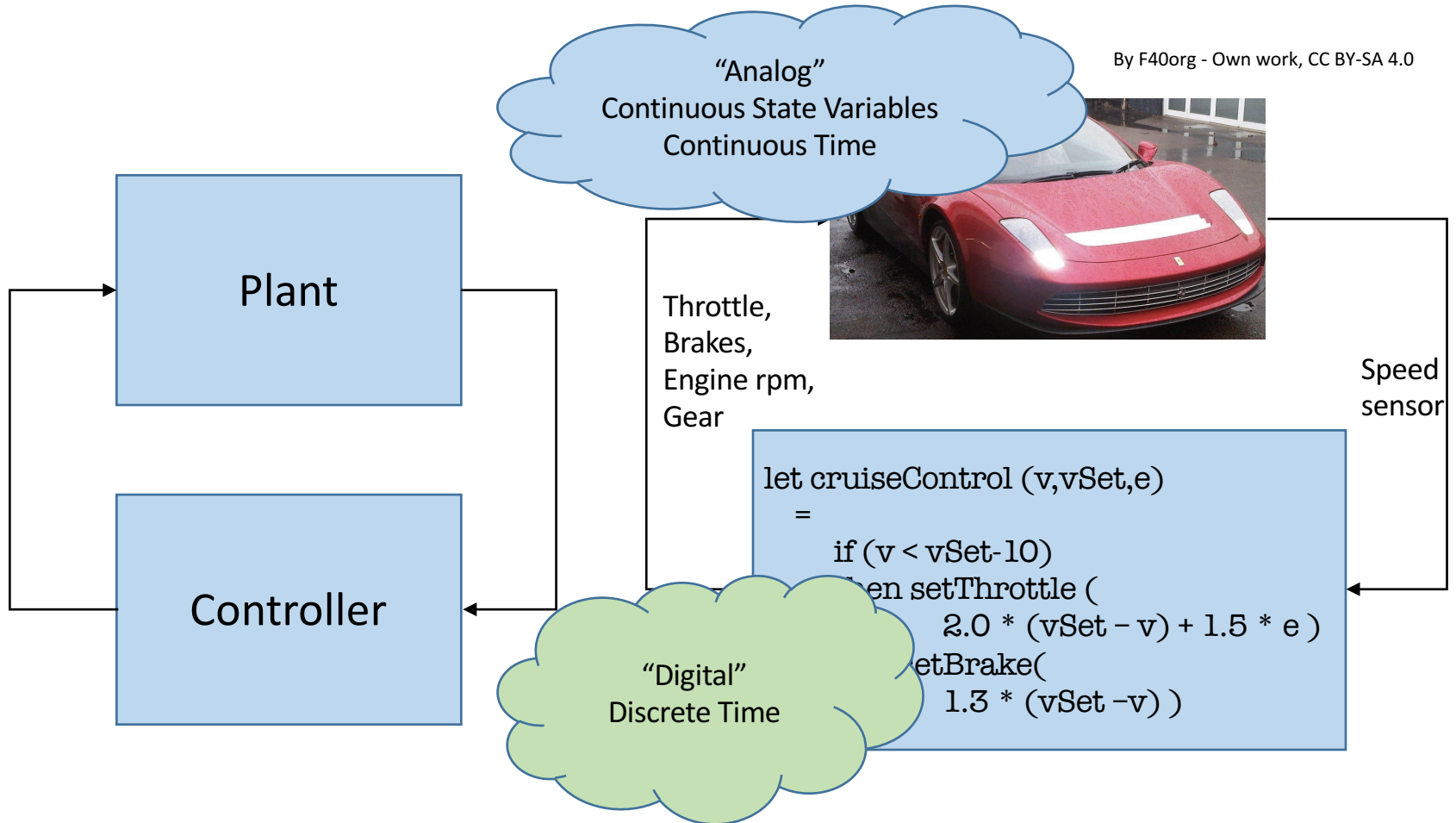


# Outline

- **Cyber-Physical Systems.**
- Theoretical Challenges.
- Application Challenges.
- Research Directions.
- PhD in formal methods for CPS.

# Control Systems

By F40org - Own work, CC BY-SA 4.0

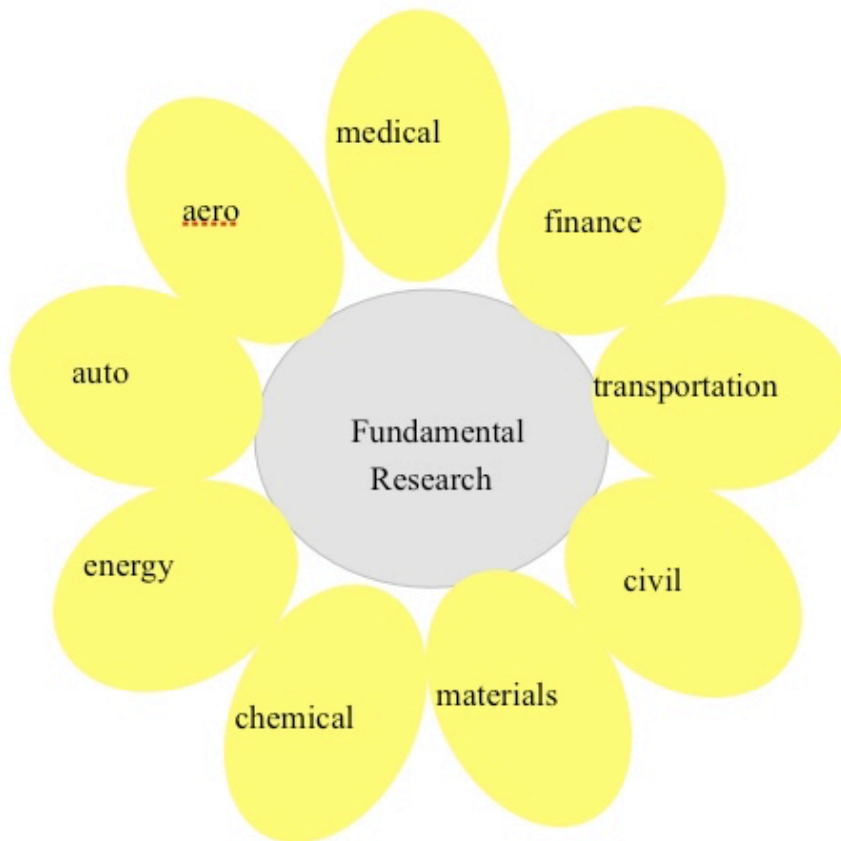


# Cyber-Physical Systems (CPS)

- Computer Control of a Physical Process.
  - “Deeply” embedded, real-time computation.
- Discrete-time (digital) system interacting with continuous time (analog) system.
- Natural systems: physics, biology, ecology, climate,...
- Common examples around us!

# CPS Application Domains

Large variety of application domains.



“CPS Flower” by P.R. Kumar and Jeannette Wing.



# Safety Critical Systems

Failure => (Injury | Death)

# Verification of CPS: Automotive Systems

**NHTSA Campaign Number:** 12V504000

BMW 7-series model years 2005-2007

Due to a **software problem**, the **doors may appear to be closed** and latched, but, in fact, may inadvertently open.

**CONSEQUENCE:**

The door may unexpectedly open due to road or driving conditions or occupant contact with the door.

The sudden opening **may result in occupant ejection** or increase the **risk of injury** in the event of a crash.

**NHTSA Campaign Number:** 11V248000

Honda CR-Z model year 2011

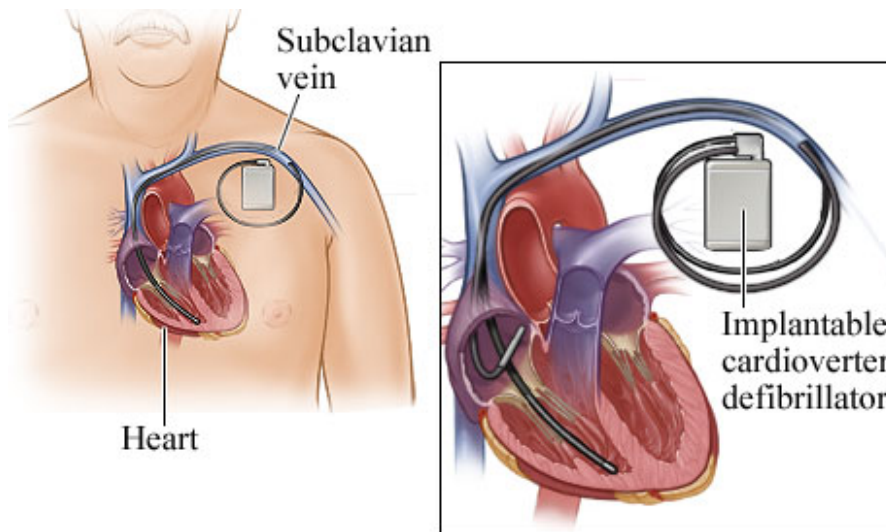
There is possibility that the **engine control unit software may cause** the electric motor of the hybrid system to **move the vehicle unexpectedly in the opposite direction of the selected gear**.

**CONSEQUENCE:**

Unexpected Vehicle Movement could increase the risk of a crash or **personal injury** to the persons in the path of the moving vehicle.

Source: [safercar.gov](http://safercar.gov)

# Cardioverter Defibrillators (ICD)



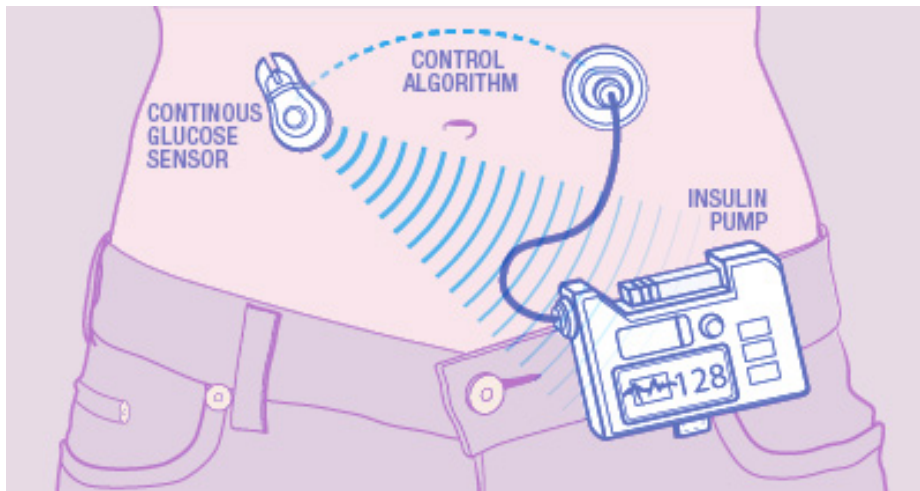
© Healthwise, Incorporated

Image source: webmd.com

- Injuries/Deaths due to
- Inappropriate shocks delivered.
  - Appropriate shock not delivered.

[Smolka, Grosu et al., Mangharam et al.]

# Artificial Pancreas



Source: MayoClinic.com

Too much insulin delivery:  
loss of consciousness, coma, death.

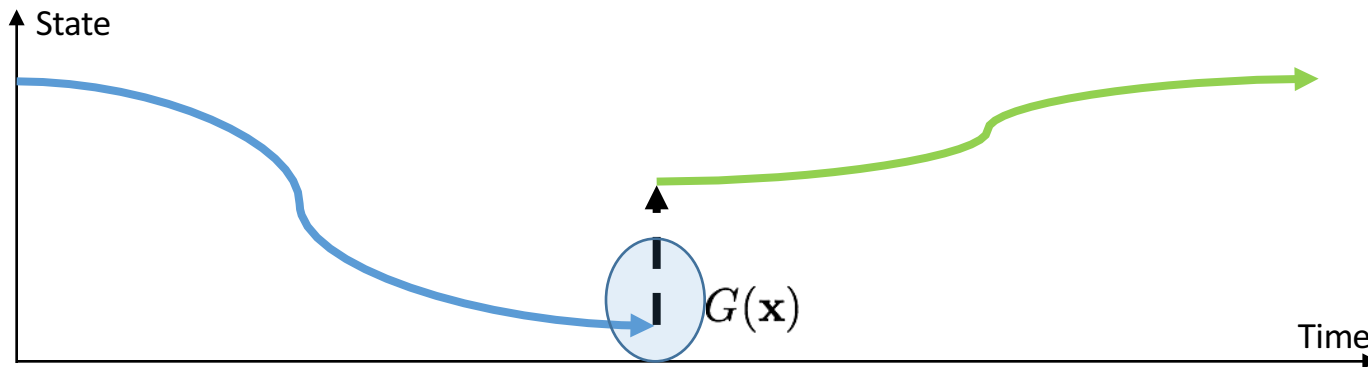
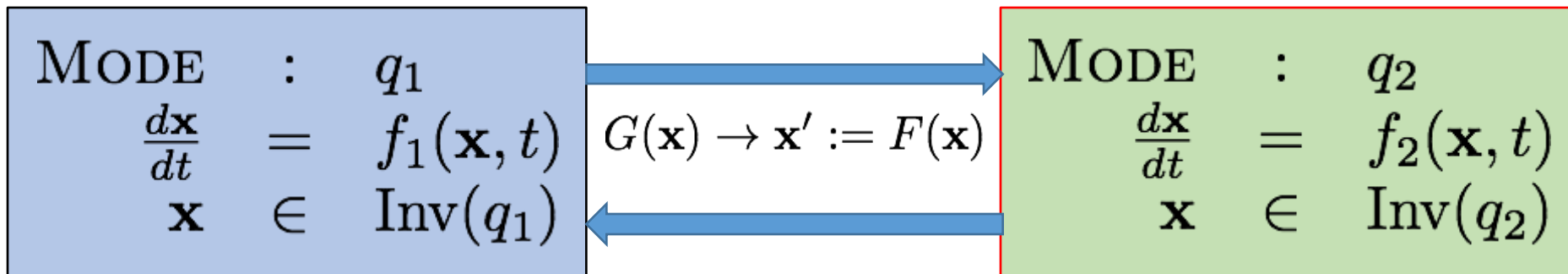
Too little insulin delivered:  
diabetic ketacidosis

[Sankaranarayanan et al.'15,'16; Sanjian Chen et al.'15]

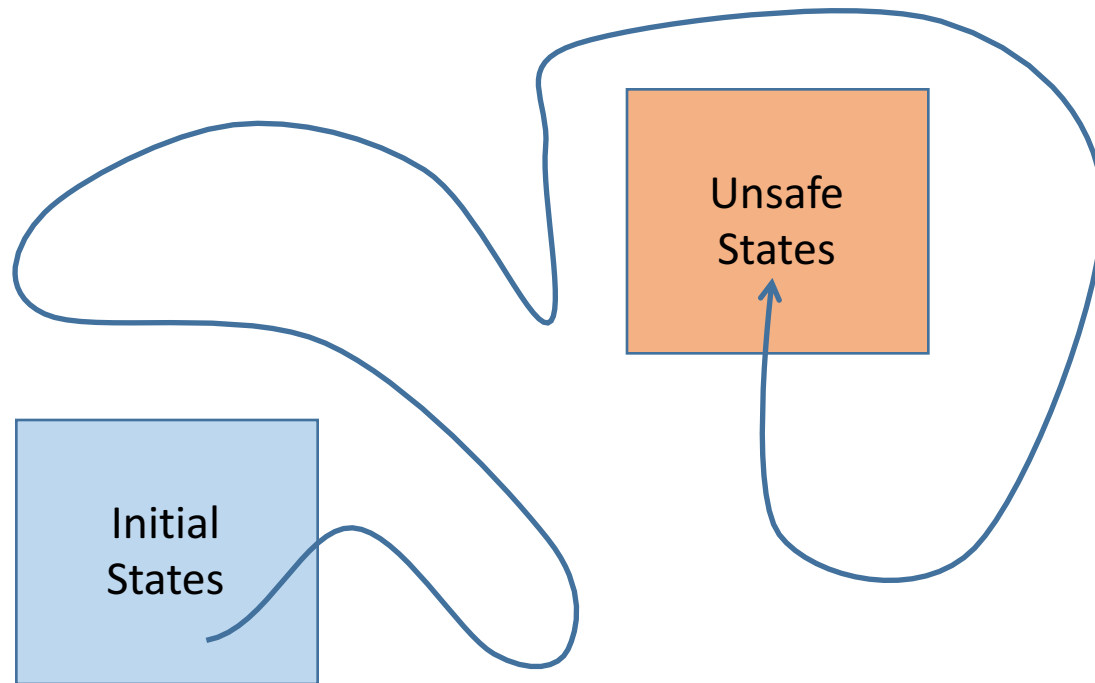
# Foundations of CPS

# Hybrid Automata

[Alur, Henzinger,  
Dill, Pnueli,  
Manna, Maler,  
Sastry, Lygeros, Tomlin,...]

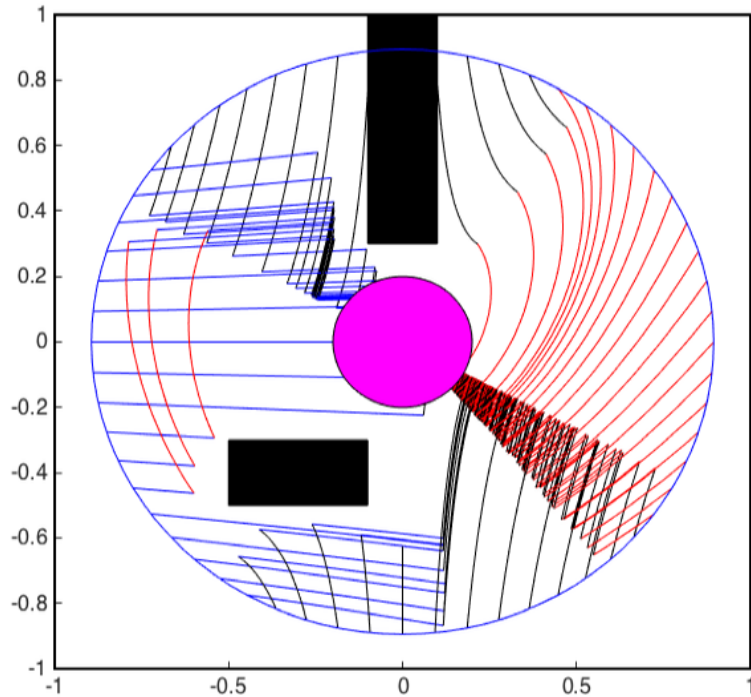


# Specifications



Reachability: trajectory from initial to unsafe state?

# Stability



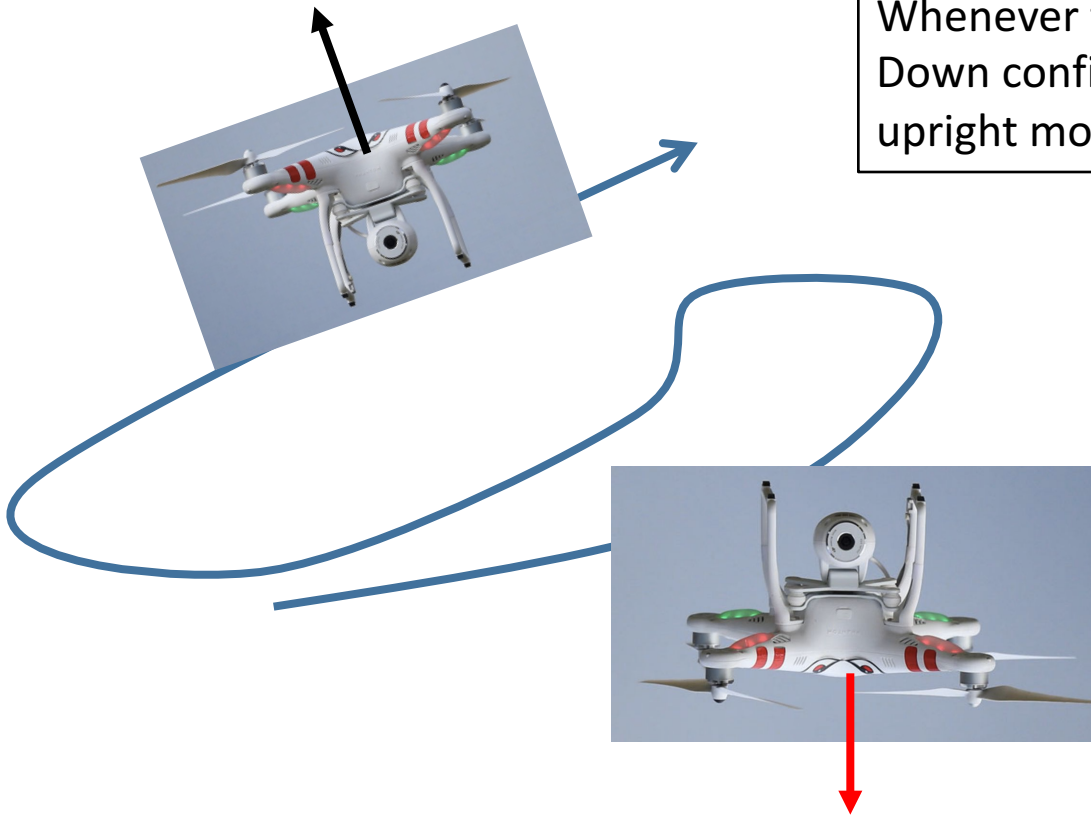
## Stability

1. System converges towards an equilibrium
2. Trajectories that start close to the equilibrium remain close.



# Real-Time Temporal Properties

Whenever the UAV reaches an Upside-Down configuration, it must reach an upright mode within 2 seconds.



# Verification Techniques

# Challenges

Murphy's Law for Hybrid Systems

Class of Hybrid Systems  
with "interesting"  
examples.



Verification Problems are  
Undecidable.

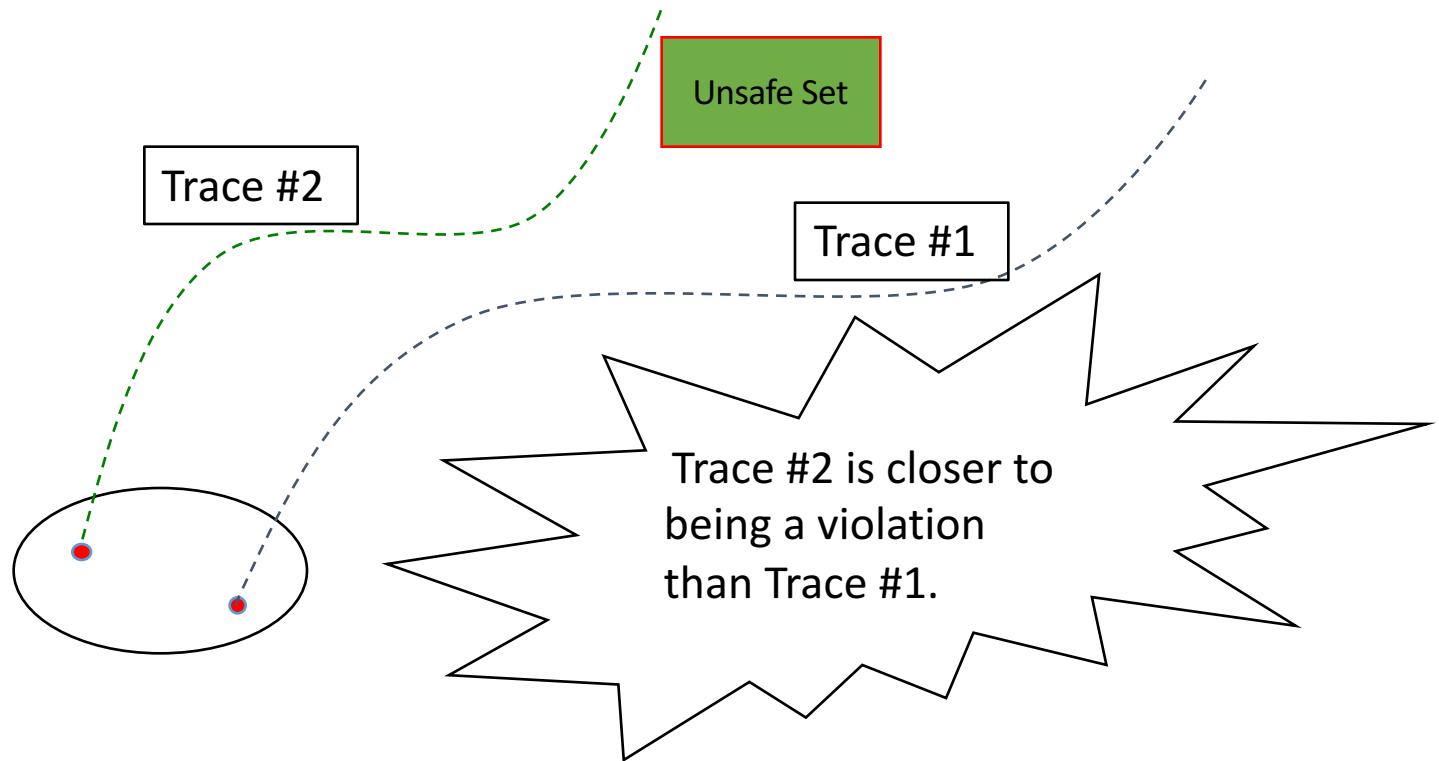
[Ken's One System Per Paper Syndrome]

# Falsification Approaches

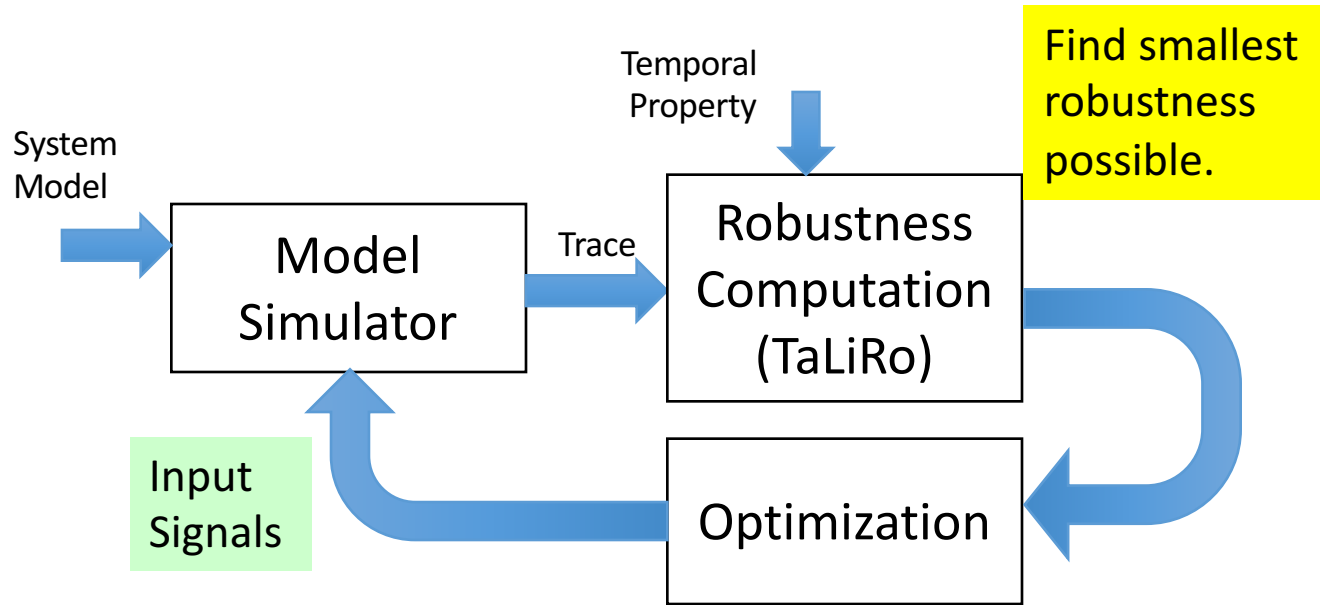
Lower Expectations, Throw away  
the Baby.

- Best effort search to find counterexamples.
- No correctness proofs.
- Falsification tools:
  - S-Taliro [Fainekos+S+Others],
  - Breach [Donze+Others].

# Robustness: Idea

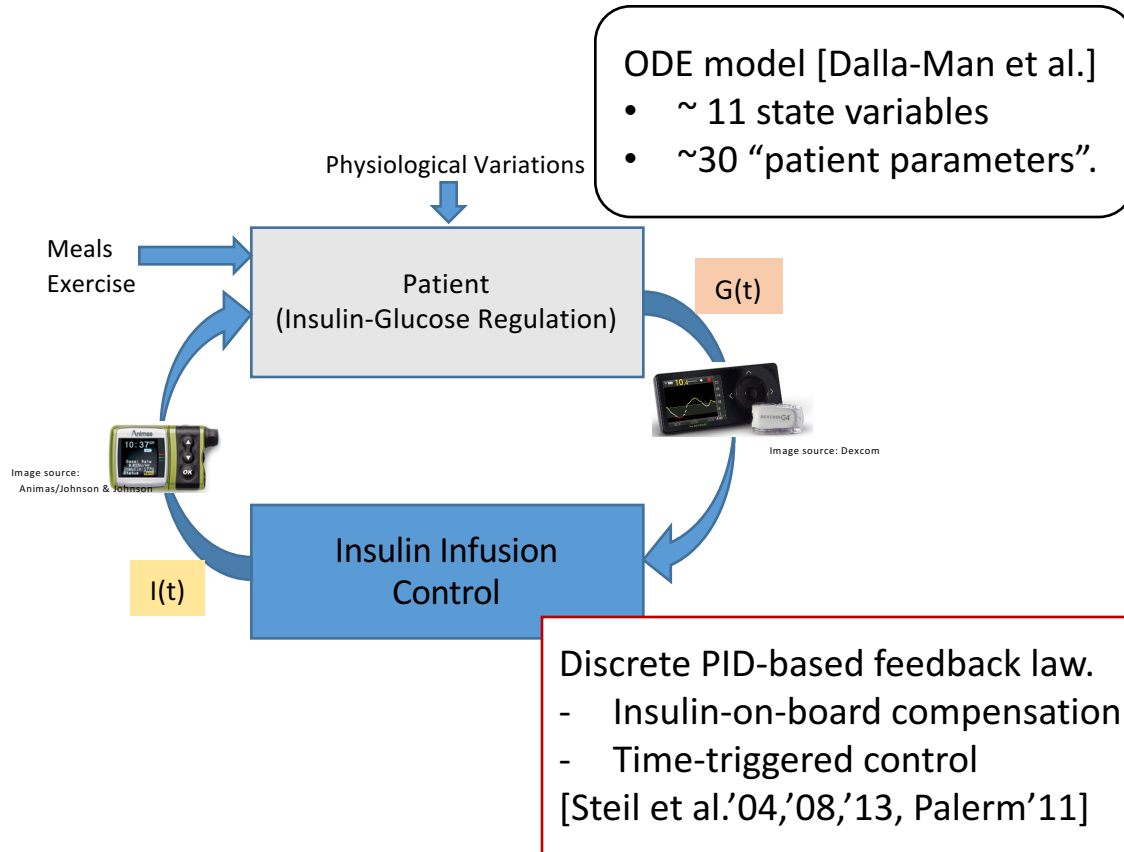


# S-Taliro: Falsification Using Robustness



[Fainekos+S+Abbas+Ivancic+Gupta]

# Case-Study: Artificial Pancreas Control.



# S-Taliro Results

[Cameron et al.'15, S. et al. '16]

1. Hypoglycemia: Can blood glucose level go below 70 mg/dl?

“Near” Violation!

2. Hyperglycemia: Can blood glucose level go above 350 mg/dl?

Violation!

3. Insulin infusion below target: Can controller deliver insulin below target level of 90 mg/dl?

Violation!

4. Can “wakeup” hyperglycemia above 200 mg/dl happen?

No Violation!

5. Can the patient suffer a “prolonged” hyperglycemic episode?

Violation!

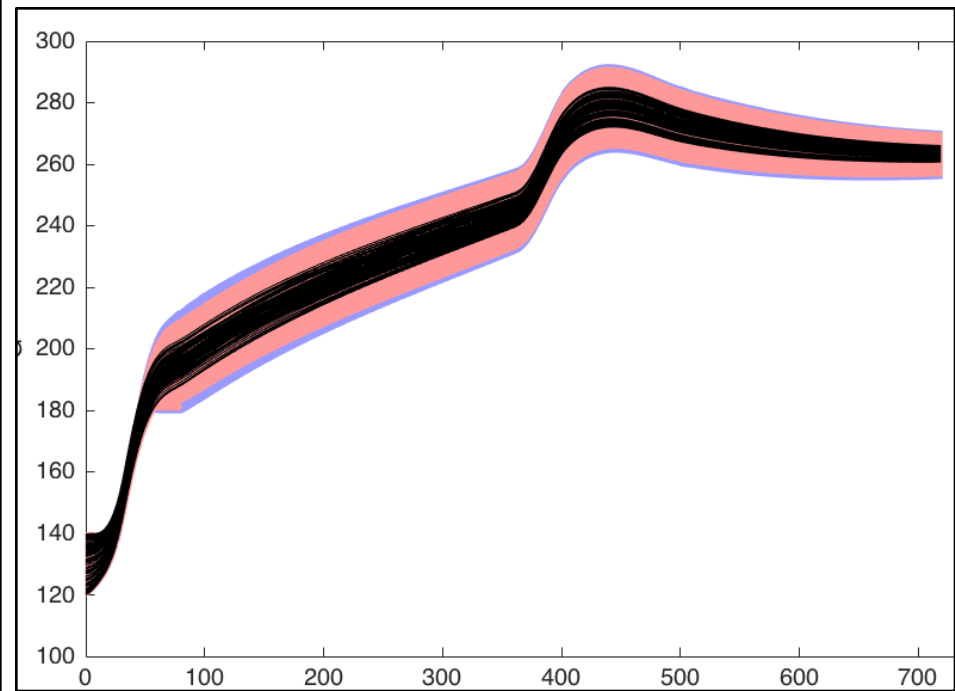
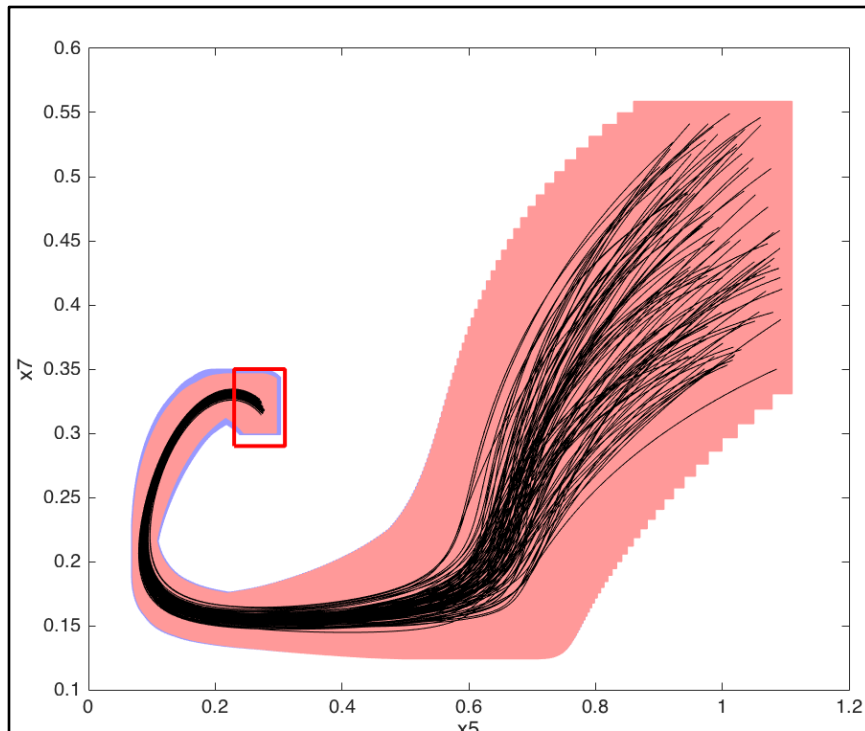
~ 3.5 hours of wall clock time.



# Formal Verification

- Goal #1: Establish presence or absence of bugs.
- Goal #2: handle semantics with mathematical precision.

# Approach #1: Flowpipe Construction



**Goal:** explore all reachable states up to a finite time horizon.

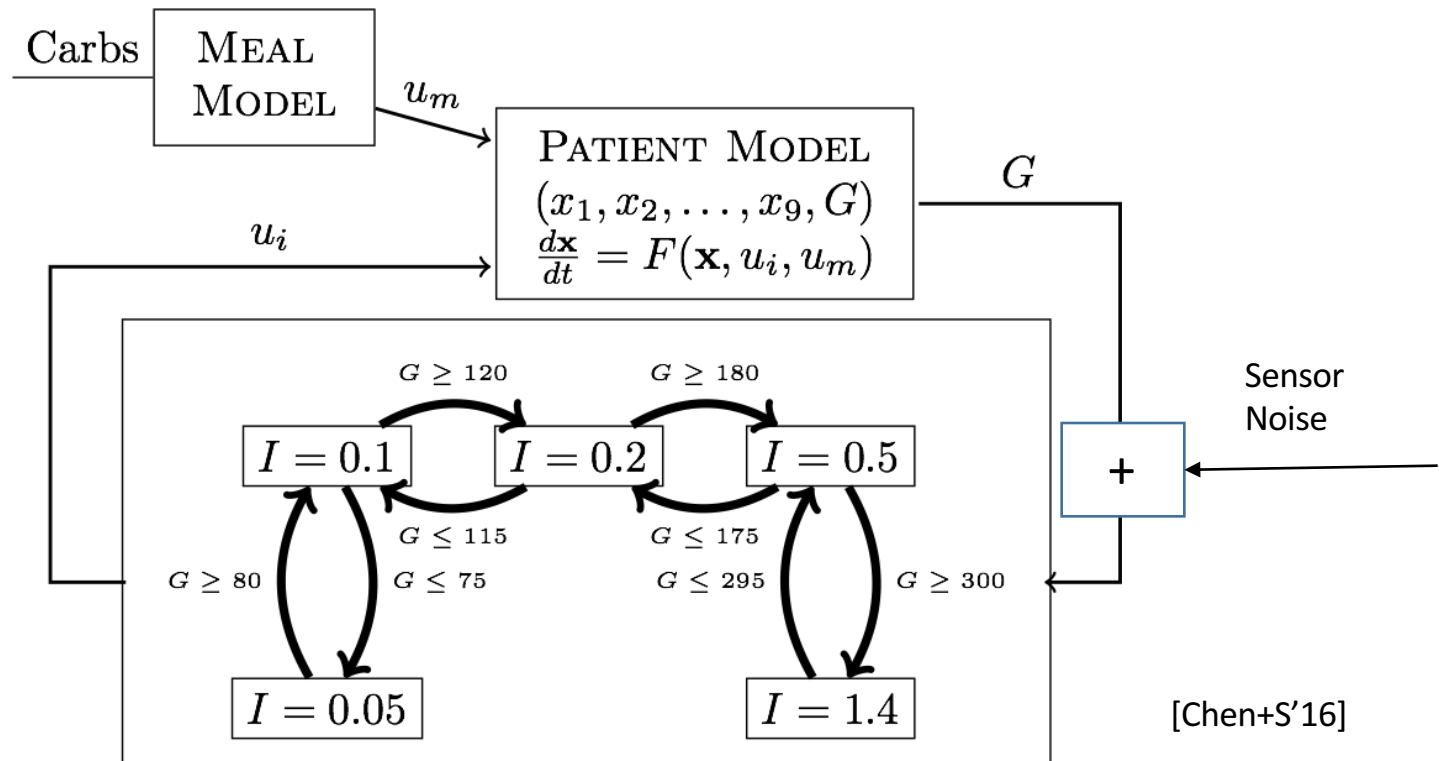
# Symbolic Model Checkers For Hybrid Systems

- Linear hybrid systems:
  - SpaceEx [Frehse+Others, Verimag].
- Nonlinear hybrid systems:
  - Flow\* [Chen+Abraham+S, Univ. of Colorado + RWTH Aachen University].
  - Cora [Althoff et al., TU Munich].
  - iSAT [Franzle+Others, Oldenburg].
  - dReach(\*) [Gao+Kong+Clarke, MIT/CMU].

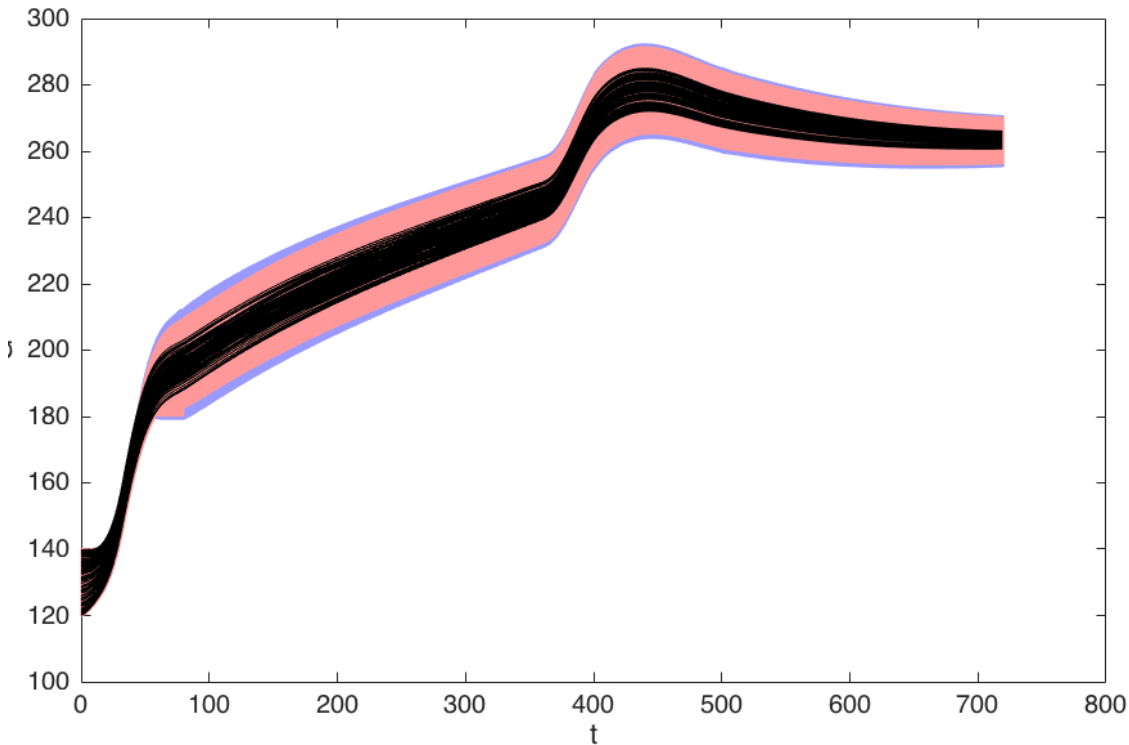
# State-of-the-Art

- SpaceX: linear systems with 50+ state variables.
  - Interesting commercial applications at this scale.
- Nonlinear Systems:
  - Smaller systems: 5-10 variables.
- Recent work by Xin Chen + **S**:
  - 30+ variables if “structurally sparse”

# Verification of Multi-Basal Artificial Pancreas System



# Flow\* Verification



- ✓ Absence of hypoglycemia
- ✓ No ketacidosis ( $G \geq 300$  mg/dl)
- Blood glucose does not settle inside range [70,180]

# dReal/dReach

- Delta-Complete decision procedure over reals.
- Reason about type-2 computable real functions.
  - Solutions to Ordinary Differential Equations.
- Numerous applications:
  - Artificial Pancreas parameter selection [Chen+Lee'15]
  - ICD (Cardiac) Devices [Islam+Smolka+Grosu+Others]

# Theorem Proving

- Early work on STeP Theorem Prover [Manna+Sipma..'90s]
- Keymera [ Platzzer+Others' CMU]
  - Differential Dynamic Logic.
  - Integrates with decision procedures over reals.
  - Extensions to concurrent and parameterized systems.
- Successes of KeyMera:
  - ACAS-X collision avoidance protocol verification.
  - Numerous application case studies.



# Combining Simulations + Proofs

- C2E2 [ Duggirala+Mitra + Viswanathan , UIUC]
- Testing + Proofs using discrepancy functions.
  - Compositional reasoning.
  - Inference of discrepancy functions from simulations.
- **Applications:** medical devices, airtraffic management protocols, automotive systems.

# Formal Synthesis

# Formal Synthesis for CPS

- Interesting combination of CS + Control theory approaches for synthesis.
- Control approach: find feedback functions to obtain reachability/stability properties.
- CS approach: synthesize control for temporal objectives.

# Tutorial Tomorrow



CAV 2016 tutorial on synthesis by Paulo Tabuada

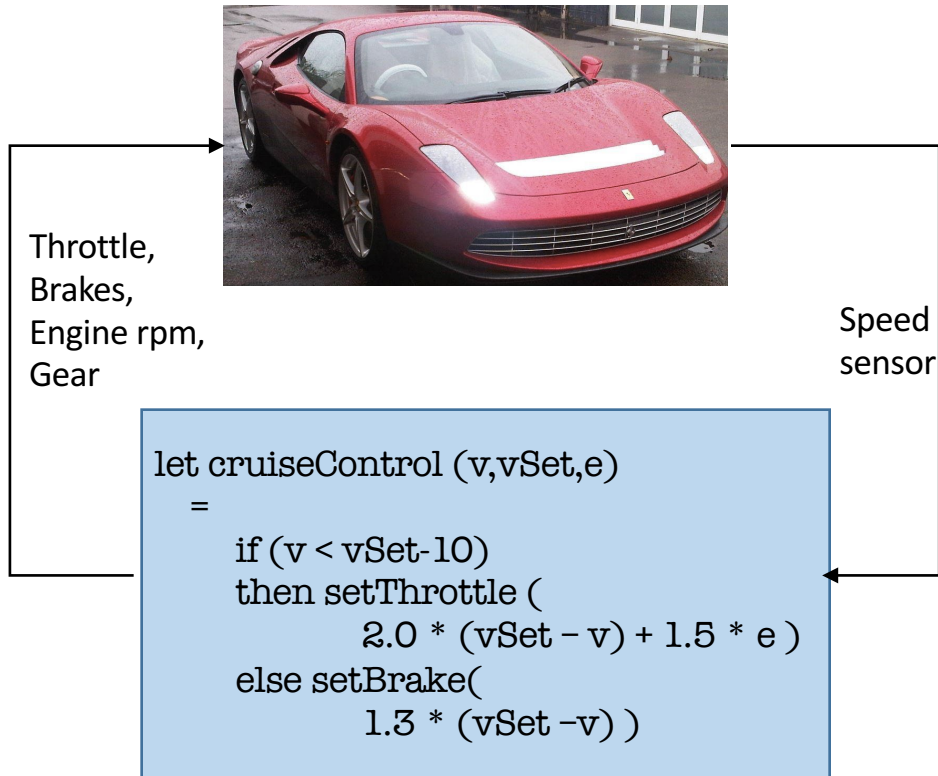
Tomorrow afternoon.

# “Hot” Research Directions

# Foundations of CPS

- Understanding properties of hybrid systems.
  - Invariant Sets.
  - Lyapunov Functions.
  - Equivalences between systems.
- Efficient inference of properties.
  - Invariant Synthesis for nonlinear systems.
  - Nonlinear Lyapunov synthesis.
- Formal synthesis.
- Stochastic Hybrid Systems

# From the model to implementation.

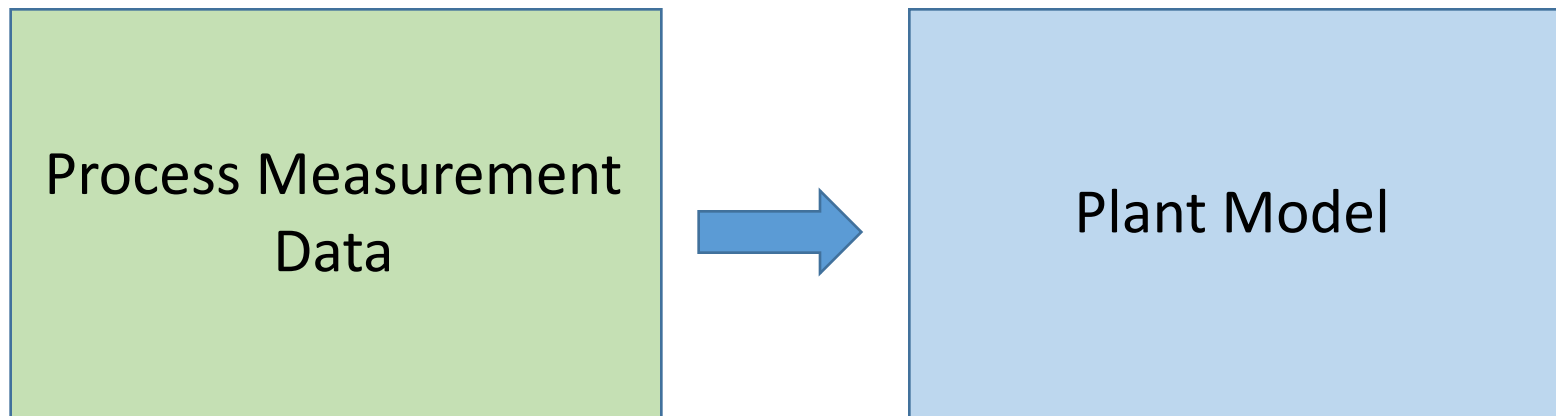


## Goal

Integrate code analysis  
with models of physical  
dynamics.

# Constructing Models from Data

## System Identification



Goal: Identify models more suitable for verification.



# Reasoning about pattern recognition/machine learning.



Image Credit: google.com

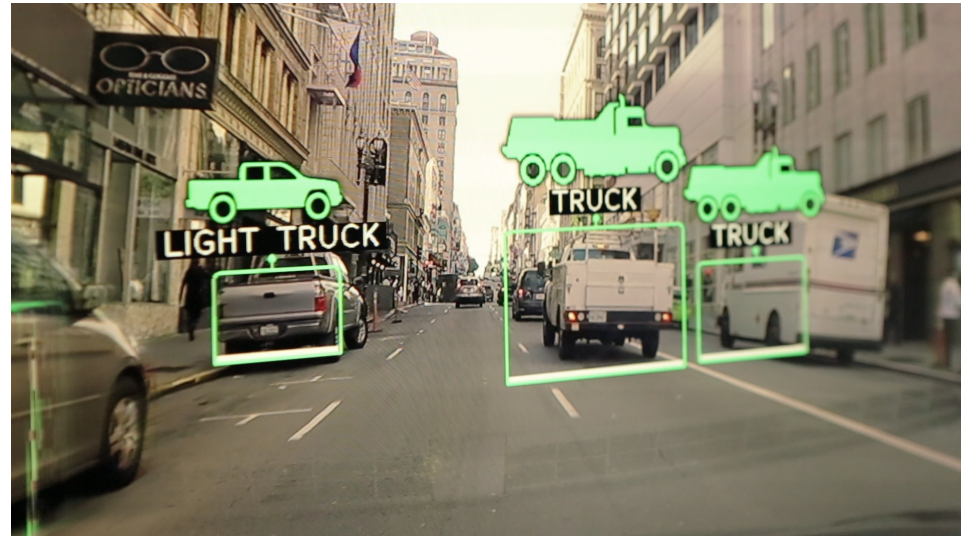


Image Credit: nvidia.com

# Closed-Loop Medical Systems

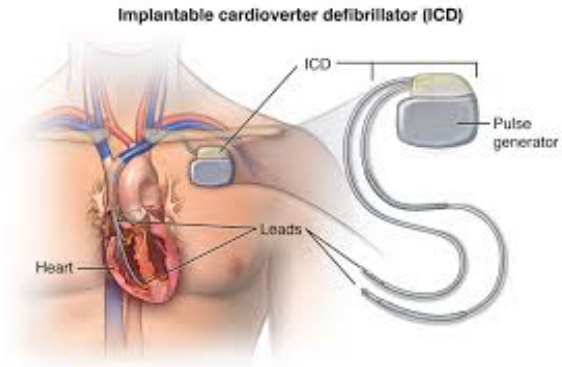
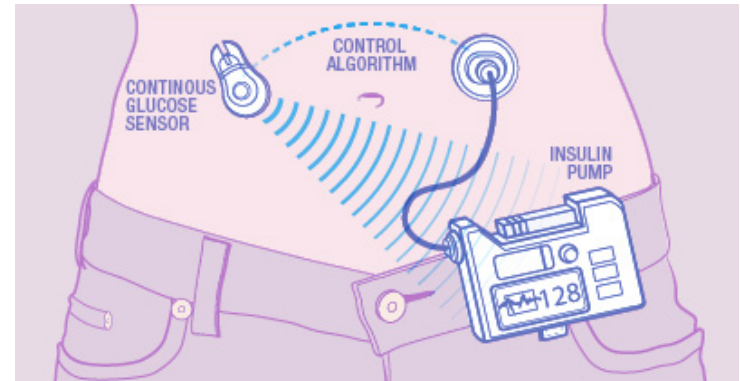


Image source: hopkinsmedicine.com



Source: MayoClinic.com



# Resources for Closed Loop Medical Devices

<https://www.cs.colorado.edu/~srirams/courses/clMedDevices-spr16/clDevices.html>

CSCI 7000 Spring 2015

[Basic information](#)

[News](#)

[Closed-Loop Topics](#)

[Schedule](#)

[Topics](#)

[Course work](#)

## Closed-Loop Devices

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I have attempted to organize some basic references for various devices below. Your task for Week 1 is to expand and enlarge the list by doing a literature search.

### Artificial Pancreas

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The artificial pancreas is a great example of a closed loop medical device. A large portion of this class will focus on this device.

#### Scientific Survey Articles

- Claudio Cobelli, Eric Renard and Boris Kovatchev, [Artificial Pancreas: Past, Present, Future](#), Diabetes November 2011 vol. 60 no. 11 2672-2682.
- B. Kovatchev, M. Breton, C. Dalla Man and C. Cobelli, [In Silico Preclinical Trials: A Proof of Concept in Closed-Loop Control of Type 1 Diabetes](#), J Diabetes Sci Technol. Jan 2009; 3(1): 44-55.
- Francis J. Doyle, Lauren M. Huyett, Joon Bok Lee, Howard C. Zisser and Eyal Dassau, [Closed-Loop Artificial Pancreas Systems: Engineering the Algorithms](#), Diabetes Care, May 2014 vol. 37 no. 5 1191-1197.
- Hovorka R, Canonico V, Chassin LJ, Haueter U, Massi-Benedetti M, Orsini Federici M, Pieber TR, Schaller HC, Schaupp L, Vering T, and Wilinska ME. [Nonlinear model predictive control of glucose concentration in subjects with type 1 diabetes](#). Physiol Meas. 2004 Aug;25(4):905-20.

#### Videos

- [Bruce Buckingham's TEDxDelMar Talk](#).
- [Ed Damiano's TEDx Talk](#).

#### Press Articles

# Human-in-the-loop CPS

- Many CPS are operated by humans in the loop.
- How do we reason formally about humans?
  - Modeling human behavior: ideas from psychology/cognitive science.
  - Collecting data about human actions and mistakes.

How to get started?

# CPS-focused conferences

- CAV: plenty of papers on hybrid systems.
  - ETAPS/TACAS.
- CPSWeek: multiple conferences under single umbrella.
  - Hybrid Systems: Computation and Control (part of CPSWeek).
- ESWeek.
  - ACM/IEEE conference on Embedded Software (EMSOFT)
- RTSS:
  - IEEE Real Time Systems Symposium.

# CPS Courses

- Formal Methods.
- Control Theory and Dynamical Systems.
- Real-Time and Embedded Systems.
- Convex Optimization.
- Robotics.

# CPS Books

- Lee and Seshia, Introduction to Embedded Systems.
  - <http://leeseshia.org>
- Rajeev Alur, Principles of Cyber-Physical Systems.
  - <https://mitpress.mit.edu/books/principles-cyber-physical-systems>
- Andre Platzer, Logical Analysis of Hybrid Systems
- Handbook of hybrid systems (good reference).



# Acknowledgments

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