ω-regular expression

Σ^ω : $\{\sigma \mid \sigma = A_0 A_1 \ldots$ an infinite sequence of symbols over $\Sigma\}$

ω-language if $\mathcal{L} \subseteq \Sigma^+$ $\quad \varepsilon \notin \mathcal{L}$

$\mathcal{L}^\omega = \{w_1 w_2 w_3 \ldots \mid w_i \in \mathcal{L} \; i \geq 1\}$ $\quad \mathcal{L} \subseteq \Sigma^*$

$\mathcal{L}_1$ language of finite words $\quad \mathcal{L}_2 \subseteq \mathcal{L}^\omega$

$\mathcal{L}_1 . \mathcal{L}_2 = \{w\sigma \mid w \in \mathcal{L}_1$ and $\sigma \in \mathcal{L}_2\}$

ω-regular expression, G :

$$G = E_1 F_1^\omega + \ldots + E_n F_n^\omega \qquad n \geq 1$$

$E_1 \ldots E_n$, $F_1 \ldots F_n$ are regular expressions over $\Sigma \ni \varepsilon \notin \mathcal{L}(F_i)$

$$\mathcal{L}_\omega(G) = \mathcal{L}(E_1) . \mathcal{L}(F_1)^\omega \cup \ldots \cup \mathcal{L}(E_n) . \mathcal{L}(F_n)^\omega$$

ω-regular language $\mathcal{L} = \mathcal{L}_\omega(G)$ for ω-regular exp. G

e.g. $(B^* A)^\omega$ contains infinitely many A's

ω-regular property Linear time prop. P over AP if P is an ω-reg lang. over 2^{AP}

e.g. 1 $AP = \{a, b\}$ $\quad P_{inv}$ induced by $\Phi = a \vee \neg b$ is ω-reg. prop. :

$P_{inv} = \{A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \mid \forall i \geq 0 \; a \in A_i$ or $b \notin A_i)\}$

Let $G = E = (\{\} + \{a\} + \{a, b\})^\omega$ over $\Sigma = 2^{AP} = \{\{\}, \{a\}, \{b\}, \{a,b\}\}$

Nondeterministic Büchi Automata (NBA)

accept ω-regular languages (ω-automata)

NBA; $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

Q states (finite)

Σ alphabet (finite?)

$\delta : Q \times \Sigma \to 2^Q$

$Q_0 \subseteq Q$ start states

$F \subseteq Q$ accept states

$|\mathcal{A}| = |Q| + |Q \times \Sigma|$

transition relation $\to \subseteq Q \times \Sigma \times Q$

$q \xrightarrow{A} p$ iff $p \in \delta(q, A)$

propositional logic used for transitions:

$AP = \{a, b\}$ $\quad q \xrightarrow{a \vee b} p$ means transition for $\{a\}, \{b\}, \{a, b\}$

persistence property over AP is an LT prop. $P_{pers} \subseteq (2^{AP})^\omega$

"eventually forever Φ" for some prop. form. over AP

$P_{pers} = \{A_0 A_1 \ldots \in (2^{AP})^\omega \mid \overset{\infty}{\forall} j \; A_j \models \Phi\}$ $\quad \Phi$ a propositional formula over AP

TS a finite transition system over AP & P_{pers} a persistent property

Nested depth-first search returns no iff TS $\not\models P_{pers}$

LTL : specify LT properties

LT property over AP is a subset of $(2^{AP})^{\omega}$

<u>satisfaction relation</u>

P an LT prop. over AP & $TS = (S, Act, \rightarrow, I, AP, L)$

a transition system w/o terminal states. Then

TS <u>satisfies</u> P $\quad TS \models P$ iff $Traces(TS) \subseteq P$

S: states $\quad Act$ actions $\quad \rightarrow \subseteq S \times Act \times S$

I: initial states, AP atomic prop. $\quad L: S \to 2^{AP}$ labeling function

directed graphs

propositional temporal logic $\qquad$ time-abstract (synchronous)

not real-time (Timed CTL)

vs. CTL Computation Tree Logic

$\Box$ always

$\Diamond$ eventually

<u>atomic propositions</u> state labels $a \in AP$ in transition system
(assertions about control variables)

<u>Boolean operators</u>

<u>temporal operators</u>:

$\bigcirc$ next $\quad \bigcirc \varphi \quad \varphi$ holds in next step

U until $\quad \varphi_1 U \varphi_2 \quad \varphi_2$ holds at some future time & φ_1 holds until then

$a \in AP$ [co-safe LTL] finite good prefix

<u>LTL formulae</u>

$\varphi ::= true \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg \varphi \mid \bigcirc \varphi \mid \varphi_1 U \varphi_2$

<u>precedence</u>: unary over binary; $\neg$ & $\bigcirc$ equal

U over $\wedge, \vee, \rightarrow$

U is right associative

$\varphi_1 U \varphi_2 U \varphi_3 = \varphi_1 U (\varphi_2 U \varphi_3)$

$\Diamond \varphi \stackrel{def}{=} true\, U \varphi$

$\Box \varphi \stackrel{def}{=} \neg \Diamond \neg \varphi$

$\Box \Diamond \varphi$ means infinitely often φ

$\Diamond \Box \varphi$ " eventually forever φ

LTL model checking is PSPACE-hard

co-safe LTL formulas always have a finite good prefix

Given φ & $w = w_0 w_1 \ldots \in (2^{AP})^\omega \ni w \models \varphi$

w has a good prefix if $\exists n \in \mathbb{N} \ni w|_n = w_0 w_1 \ldots w_n \cdot w' \models \varphi$

$\forall w' \in (2^{AP})^\omega$

for any co-safe LTL formula φ over AP $\exists$ DFA

$$A_\varphi = (\underset{\text{states}}{Q}, \underset{\text{initial state}}{\bar{q}}, \underset{\text{accepting states}}{Q_F}, \underset{\text{alphabet}}{2^{AP}}, \delta_{A_\varphi}) \qquad \delta_{A_\varphi} : Q \times 2^{AP} \to Q$$

& A_φ accepts the good prefixes for φ.

extend δ_{A_φ} to finite sequences $\delta^+_{A_\varphi} : Q \times (2^{AP})^* \to Q$

by applying sequentially to finite sequence

an accepting state $q_F \in Q_F$ is $\ni$ $\delta_{A_\varphi}(q_F, \alpha) \in Q_F \ \forall \alpha \in 2^{AP}$.