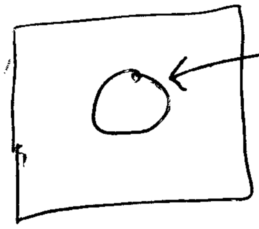
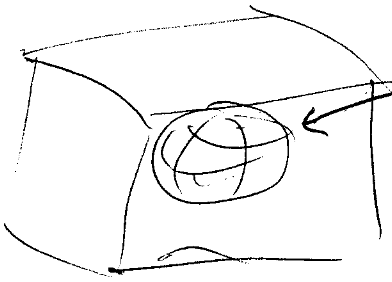
shape : defined by boundary (in 2D or 3D)

in 2D



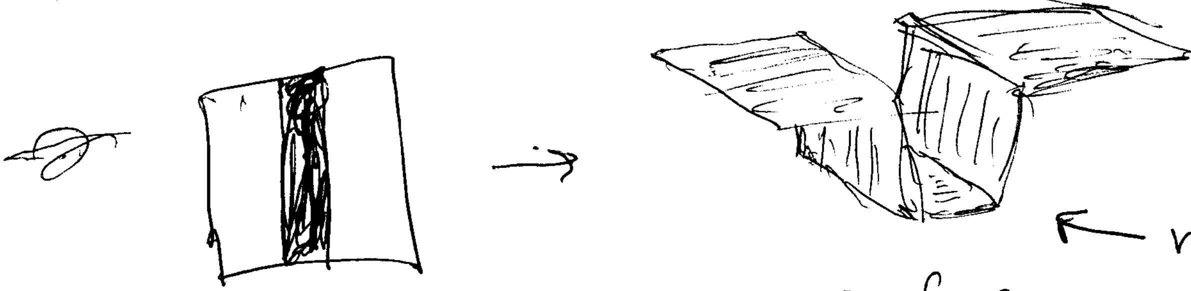
1D curve

in 3D



2D surface

also, a property of the boundary : curvature

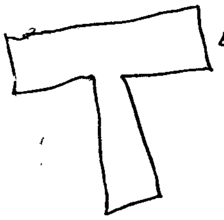


image

surface

ravine

2D shape:



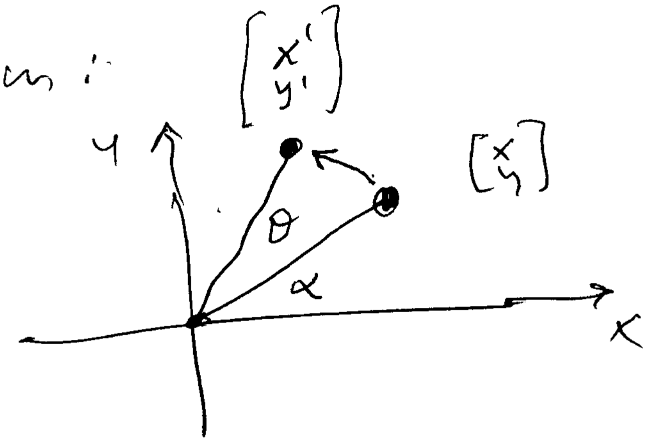
points on boundary

$$\begin{bmatrix} x_1 y_1 \\ x_2 y_2 \\ \\ \\ x_n y_n \end{bmatrix}$$

invariant to: scale, translation and rotation

Shape Transformations

Rotation :



$\left\| \begin{bmatrix} x \\ y \end{bmatrix} \right\| = r$

$$x = r\cos\alpha$$
$$y = r\sin\alpha$$

$$x' = r\cos(\alpha+\theta) \qquad y' = r\sin(\alpha+\theta)$$

$$x' = r\left[\cos\alpha\cos\theta - \sin\alpha\sin\theta\right] \qquad y' = r\left[\sin\alpha\cos\theta + \cos\alpha\sin\theta\right]$$

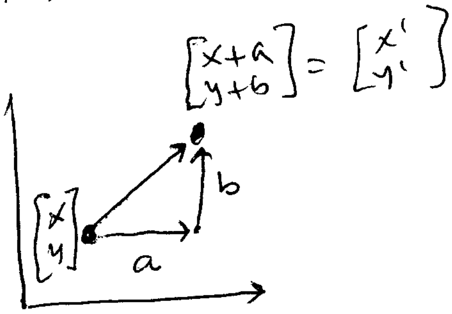
$$= r\cos\alpha\cos\theta - r\sin\alpha\sin\theta \qquad y' = r\sin\alpha\cos\theta + r\cos\alpha\sin\theta$$

$$\boxed{x' = x\cos\theta - y\sin\theta \qquad y' = y\cos\theta + \cancel{y}x\sin\theta}$$

Makes 2 equations in 2 ~~unknowns~~ variables !

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Translation:



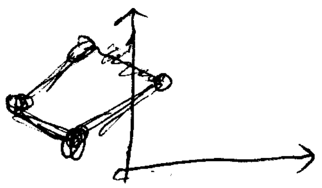
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \end{bmatrix}$$

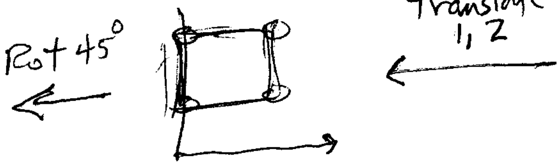
Use homogeneous coordinates so:

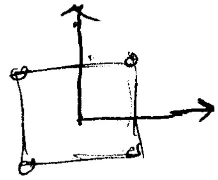
$$\begin{bmatrix} x+a \\ y+b \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$\boxed{\text{addition done as multiplication}}$

Combine transforms:





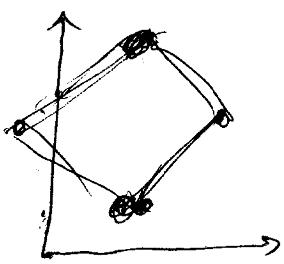


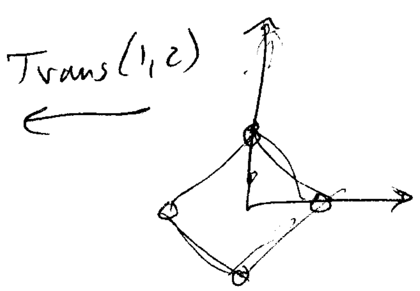
$$X = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

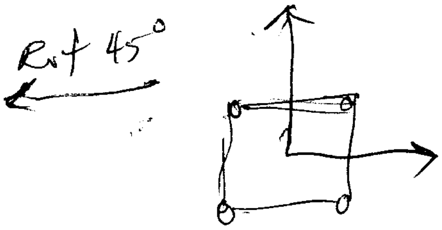
$$\begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} X \leftarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * X \leftarrow X$$

$T2*T1*X$

$\neq$







$T1*T2*X$ $\qquad$ $T2*X$ $\qquad$ X

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Scaling (not a rigid transformation)

$$\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Affine transform:

$$T = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

<u>Nonlinear Transformations</u>

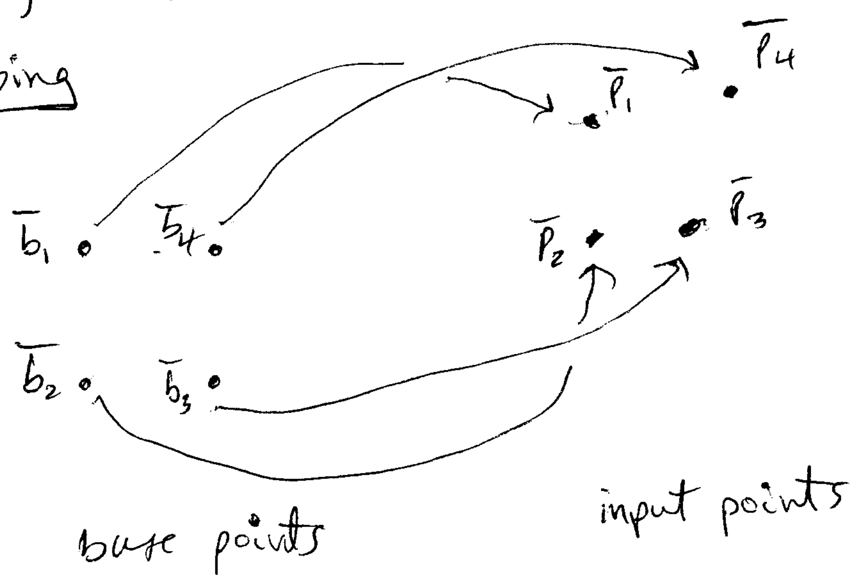
$$x' = T_x(x, y) \qquad y' = T_y(x, y)$$

E.g., 2nd order polynomial

$$x' = T_x(x,y) = a_0 x^2 + a_1 xy + a_2 y^2 + a_3 x + a_4 y + a_5$$

$$y' = T_y(x,y) = b_0 x^2 + b_1 xy + b_2 y^2 + b_3 x + b_4 y + b_5$$

<u>Warping</u>



Choose some functional form for the transformation, e.g.: cubic polynomial

$\bar{b}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $\bar{b}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ $\bar{b}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$ $\bar{b}_4 = \begin{bmatrix} x_4 \\ y_4 \end{bmatrix}$

$\bar{P}_1 = \begin{bmatrix} x_1' \\ y_1' \end{bmatrix}$ $\bar{P}_2 = \begin{bmatrix} x_2' \\ y_2' \end{bmatrix}$ $\bar{P}_3 = \begin{bmatrix} x_3' \\ y_3' \end{bmatrix}$ $\bar{P}_4 = \begin{bmatrix} x_4' \\ y_4' \end{bmatrix}$

So

$$x_i' = a_1 \cdot 1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 y^2 + a_7 yx^2 + a_8 xy^2 + a_9 x^3 + a_{10} y^3$$

$$y_i' = b_1 \cdot 1 + b_2 x + b_3 y + b_4 xy + b_5 x^2 + b_6 y^2 + b_7 yx^2 + b_8 xy^2 + b_9 x^3 + b_{10} y^3$$

Solve

$$\begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 & x_1^2 & y_1^2 & y_1 x_1^2 & x_1 y_1^2 & x_1^3 & y_1^3 \\ 1 & x_2 & y_2 & x_2 y_2 & x_2^2 & y_2^2 & y_2 x_2^2 & x_2 y_2^2 & x_2^3 & y_2^3 \\ & & & & \vdots & & & & & \\ 1 & x_n & y_n & x_n y_n & x_n^2 & y_n^2 & y_n x_n^2 & x_n y_n^2 & x_n^3 & y_n^3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix}$$

similarly for $[b_1, b_2, \ldots, b_{10}]^T$

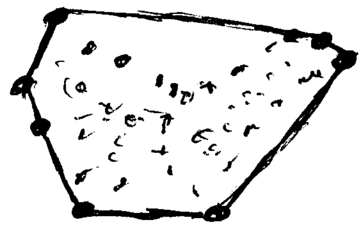
See examples for cpselect and cp2tform.

cs6640_week8

Some useful geometry

Convex Hull:

A set of points, S, is convex if for every pair of points in the set, $\bar{p}_1$ and $\bar{p}_2$, the set of points on the line segment from $\bar{p}_1$ to $\bar{p}_2$ is also in S.



} set of points

} convex hull boundary

The convex hull is the smallest convex set that contains the given set of points.

Voronoi diagram

Given a set of n points, the Voronoi diagram is a set of convex polygons such that each polygon contains exactly one generating point and every point in a given polygon is closer to its generating point than to any other

(Wolfram)

Delaunay triangulation

for a given set of points, S, in the plane, the Delaunay triangulation is one such that no point in P is inside the circumcircle of any triangle in the triangulation.

(after wikipedia)

Harris corner detector

Given an image, im, the Harris corner detector works as follows:

```
[dx, dy] = gradient(im);
[dxx, dxy] = gradient(dx);
[dyx, dyy] = gradient(dy);
```

Then, at every pixel (r, c) in im:

* form $H = \begin{bmatrix} dxx(r,c) & dxy(r,c) \\ dxy(r,c) & dyy(r,c) \end{bmatrix}$

* Find eigenvectors and eigenvalues

  [V, D] = eigs(H)

* if $|D(1,1)|$ and $|D(2,2)|$ are > 0

  corner exists