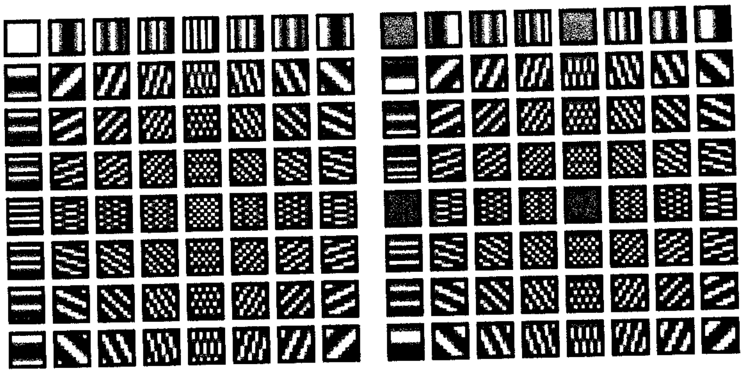
$$
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \omega & -j & -j\omega & -1 & -\omega & j & j\omega \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -j\omega & j & \omega & -1 & j\omega & -j & -\omega \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -\omega & -j & j\omega & -1 & \omega & j & -j\omega \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & j\omega & j & -\omega & -1 & -j\omega & -j & \omega
\end{bmatrix}
$$



c

...6.7 (a) Tranformation matrix $\mathbf{A}_F$ of the discrete Fourier transform for $N = 8$, where $\omega = e^{-j2\pi/8}$ or $(1-j)/\sqrt{2}$. ...nd (c) The real and imaginary parts of the DFT basis images of size 8×8. For clarity, a black border has been ...d around each basis image. For 1-D transforms, matrix $\mathbf{A}_F$ is used in conjunction with Eqs. (6-43) and (6-44); ...-D transforms, it is used with Eqs. (6-41) and (6-42).

than complex-valued—the *discrete Hartley transform*, *discrete cosine transform*, and *discrete sine transform*. All three transforms avoid the computational complexity of complex numbers and can be implemented via fast FFT-like algorithms.

THE DISCRETE HARTLEY TRANSFORM

The transformation matrix of the *discrete Hartley transform* (DHT) is obtained by substituting the inverse transformation kernel

...ion cas, an acronym ...cosine-and-sine ...on, is defined as ...$= \cos(\theta) + \sin(\theta)$.

$$
\begin{aligned}
s(x,u) &= \frac{1}{\sqrt{N}} \operatorname{cas}\left(\frac{2\pi ux}{N}\right) \\
&= \frac{1}{\sqrt{N}} \left[\cos\left(\frac{2\pi ux}{N}\right) + \sin\left(\frac{2\pi ux}{N}\right) \right]
\end{aligned}
\tag{6-78}
$$

whose separable 2-D counterpart is

...ll not consider ...on-separable form ...$(u,v) =$ ...$\operatorname{cas}\left(\frac{2\pi(ux+vy)}{N}\right)$.

$$
s(x,y,u,v) = \left[\frac{1}{\sqrt{N}} \operatorname{cas}\left(\frac{2\pi ux}{N}\right) \right] \left[\frac{1}{\sqrt{N}} \operatorname{cas}\left(\frac{2\pi vy}{N}\right) \right] \tag{6-79}
$$

into Eqs. (6-22) and (6-24). Since the resulting DHT transformation matrix—denoted $\mathbf{A}_{HY}$ in Fig. 6.8—is real, orthogonal, and symmetric, $\mathbf{A}_{HY} = \mathbf{A}_{HY}^T = \mathbf{A}_{HY}^{-1}$ and $\mathbf{A}_{HY}$ can be used in the computation of both forward and inverse transforms. For 1-D transforms, $\mathbf{A}_{HY}$ is used in conjunction with Eqs. (6-28) and (6-29) of Section 6.2; for 2-D transforms, Eqs. (6-35) and (6-36) are used. Since $\mathbf{A}_{HY}$ is symmetric, the forward and inverse transforms are identical.