

- Summary of Fourier Transform

for  $u=0:M-1$  and  $v=0:N-1$

$$F(u,v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-i2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)}$$

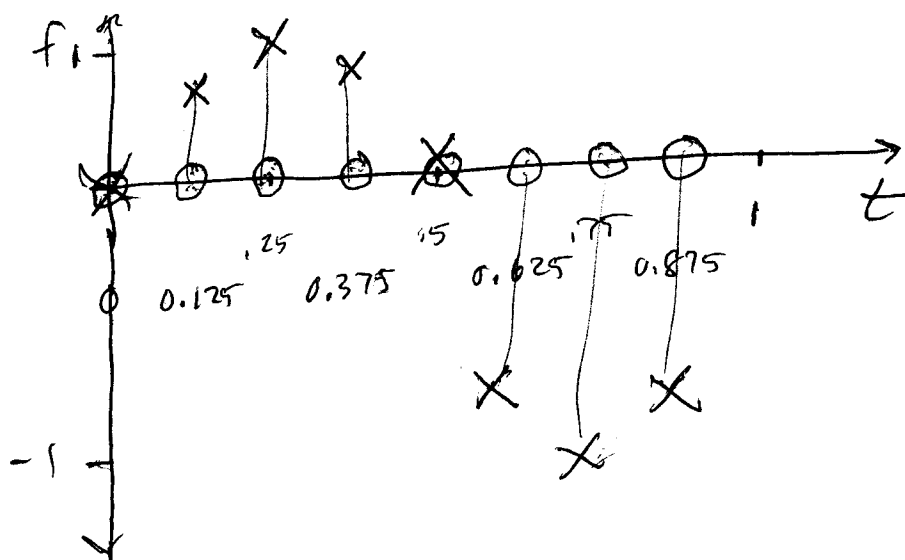
$$f(x,y) = \frac{1}{\sqrt{MN}} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{i2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)}$$

Consider a simple function in 1D:

$$f(x) = \sin(2\pi x)$$

(see CS6640-Week6.m)

sample at 8 points across  $[0, 2\pi]$



$$f = [0.7071, 1, 0.7071, 0, -0.7071, -1, -0.7071]$$

Then :

$$F(u) = \begin{cases} \sum_{x=0}^7 f(x) e^{-i2\pi u x / 8} & \leftarrow \text{Matlab} \\ \frac{1}{2.8284} \sum_{x=0}^7 f(x) e^{-i2\pi u x / 8} \end{cases}$$

depending on implementation

$\frac{1}{\sqrt{M}}$

$$\text{let } \bar{B}_u = [e^{-i2\pi u \cdot 0 / 8}, e^{-i2\pi u \cdot 1 / 8}, \dots, e^{-i2\pi u \cdot 7 / 8}]^T$$

$$\text{then } F(u) = \frac{1}{2.8284} \bar{f} \cdot \bar{B}_u$$

This coefficient is the projection of  $\bar{f}$  onto  $\bar{B}_u$

$$\text{e.g.) } \bar{B}_0 = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]^T$$

$$\bar{f} \cdot \bar{B}_0 \approx 0 = \sum f = \sum \begin{cases} 0, 0 \\ 0.7071, 0.7071 \\ 1 \\ -0.7071, -0.7071 \\ -1 \end{cases}$$

In general, the basis vectors are complex

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Matrix form:

$$F \begin{bmatrix} x \\ 0 \\ \vdots \\ 0 \end{bmatrix} = u$$

$A$

$M-1$

$$f \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$M-1$

$$F \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7071 \\ 1 \\ 0.7071 \\ 0 \\ -0.7071 \\ -1 \\ -0.7071 \end{bmatrix}$$

P2

Maps

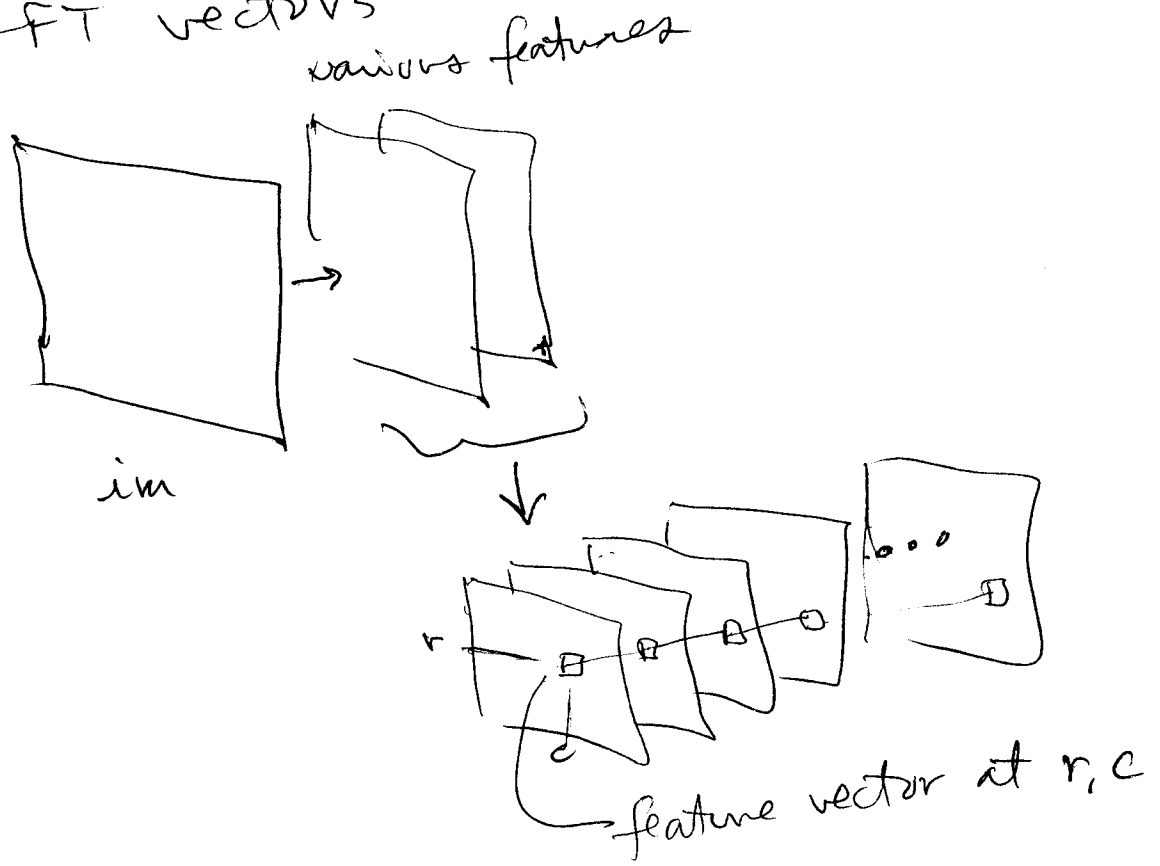
find edges + orientations + use properties

$$[dx, dy] = \text{gradient}(\text{double}(\text{map} | g));$$

$$\text{mag} = \text{sqrt}(dx.^2 + dy.^2);$$

$$\text{ori} = \text{atan2}(dy, dx);$$

- feature vectors (to be clustered by Kmeans)
- \* use mag histogram of subwindow around each pixel
- \* " ori " "
- \* find representative vectors + use Euclidean distance to classify
- \* Laws vectors
- \* FFT vectors



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# bottle inspection

e.g., cap detection:

\* use window around pixel in  $r, g + b$  channels

