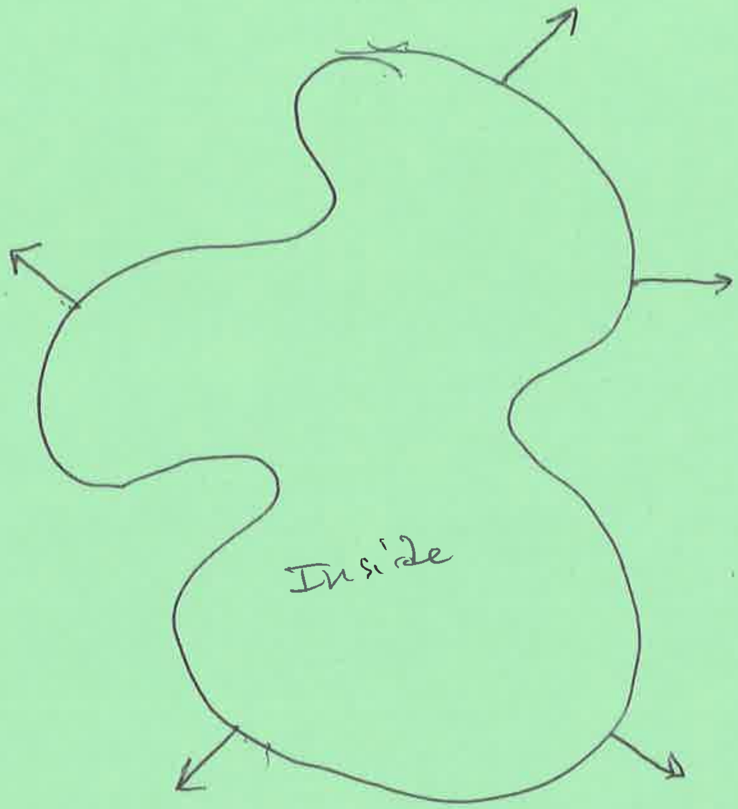
CS 6640, Week 12   5 Nov '18   (from Sethian)



independent props
e.g., transport motion

$F = F(L, G, I)$

L: local properties
curvature, normal

G: global properties
integrals/ diff eqn's
along front

outside

boundary : curve in 2D

moves in normal direction with speed F

goal: track motion

Assume F known

Assume $F > 0$ (front moves outward)

compute arrival time $T(x,y)$

in 1-D $\quad 1 = F\frac{dT}{dx}$ $\quad \begin{cases} d = rt \\ x = F\,T(x) \\ \frac{dx}{dx} = F\,\frac{T(x)}{dx} \end{cases}$

$>$ 1-D $\quad 1 = F\,|\nabla T| \quad T = 0$ on Γ

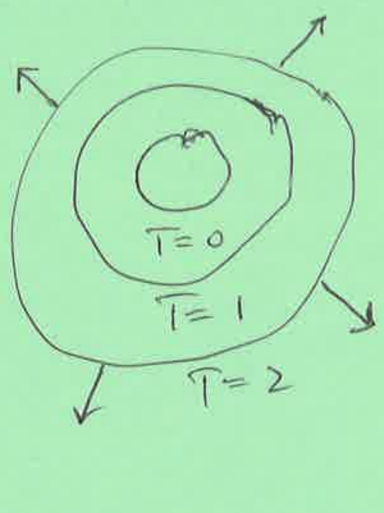
$\uparrow$

initial boundary

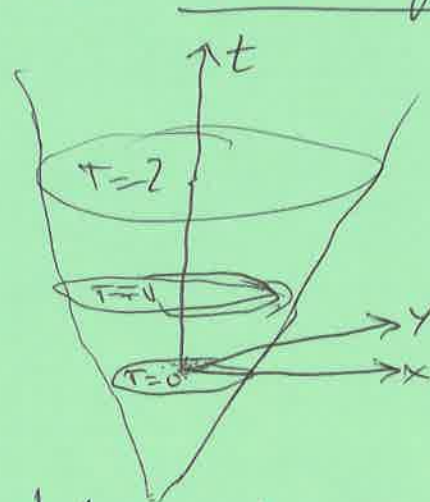
<u>boundary value problem</u>

if F depends only on position, then <u>Eikonal eqn</u>

Consider circle:



since F may be $\pm$ embed in t space



$\phi(x(t), t) = 0$

zero level set of ϕ is boundary

$$\phi_t + \nabla\phi(x(t), t) \cdot x'(t) = 0$$ by chain rule

Let the normal to curve be:

$$n = \frac{\nabla\phi}{|\nabla\phi|}$$

Then $x'(t) \cdot n = F$

Results in

$$\phi_t + F|\nabla\phi| = 0$$

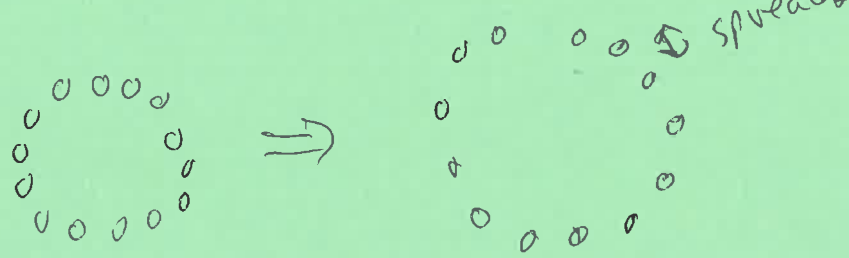
given $\phi(x, t=0)$

Summary

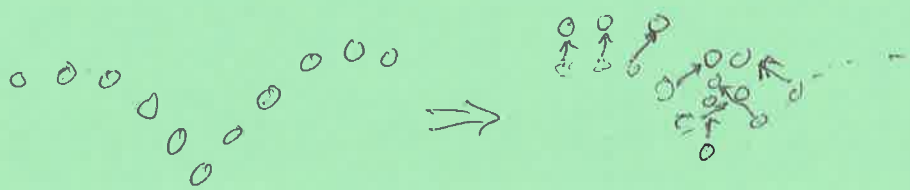
Let Γ be a curve in 2D propagating with speed $F \ni \Gamma(t)$ gives position of front at t

Boundary Value Formulation	Initial Value Formulation
$\lvert\nabla T\rvert F = 1$	$\phi_t + F\lvert\nabla\phi\rvert = 0$
Front $= \Gamma(t) = \{(x,y) \mid T(x,y) = t\}$	Front $= \Gamma(t) = \{(x,y) \mid \phi(x,y,t) = 0\}$
$F > 0$	arbitrary F

Contrast to tracking points



also, interacting points



Also, accurate computation:

h: mesh spacing

(i,j): grid node locations

$\phi^n_{ij} \approx \phi(ih, jh, n\Delta t)$

$$\frac{\phi^{n+1}_{ij} - \phi^{n}_{ij}}{\Delta t} + F\,|\underbrace{\nabla_{ij}\phi^{n}_{ij}}_{\text{gradient}}| = 0$$

$$\bar{n} = \frac{\nabla\phi}{|\nabla\phi|} \qquad \bar{n} = \frac{\nabla T}{|\nabla T|}$$

$$k = \begin{cases} \nabla\cdot\frac{\nabla\phi}{|\nabla\phi|} = \phi_{xx}\phi_y^2 - 2\phi_x\phi_y\phi_{xy} + \phi_{yy}\phi_x^2 / (\phi_x^2+\phi_y^2)^{3/2} \\ \nabla\cdot\frac{\nabla T}{|\nabla T|} = T_{xx}T_y^2 - 2T_xT_yT_{xy} + T_{yy}T_x^2 / (T_x^2+T_y^2)^{3/2} \end{cases}$$