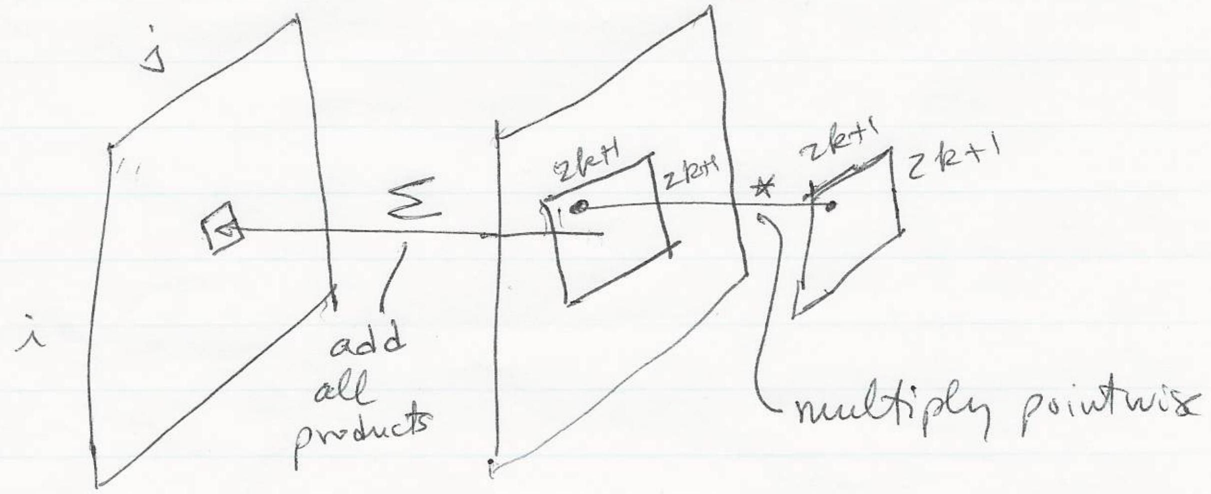
Chapter 4 Linear Filters

Simple view:

- An image is a 2D array of discrete values
- A mask is given which is a $2k+1 \times 2k+1$ array



$$R(i,j) = \sum_{u=i-k}^{i+k} \sum_{v=j-k}^{j+k} H(u-(i-k)+1, v-(j-k)+1)\, F(u,v)$$

The average value at a pixel is given by using:

H = ones(2*k+1, 2*k+1)/(2*k+1)^2

<u>shift invariant</u>: depends on region, not location

<u>linear</u>: convolve (F+G) = conv(F)+conv(G)

apply filter

<u>linear filtering</u>

window: kernel

blurring with Gaussian

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{x^2+y^2}{2\sigma^2}\right) \qquad \mathbb{R}^2$$

σ is standard deviation units: pixels

discrete version

$$H_k(i,j) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{((i-k-1)^2+(j-k-1)^2)}{2\sigma^2}\right)$$

for many kernels, use Matlab fspecial
e.g.,

```
H = fspecial('gaussian', 21, 2)
surf (H)     % looks like p.109
```

Derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon,y)-f(x,y)}{\varepsilon}$$

Set ε to 1 $\approx \dfrac{f(x+1,y)-f(x,y)}{1}$

better to use central symmetric difference:

$$f(x+1,y)-f(x-1,y)$$

& use kernel $H = [-1 \;\; 0 \;\; +1]$ (book has it backward)

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There is a function to apply kernels to images:

filter2 (H, F)

try on trees $\quad H = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

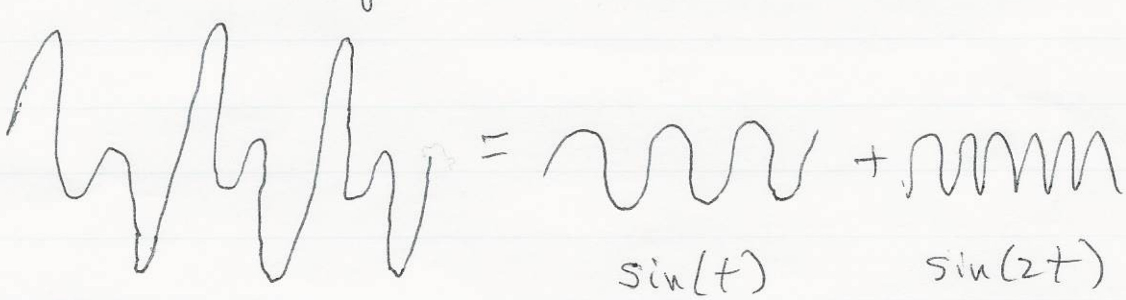
<u>Shift Invariant Linear Systems</u>

book shows that response of a system (camera) to any signal can be characterized by its response to a simple input (impulse response)

(Image Processing goes deeper into the math of this and Fourier Transforms)

<u>Simple use of Fourier Transform</u>

Given a function $f(x)$, it can be represented as a sum of basis functions: (sines, cosines) of various frequencies.



FT helps find which basis functions contribute to function (see Matlab)

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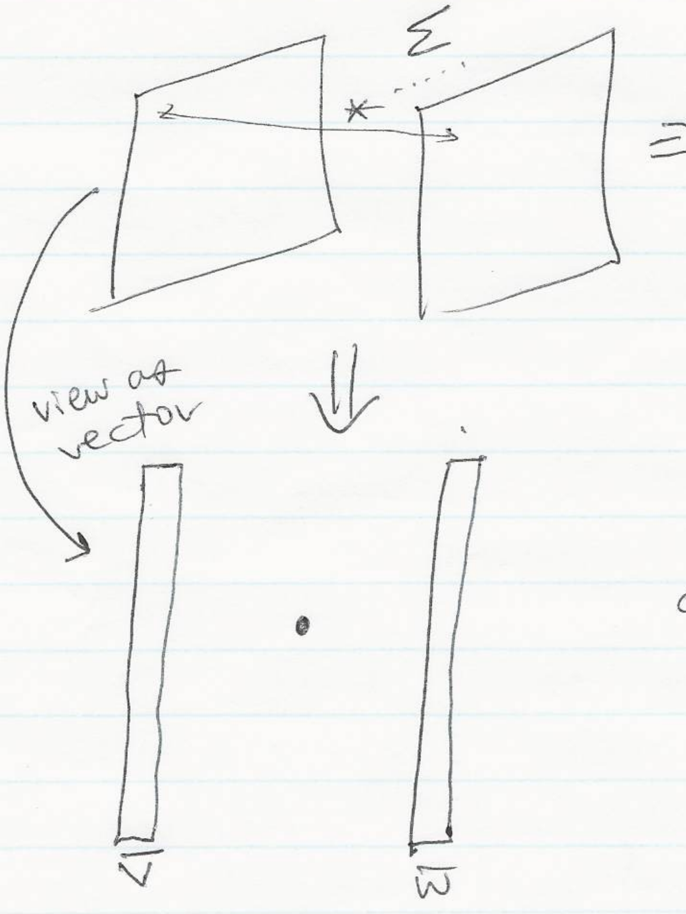
FT can be zeroed out for higher frequencies in order to smooth the image (or signal)

Filters as templates



dot product

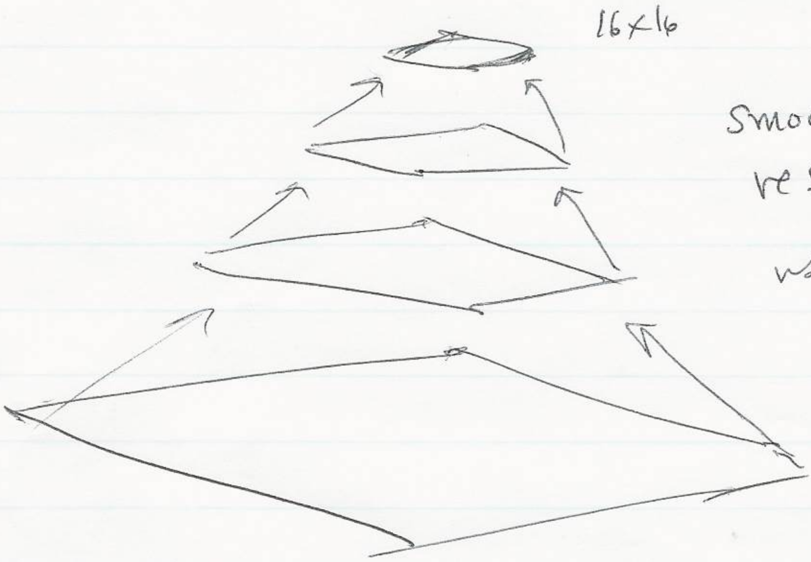
$\Longrightarrow$ if $\bar{v}$ + $\bar{w}$ are similar then:

$$\cos\theta = \underbrace{\frac{\bar{v}\cdot\bar{w}}{\|\bar{v}\|\|\bar{w}\|}}$$

is near 1 when similar
near −1 when opposite

See Matlab

Gaussian Pyramid



$1 < \sigma < 2$

smooth & resample

1/2 size

use nearest $2^n \times 2^n$