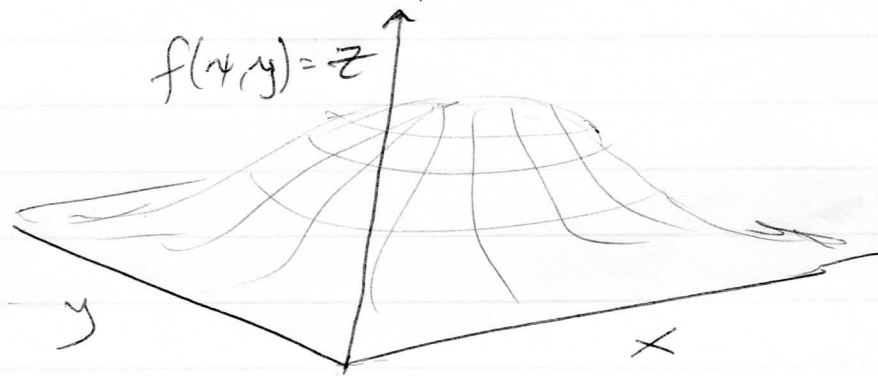
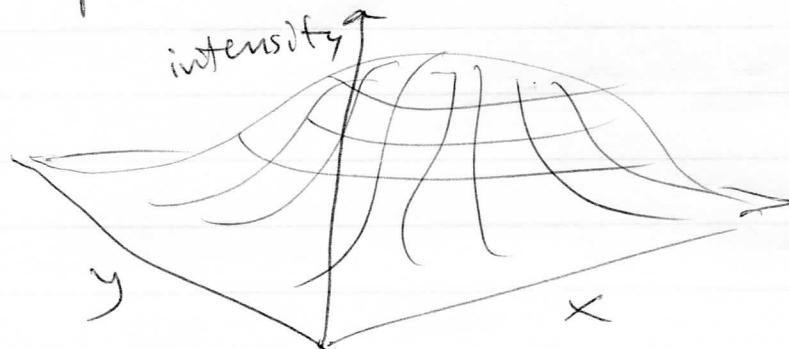


Photometric Stereo

use multiple illuminations to recover ~~the~~ reflectance ~~radiance~~
 Build a depth map for surface:



Note, an intensity image may have the same (qualitative) function:



illuminate surface from different locations

$$B(\bar{x}) = \rho(\bar{x}) \bar{N}(\bar{x}) \cdot \bar{S}_i$$

$$I(x, y) = k B(\bar{x}) = k B(x, y)$$

$$= k \rho(x, y) \bar{N}(x, y) \cdot \bar{S}_i$$

$$= \bar{g}(x, y) \cdot \bar{V}_i \quad \Rightarrow \quad \bar{g}(x, y) = \rho(x, y) \bar{N}(x, y)$$

$$\bar{V}_i = k \bar{S}_i$$

5 images p. 48 of hemisphere

For n sources, \bar{V}_i

$$V = \begin{bmatrix} \bar{V}_1^T \\ \bar{V}_2^T \\ \vdots \\ \bar{V}_n^T \end{bmatrix}$$

For each image point, create measurement vector:

$$\bar{i}(x, y) = [I_1(x, y); I_2(x, y); \dots; I_n(x, y)]$$

So,

$$\bar{i}(x, y) = V \bar{g}(x, y)$$

Obtain \bar{g} by solving system.

Then, albedo \Rightarrow $\rho(x, y) = |\bar{g}(x, y)|$
 $0 \leq \rho \leq 1$ else error

$$\bar{N}(x, y) = \frac{\bar{g}(x, y)}{|\bar{g}(x, y)|}$$

Surface is: $(x, y, f(x, y))$, so

$$\bar{N}(x, y) = \frac{1}{\sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}} \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ 1 \end{bmatrix}$$

Assume $\vec{N}(x,y) = \begin{bmatrix} a(x,y) \\ b(x,y) \\ c(x,y) \end{bmatrix}$

Then:

$$\frac{\partial f}{\partial x} = \frac{a(x,y)}{c(x,y)} \quad \frac{\partial f}{\partial y} = \frac{b(x,y)}{c(x,y)}$$

Can check that data is good since

$$\frac{\partial^2 f}{\partial x \partial y} \text{ should } = \frac{\partial^2 f}{\partial y \partial x}$$

or
$$\frac{\partial \left(\frac{a(x,y)}{c(x,y)} \right)}{\partial y} = \frac{\partial \left(\frac{b(x,y)}{c(x,y)} \right)}{\partial x}$$

$$f(x,y) = \int_C \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \cdot d\vec{l} + C$$

