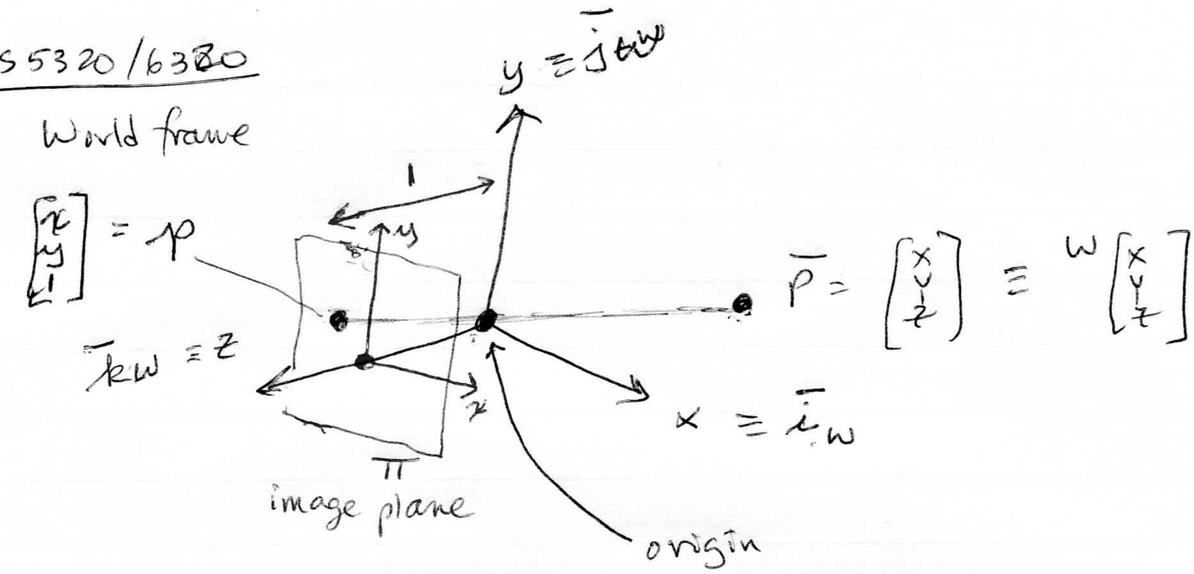
CS 5320/6380

World frame



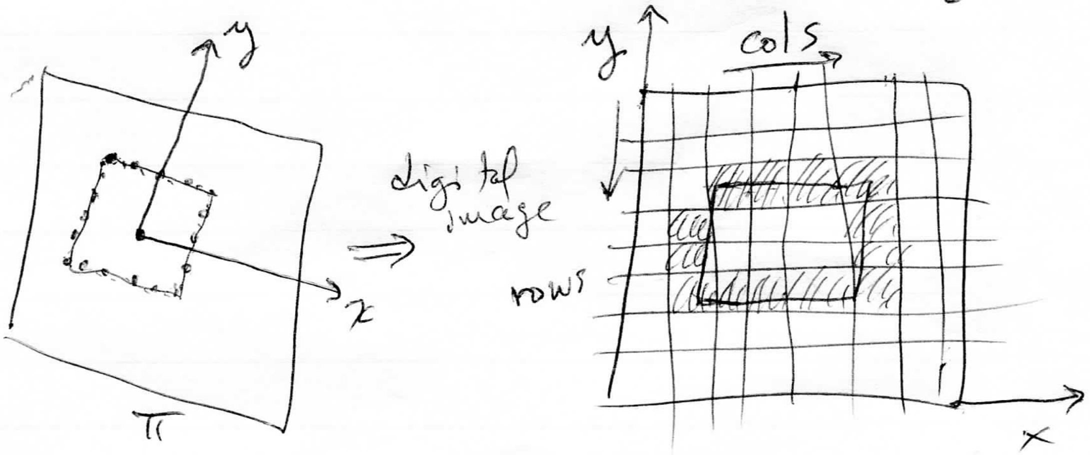
a point in world frame: $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \bar{P} \equiv X\bar{i}_w + Y\bar{j}_w + Z\bar{k}_w$

Put a camera

Point projects through optic center (pinhole) to π: to point $p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv {}^w\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

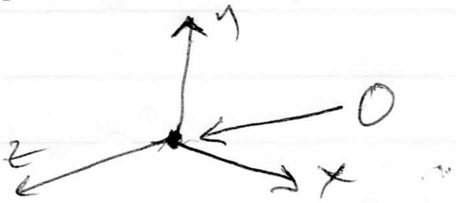
Note: $\overrightarrow{Op} = \lambda\, \overrightarrow{OP}$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda X \\ \lambda Y \\ \lambda Z \end{bmatrix} \Rightarrow \frac{x}{X} = \frac{y}{Y} = \frac{1}{Z} \Rightarrow x = \frac{X}{Z}, \quad y = \frac{Y}{Z}$$



Frames and Coordinates

frame: point + x, y, z directions ($\bar{i}, \bar{j}, \bar{k}$)
origin — names of directions — vectors



any point $\bar{P} = x\bar{i} + y\bar{j} + z\bar{k} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

transformation: rigid motion of points

* translate $\bar{P} + \bar{t}$ $\quad t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$

* rotate: use matrix

e.g., to rotate θ degrees about z-axis

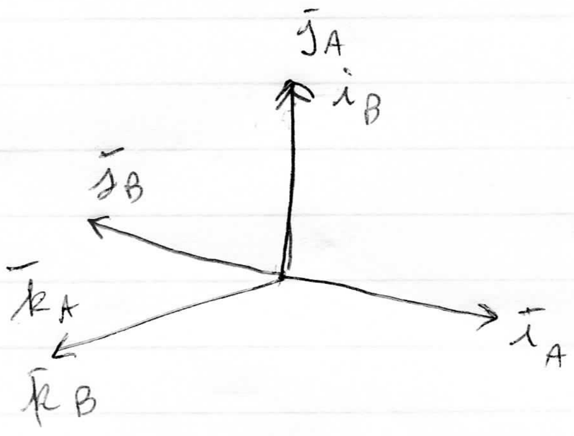
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Try 90 degrees: $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

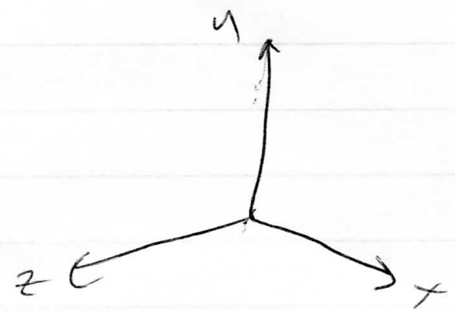
Given 2 frames $\bar{i}_A, \bar{j}_A, \bar{k}_A$ & $\bar{i}_B, \bar{j}_B, \bar{k}_B$

$R = \begin{bmatrix} {}^A\bar{i}_B & {}^A\bar{j}_B & {}^A\bar{k}_B \end{bmatrix} = R_B^A$ maps ${}^A p$ to ${}^B p$

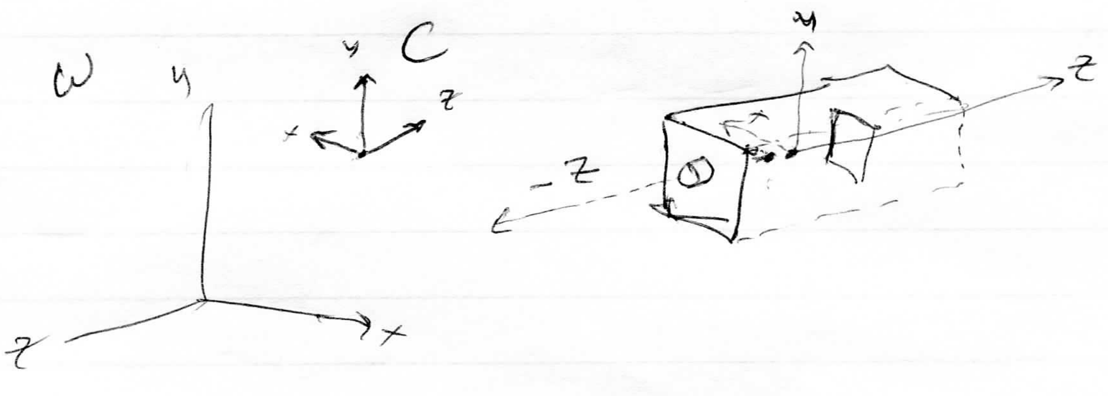
$R_B^A \, {}^B p = {}^A p$



We're doing this because the camera has a frame:



which will be moved around in space:



Suppose $\quad {}^{A}\bar{i}_B = \bar{j}_A \qquad {}^{A}\bar{j}_B = -\bar{i}_A \qquad {}^{A}\bar{k}_B = \bar{k}_A$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_B^A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_B^A \overset{\bar{i}_B}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} = \overset{\bar{j}_A}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} \qquad R_B^A \overset{\bar{j}_B}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} = \overset{-\bar{i}_A}{\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}}$$

Combine translate & rotate:

$${}^{A}\bar{p} = R_B^A \, {}^{B}\bar{p} + \bar{t}$$

Using homogeneous coordinates,

$${}^{A}\bar{p} = T_B^A \, {}^{B}\bar{p}$$

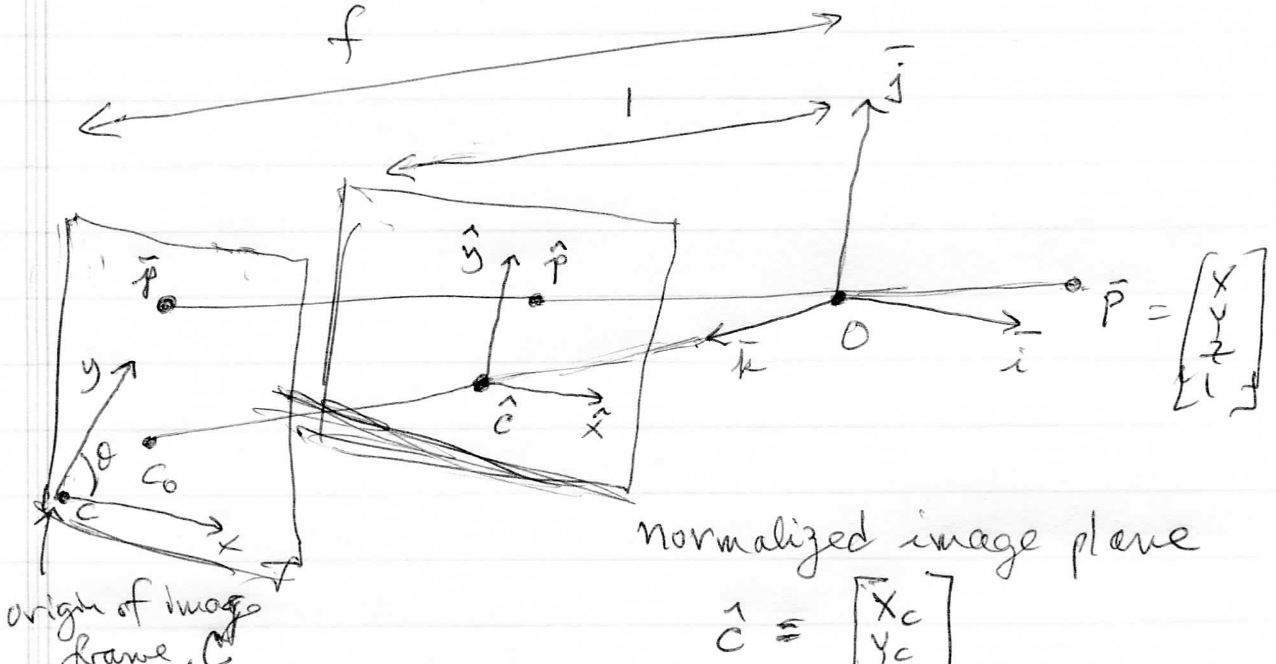
where $\quad T_B^A = \begin{bmatrix} R_B^A & \bar{t} \\ \bar{0} & 1 \end{bmatrix}$ (4, 4) $\quad$ 4×4

need points with homogeneous coord

We use this representation to describe image formation:

$$\bar{p} = \frac{1}{Z} M \bar{P}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



$$\hat{c} = \begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix}$$

Then:

$$\hat{x} = \frac{X}{Z} \iff \hat{p} = \frac{1}{Z}\left(Id \;\; \bar{0}\right)\bar{P} \qquad (1.11)$$

$$\hat{y} = \frac{Y}{Z}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$f \equiv$ focal length e.g., meters

pixels may be rectangular, so have dimension $\frac{1}{k} \times \frac{1}{l}$

$k, l \quad \frac{\text{pixel}}{m}$

$$x = kf\frac{X}{Z} = kf\hat{x}$$

$$y = kf\frac{Y}{Z} = lf\hat{y}$$

Can use $\alpha = kf \quad \beta = lf$ in pixel units

$C_0 = [x_0, y_0]$ is image frame

(11) becomes

$$x = \alpha\hat{x} + x_0 \qquad (1.12)$$

$$y = \beta\hat{y} + y_0$$

The image axes may be skewed, i.e., $\theta \neq 90°$

$$x = \alpha \hat{x} - \alpha \cot\theta \, \hat{y} + x_0$$

$$y = \frac{\beta}{\sin\theta} \hat{y} + y_0$$

point in image coords

$$\bar{p} = K \hat{p}$$

$$\underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha\cot\theta & x_0 \\ 0 & \frac{\beta}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{calibration matrix internal}} \begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix}$$

$$\bar{p} = \frac{1}{Z} K (Id \; \bar{O}) \bar{P} = \frac{1}{Z} M \bar{P}$$

$$M = (K \; \bar{O}) = \begin{bmatrix} \alpha & -\alpha\cot\theta & x_0 & 0 \\ 0 & \beta/\sin\theta & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\alpha, \beta, \theta, x_0, y_0$ are the intrinsic parameters of the camera

E.g. let $\alpha = \beta = 1$ $\theta = 90$ $x_0 = 1$ $y_0 = 1$ $\quad P = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

$$\bar{p} = \frac{1}{-1} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

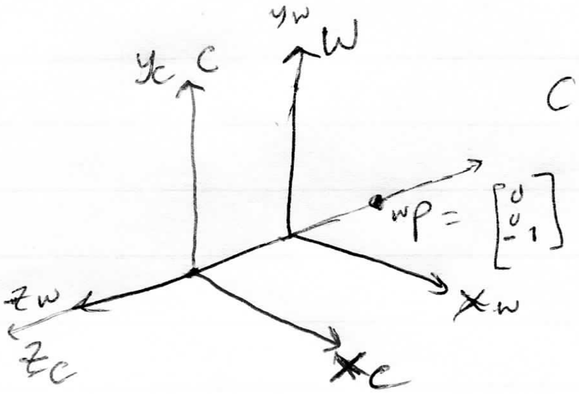
Extrinsic parameters

If $\hat{p}$ is known, then $\bar{p} = K\hat{p}$
so need to transform ${}^{w}\bar{p}$ into ${}^{c}\bar{p}$

Suppose the camera is rotated and translated, then its frame is:

$$C \equiv T_c = \begin{bmatrix} R & \bar{t} \\ \bar{0} & 1 \end{bmatrix} \quad \text{a } 4 \times 4$$

E.g.:



$$C \equiv T_c = \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 1 \\ & & & 1 \end{bmatrix}$$

$\bar{t}$

$R = I_{3\times3}$

$\bar{t} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Then a point in world coords is transformed to camera coords as:

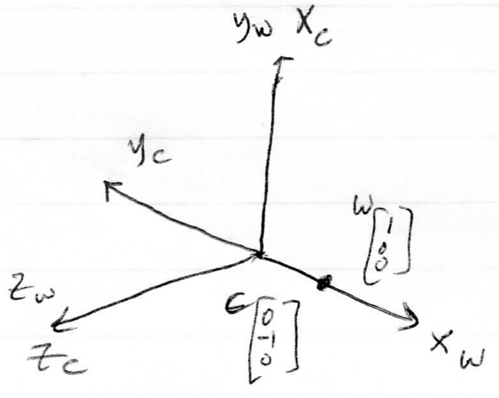
$${}^{c}p = R^{T}({}^{w}p - \bar{t}) = T^{-1}\,{}^{w}p$$

Consider ${}^{w}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ then ${}^{c}\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} = I_{3\times3}\left(\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) =$

Also,

$$\begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & -1 \\ & & & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Consider a simple rotation: $R_z(\frac{\pi}{2}) \equiv \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



$C \equiv T_c = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

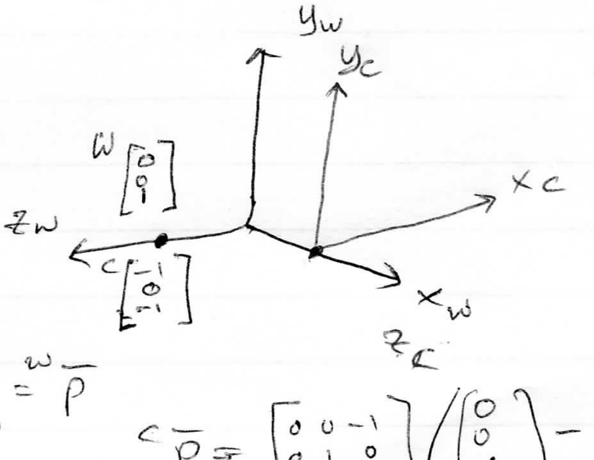
$R^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Consider ${}^{w}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = {}^{w}\bar{p}$

$${}^{c}\bar{p} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$${}^{c}\bar{p} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Now, bring it to full rotate and translate:
rotate about y $90°$, then translate in x by 1.



$C \equiv T = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T^{-1} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Given ${}^{w}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = {}^{w}\bar{p}$

$${}^{c}\bar{p} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Camera performs:

$$\bar{p} = \frac{1}{Z} M \bar{P} \qquad M = K(R \;\; \bar{t})$$

$\uparrow$ is in camera coords

$$\bar{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{m}_1 \cdot \bar{P}}{\bar{m}_3 \cdot \bar{P}} \\ \dfrac{\bar{m}_2 \cdot \bar{P}}{\bar{m}_3 \cdot \bar{P}} \\ 1 \end{bmatrix}$$

Let $R = [\bar{r}_1^T ; \bar{r}_2^T ; \bar{r}_3^T]$

Then

$$M = \begin{bmatrix} \alpha \bar{r}_1^T - \alpha \cot\theta \, \bar{r}_2^T + x_0 \bar{r}_3^T & \alpha t_1 - \alpha \cot\theta \, t_2 + x_0 t_3 \\ \dfrac{\beta}{\sin\theta} \bar{r}_2^T + y_0 \bar{r}_3^T & \beta/\sin\theta \, t_2 + y_0 t_3 \\ \bar{r}_3^T & t_3 \end{bmatrix}$$

Use this for CS5320_camera

Camera Calibration

Given a correspondence between a set of image points $\{x_i, y_i\}$ and world points $\bar{P}_i$

calibration means: find parameters $\xi \ni$

$$x_i = \frac{\bar{m}_1(\xi)\cdot \bar{P}_i}{\bar{m}_3(\xi)\cdot \bar{P}_i}$$

$$y_i = \frac{\bar{m}_2(\xi)\cdot \bar{P}_i}{\bar{m}_3(\xi)\cdot \bar{P}_i}$$

ξ : $\alpha, \beta, \theta, x_0, y_0, R, \bar{t}$

$\bar{m}_i^T$ is ith row of M

Each equation above yields one equation in $\bar{m}_1, \bar{m}_2, \bar{m}_3$

$$x_i = \frac{\bar{m}_1 \bar{P}_i}{\bar{m}_3 P_i} \Rightarrow x_i \bar{m}_3 \cdot P_i = \bar{m}_1 \cdot P_i \Rightarrow \bar{m}_1 P_i - x_i \bar{m}_3 \cdot P_i = 0$$

$$\Rightarrow (m_1 - x_i \bar{m}_3)\cdot P_i = 0 \Rightarrow \bar{P}_i^T \bar{m}_1 + \bar{0}^T \bar{m}_2 - x_i \bar{P}_i^T \bar{m}_3 = 0$$

$$\begin{bmatrix} P_{i1} & P_{i2} & P_{i3} & 1 & 0 & 0 & 0 & 0 & -x_i P_{i1} & -x_i P_{i2} & -x_i P_{i3} & -x_i \end{bmatrix} \bar{m}$$

$$\bar{m} = \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ \vdots \\ m_{34} \end{bmatrix}$$

Solving:

Put all equations in matrix P

$$P\bar{m} = 0 \qquad + \text{ solve}$$

Find eigenvalues of $P^T P$
then eigenvector associated with smallest eigenvalue
is $\bar{m}$ solution

```
[V,D] = eigs(P'P, 12)
[vals,indexes] = sort(diag(D));
index = indexes(1)
M = [V(1:4,index)'; V(5:8,index)'; V(9:12,index)'];
```

Now, write $M = (A\ \bar{b})$
and

$$\bar{a}_1^T = A(1,1{:}3) \quad \bar{a}_2^T = A(2,1{:}3) \quad \bar{a}_3^T = A(3,1{:}3)$$

Use equations p. 26 to get calibration parameters:

$$\rho = 1/\|\bar{a}_3\|$$

$$\bar{r}_3 = \rho\bar{a}_3$$

$$x_0 = \rho^2(\bar{a}_1 \cdot \bar{a}_3)$$

$$y_0 = \rho^2(\bar{a}_2 \cdot \bar{a}_3)$$

$$\theta = \cos^{-1}\left(-\frac{(\bar{a}_1 \times \bar{a}_3)\cdot(\bar{a}_2 \times \bar{a}_3)}{\|\bar{a}_1 \times \bar{a}_3\|\|\bar{a}_2 \times \bar{a}_3\|}\right)$$

$$\alpha = \rho^2\|\bar{a}_1 \times \bar{a}_3\| \sin\theta$$

$$\beta = \rho^2\|\bar{a}_2 \times \bar{a}_3\| \sin\theta$$

$$r_1 = \frac{\bar{a}_2 \times \bar{a}_3}{\|\bar{a}_2 \times \bar{a}_3\|}$$

$$r_2 = \bar{r}_3 \times \bar{r}_1$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & x_0 \\ 0 & \beta/\sin\theta & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{t} = \rho K^{-1}\bar{b}$$

where

$$\bar{b} = M(1:3,4)$$

Show im =

[alpha, ...