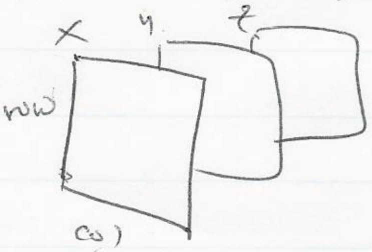
Chapter 14 Range Data

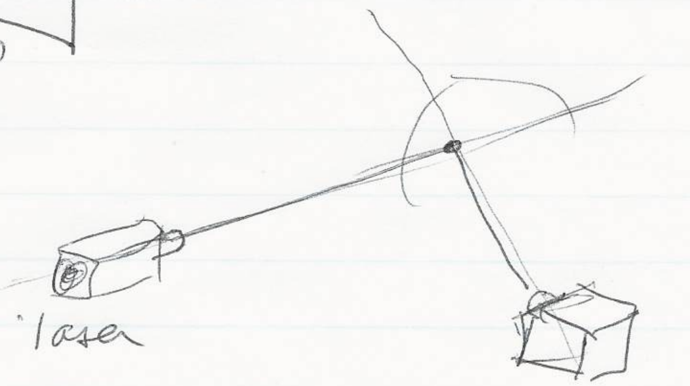
depth maps: depth (row, col)

3D points:



Sensors

Triangulation:

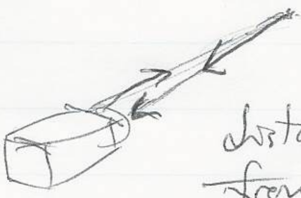


intersect 2 lines

Patterns

use pattern + camera parameters

Time-of-Flight



distance derived from time of flight

All Required functions

CS5320_range2pts (range_im)

depth image (m×n×1) → x,y,z (m×n×3)
use columns & channel 1
num rows − rows +1 : channel 2
depth : channel 3

CS5320-18April2016 All

CS5320_normals (pts_im, k)

points (m×n×3) → normals (m×n×3)
PCA vs cross products vs. LS

CS5320_planes (pts_im, normals, k)

→ planes (m×n×5) a b c d error

PCA

CS5320_topo (pts_im, normals, planes, k)

→ (m×n×7) probability
of each topo class

Range Data Segmentation

based on analytical differential geometry

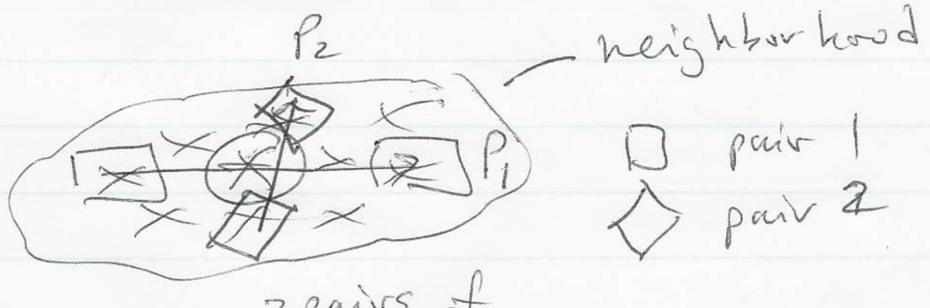
a surface $\bar{x} : U \subset \mathbb{R}^2 \to \mathbb{R}^3$

$$\bar{x}(u,v) = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\bar{x}_u \equiv \frac{\partial \bar{x}}{\partial u} \qquad \bar{x}_v \equiv \frac{\partial \bar{x}}{\partial v}$$

$$\bar{N} = \frac{1}{\| \bar{x}_u \times \bar{x}_v \|} (\bar{x}_u \times \bar{x}_v)$$ surface normal

How can we find a normal vector at a point in a ~~depth~~ range image?



method 1 : Find 2 pairs of neighbors each pair is on opposite sides of pt & 2 pairs are roughly orthogonal

take cross product (direction of normal)?

method 2 : Fit a plane $ax + by + cz + d = 0$

which way does normal face? ? ~~see lecture notes~~ Chapter ~~10, p. 4~~

$[a, b, c]' = \bar{N}$

$$(1)\ a\overline{x^2} + b\,\overline{xy} + c\,\overline{x} = \lambda 2a$$
$$(2)\ a\,\overline{xy} + b\,\overline{y^2} + c\,\overline{y} = \lambda 2b$$

substitute

$$(3)\ a\overline{x} + b\overline{y} + c = 0 \Rightarrow c = -(a\overline{x} + b\overline{y})$$

$$(1')\ a\overline{x^2} + b\,\overline{xy} + (-(a\overline{x} + b\overline{y}))\overline{x} = 2\lambda a$$
$$(2')\ a\,\overline{xy} + b\,\overline{y^2} + (-(a\overline{x} + b\overline{y}))\overline{y} = 2\lambda b$$

$$\Rightarrow\ a\overline{x^2} + b\overline{xy} - a\overline{x}\,\overline{x} - b\overline{x}\,\overline{y} = 2\lambda a$$
$$a\overline{xy} + b\overline{y^2} - a\overline{x}\,\overline{y} - b\overline{y}\,\overline{y} = 2\lambda b$$

$$\Rightarrow\ a(\overline{x^2} - \overline{x}\,\overline{x}) + b(\overline{xy} - \overline{x}\,\overline{y}) = \mu a$$
$$a(\overline{xy} - \overline{x}\,\overline{y}) + b(\overline{y^2} - \overline{y}\,\overline{y}) = \mu b$$

(1) $a\overline{x^2} + b\overline{xy} + c\overline{xz} + (-a\bar{x} - b\bar{y} - c\bar{z})\bar{x} = 2\lambda a$

(2) $a\overline{xy} + b\overline{y^2} + c\overline{yz} + (-a\bar{x} - b\bar{y} - c\bar{z})\bar{y} = 2\lambda b$

(3) $a\overline{xz} + b\overline{yz} + c\overline{z^2} + (-a\bar{x} - b\bar{y} - c\bar{z})\bar{z} = 2\lambda c$

(1') $a(\overline{x^2} - \bar{x}\bar{x}) + b(\overline{xy} - \bar{y}\bar{x}) + c(\overline{xz} - \bar{z}\bar{x}) = 2\lambda a$

(2') $a(\overline{xy} - \bar{x}\bar{y}) + b(\overline{y^2} - \bar{y}\bar{y}) + c(\overline{yz} - \bar{y}\bar{z}) = 2\lambda b$

(3') $a(\overline{xz} - \bar{x}\bar{z}) + b(\overline{yz} - \bar{y}\bar{z}) + c(\overline{z^2} - \bar{z}\bar{z}) = 2\lambda c$

$$\begin{bmatrix} \overline{x^2} - \bar{x}\bar{x} & \overline{xy} - \bar{x}\bar{y} & \overline{xz} - \bar{x}\bar{z} \\ \overline{xy} - \bar{x}\bar{y} & \overline{y^2} - \bar{y}\bar{y} & \overline{yz} - \bar{y}\bar{z} \\ \overline{xz} - \bar{x}\bar{z} & \overline{yz} - \bar{y}\bar{z} & \overline{z^2} - \bar{z}\bar{z} \end{bmatrix}$$

$$f(r, c) = a_0 + a_1 r + a_2 r^2 + a_3 c + a_4 c^2 + a_5 rc$$

$$15.8359 + -0.1749 r + 0.0015 r^2 + \underline{0.2781 c}$$

<u>89, 80</u> <u>10, 10</u>

$$err = a_0 + a_1 r + a_2 r^2 + a_3 c + a_4 c^2 + a_5 rc$$

$$S_r = \sum_{i=1}^{n} \left[a_0 + a_1 r_i + a_2 r_i^2 + a_3 c_i + a_4 c_i^2 + a_5 r_i c_i\right]^2$$

$$\frac{\partial S_r}{\partial a_0} =$$

$$\vdots$$

$$\frac{\partial S_r}{\partial a_5} =$$

<u>normal</u> $\left[-\frac{\partial f}{\partial r}, -\frac{\partial f}{\partial c}, 1\right]^T$

$$\frac{\partial f}{\partial r} = a_1 + 2a_2 r + a_5 c$$

$$\frac{\partial f}{\partial c} = a_3 + 2a_4 c + a_5 r$$

$$\bar{x}_u = \frac{\partial \bar{x}}{\partial u} \qquad \bar{x}_v = \frac{\partial \bar{x}}{\partial v} \qquad \bar{x}(u,v) = \begin{bmatrix} u \\ v \\ f(u,v) \end{bmatrix}$$

$$\bar{N} = \frac{\bar{x}_u \times \bar{x}_v}{|\bar{x}_u \times \bar{x}_v|}$$

$$E = \bar{x}_u \cdot \bar{x}_u$$
$$F = \bar{x}_u \cdot \bar{x}_v$$
$$G = \bar{x}_v \cdot \bar{x}_v$$

$$e = -\bar{N} \cdot \bar{x}_{uu}$$
$$f = -\bar{N} \cdot \bar{x}_{uv}$$
$$g = -\bar{N} \cdot \bar{x}_{vv}$$

$$S = (EG - F^2)^{-1} \begin{bmatrix} eG - fF & fG - gF \\ fE - eF & gE - fF \end{bmatrix}$$

$[V, D] = \text{eigs}(S)$

D k_1, k_2

V principal dirs

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial u^2} & \frac{\partial^2 f}{\partial u \partial v} \\ \frac{\partial^2 f}{\partial u \partial v} & \frac{\partial^2 f}{\partial v^2} \end{bmatrix} \qquad [V, D] = \text{eigs}(H)$$

λ

Consider Monge patch (range image)

$$\bar{x}(u,v) = [u, v, h(u,v)]$$

So,

$$\bar{N} = \frac{1}{(1+h_u^2+h_v^2)} (-h_u, -h_v, 1)^T$$

$$E = 1 + h_u^2 \qquad F = h_u h_v \qquad G = 1 + h_v^2$$

$$e = -\frac{h_{uu}}{(1+h_u^2+h_v^2)^{1/2}}$$

$$f = -\frac{h_{uv}}{(1+h_u^2+h_v^2)^{1/2}}$$

$$g = -\frac{h_{vv}}{(1+h_u^2+h_v^2)^{1/2}}$$

Gaussian curvature:

$$K = \frac{h_{uu}h_{vv} - h_{uv}^2}{(1+h_u^2+h_v^2)^2}$$

Mean curvature

$$H = \frac{Eg + Ge - 2Ff}{2(EG - F^2)}$$

let $A = \begin{bmatrix} \bar{x}_u \cdot \bar{x}_u & \bar{x}_u \cdot \bar{x}_v \\ \bar{x}_v \cdot \bar{x}_u & \bar{x}_v \cdot \bar{x}_v \end{bmatrix}$ $\qquad B = \begin{bmatrix} \bar{x}_{uu} \cdot \bar{n} & \bar{x}_{uv} \cdot \bar{n} \\ \bar{x}_{vu} \cdot \bar{n} & \bar{x}_{vv} \cdot \bar{n} \end{bmatrix}$

then $H = \frac{1}{2}(\mathrm{trace}(G^{-1}B))$ mean curvature

$K = \frac{|B|}{|G|}$ Gaussian curvature

$$K = \underbrace{k_1 k_2}_{} \qquad H = \frac{k_1 + k_2}{2} \quad \Rightarrow \quad k_2 = 2H - k_1$$

eigenvalues of Hessian

principal directions = eigenvectors

$$\Rightarrow \quad k_1^2 - 2Hk_1 + K = 0$$

$$k_1 = \frac{-(-2H) \pm \sqrt{4H^2 - 4K}}{2}$$

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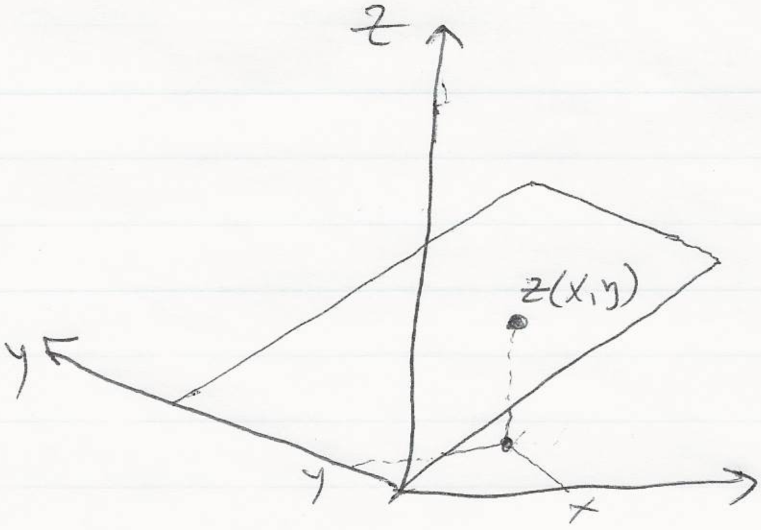
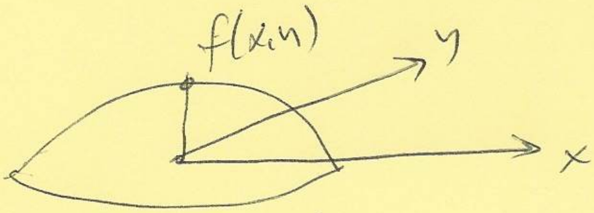


$$\bar{f}(x,y) = \begin{bmatrix} x \\ y \\ f(x,y) \end{bmatrix} = \begin{bmatrix} x \\ y \\ x \end{bmatrix}$$

$$\frac{\partial \bar{f}}{\partial x}(x,y) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \frac{\partial \bar{f}}{\partial y}(x,y) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\bar{n}(x,y) = \frac{\partial \bar{f}}{\partial x}(x,y) \times \frac{\partial \bar{f}}{\partial y} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \bar{i}\,(0-1) + \bar{j}\,(0-0) + \bar{k}\,(1-0)$$

$$\begin{bmatrix} -0.7071 \\ 0 \\ 0.7071 \end{bmatrix} \text{unit vector} \Leftarrow \quad = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$



$$f(x,y) = (R^2 - x^2 - y^2)^{1/2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(R^2 - x^2 - y^2)^{-1/2}(-2x)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(R^2 - x^2 - y^2)^{-1/2}(-2y)$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{1}{2}\cdot\frac{1}{2}(R^2 - x^2 - y^2)^{-3/2}(-2x) + \frac{1}{2}(R^2 - x^2 - y^2)^{-1/2}(-2)$$

$$\frac{\partial^2 f}{\partial x \partial y} = -\frac{1}{2}\cdot\frac{1}{2}(R^2 - x^2 - y^2)^{-3/2}(-2y)(-2x)$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{1}{2}\cdot\frac{1}{2}(R^2 - x^2 - y^2)^{-3/2}(-2y) + \frac{1}{2}(R^2 - x^2 - y^2)^{-1/2}(-2)$$

At $(0,0)$ $\quad \bar{N} = \begin{bmatrix} -\partial f/\partial x \\ -\partial f/\partial y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\|\nabla f\| = \left\| \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\| = 0$$

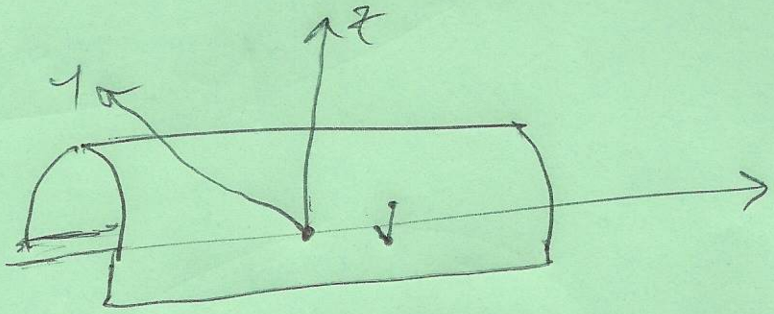
$$H = \begin{bmatrix} -\frac{1}{R} & 0 \\ 0 & -\frac{1}{R} \end{bmatrix}$$

$\lambda_1 = -.5$ $\quad K = \frac{1}{4} = \frac{1}{R^2}$

$\lambda_2 = -.5$

$\bar{d}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad \bar{d}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

14)10



$$f(x,y) = \sqrt{R^2 - y^2} = (R^2 - y^2)^{1/2}$$

$$\frac{\partial f}{\partial x} = \cancel{\tfrac{1}{2}(R^2-y^2)^{-1/2}}\ 0$$

$$(R^2)^{-3/2} = R^{-3}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(R^2 - y^2)^{-1/2}(-2y)$$

$$\frac{\partial^2 f}{\partial x^2} = 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{1}{2}\cdot\frac{1}{2}(R^2 - y^2)^{-3/2}(-2y) + \frac{1}{2}(R^2 - y^2)^{-1/2}(-2)$$

At $(0,0)$ $\|\nabla f\| = 0$

$$H = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R} \end{bmatrix} \qquad \lambda_1 = 0 \quad \lambda_2 = -\frac{1}{R} \qquad \bar{d}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \bar{d}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

14) 11
21 \ 13

Classify as follows:

1. Peak: $\|\nabla f\| = 0 \quad \lambda_1 < 0 \quad \lambda_2 < 0$
2. Pit: $\|\nabla f\| = 0 \quad \lambda_1 > 0 \quad \lambda_2 > 0$
3. Ridge:

   $\|\nabla f\| \neq 0 \quad \lambda_1 < 0 \quad \nabla f \cdot \bar{v}_1 = 0$

   $\|\nabla f\| \neq 0 \quad \lambda_2 < 0 \quad \nabla f \cdot v_2 = 0$

   $\|\nabla f\| = 0 \quad \lambda_1 < 0 \quad \lambda_2 = 0$
4. Ravine:

   $\|\nabla f\| \neq 0 \quad \lambda_1 > 0 \quad \nabla f \cdot \bar{v}_1 = 0$

   $\|\nabla f\| \neq 0 \quad \lambda_2 > 0 \quad \nabla f \cdot \bar{v}_2 = 0$

   $\|\nabla f\| = 0 \quad \lambda_1 > 0 \quad \lambda_2 = 0$
5. Saddle $\|\nabla f\| = 0 \quad \lambda_1 \cdot \lambda_2 < 0$
6. Flat $\|\nabla f\| = 0 \quad \lambda_1 = 0 \quad \lambda_2 = 0$
7. Hillside (none of the above & +)

   Slope $\lambda_1 = \lambda_2 = 0$

   Convex $\lambda_1 \geq \lambda_2 \geq 0 \;+\; \lambda_1 \neq 0$

   Concave $\lambda_1 \leq \lambda_2 \leq 0 \;+\; \lambda_1 \neq 0$

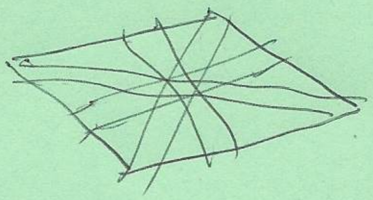
   Saddle Hill $\lambda_1 \cdot \lambda_2 < 0$

K H classier

	$H<0$	$H=0$	$H>0$
$K<0$	Saddle Ridge	Minimal Surface	Saddle Valley
$K=0$	Ridge Surface	Flat	Valley Surface
$K>0$	Peak Surf.	None	Pit Surface

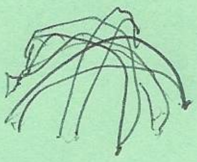
a_0	a_1x	a_2x^2
–	–	0
–	+	0
–	–	+
–	+	~

flat



4 lines

peak



4 quadratic down

pit

4 quadratic up

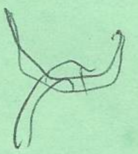
ridge

near zero in some direction
rest down

valley

near zero 1
rest up

saddle



some up some down