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Week 11 CS4640 Chapter 9 Features

boundary points

all points

Landmarks : mathematical (e.g., corners)

anatomical (e.g., eye corners)

pseudo (e.g., center of cheek)

Shape descriptors

form factor : $\frac{4\pi \times \text{Area}}{\text{Perimeter}^2}$

Roundness : $\frac{4 \times \text{Area}}{\pi \times \text{MaxDiameter}^2}$

Aspect Ratio : $\frac{\text{MaxDiameter}}{\text{MinDiameter}}$

⋮

Convexity : $\frac{\text{Convex Perimeter}}{\text{Perimeter}}$

Convexity : A set of points, P, is convex if

$\forall\ \bar{p}_1, \bar{p}_2 \in P \quad \overline{p_1 p_2} \in P$

Matlab : convhull

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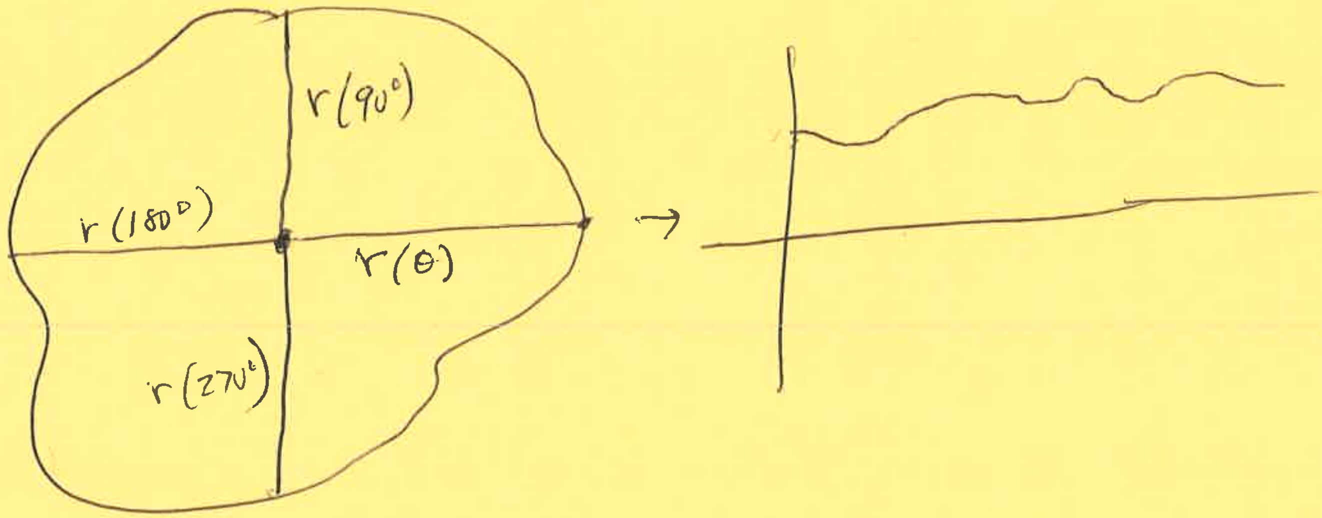
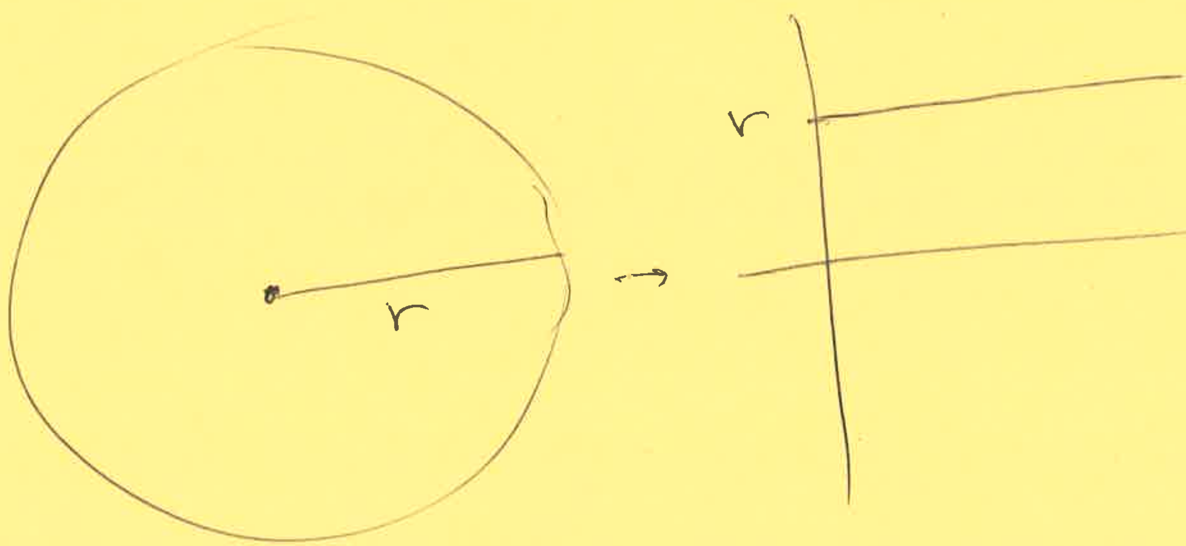
Radial Fourier Expansion





circle

Seen this previously.

Statistical Moments

$$m_n = \int_{-\infty}^{\infty} x^n p(x)\,dx \qquad n^{th} \text{ moment}$$

$$m_0 = \int_{-\infty}^{\infty} p(x)\,dx \qquad \text{area}$$

$$\mu = m_1 = \int_{-\infty}^{\infty} x\,p(x)\,dx \qquad \text{mean value}$$

Central moments : variation about mean

$$M_n = \int_{-\infty}^{\infty} (x-\mu)^n p(x)\,dx$$

e.g., $$M_2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\,dx \qquad \text{variance}$$

2D Moments

$$m_{pq} = \iint x^p y^q p(x,y)\,dx\,dy$$

$$M_{pq} = \iint (x-\mu_x)^p (y-\mu_y)^q I(x,y)\,dx\,dy$$

(p-q) th normalized central moment

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\beta}} \qquad \beta = \frac{p+q}{2} + 1$$

Hu's invariant moments

$$A_1 = \eta_{20} + \eta_{02}$$

$$A_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{1,1}^2$$

$$A_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$A_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{30})^2$$

$$A_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} - \eta_{03})^2 + (3\eta_{21} - \eta_{03})(\eta_{03} + \eta_{21})[3(\eta_{30} + \eta_{12})^2 - (\eta_{03} + \eta_{21})^2]$$

$$A_6 = (\eta_{20} - \eta_{02})[(\eta_{12} + \eta_{30})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{21} + \eta_{03})(\eta_{12} + \eta_{30})$$

$$A_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{03} + \eta_{21})^2] + (3\eta_{21} - \eta_{30})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{03} + \eta_{21})^2]$$

For the discrete case:

$$m_n = \sum_x x^n p(x)$$

$$m_{pq} = \sum_x \sum_y x^p y^q \underbrace{p(x,y)}$$

where for images, this is a gray level or binary

So m_{00} is just the area

m_{10} is the sum of x values

$\frac{m_{10}}{m_{00}}$ is mean x value

Texture Features

statistical (or other) measures over some region (typically a square window), Ω

$$\text{range}_{\Omega} = \{\max - \min\}_{\Omega}$$

$$\text{variance}_{\Omega} = \text{variance}(\Omega)$$

$$\text{mean}_{\Omega} = \text{mean}(\Omega)$$

$$\vdots$$

we've seen this previously

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PCA : Principal Component Analysis

Given a set of points, P, find rotation of P to decorrelate points.

In 2D, means minimize y values ~~of~~ new pts.

why?

$$
\begin{array}{cccc}
 & n & 2 & 2 \\
2 & \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \end{bmatrix} & n\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \\ x_n & y_n \end{bmatrix} & = 2\begin{bmatrix} & v \\ & \end{bmatrix}
\end{array}
$$

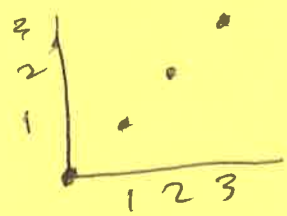
want this 0

SO not correlated

$v = x_1y_1 + x_2y_2 + \cdots x_ny_n$

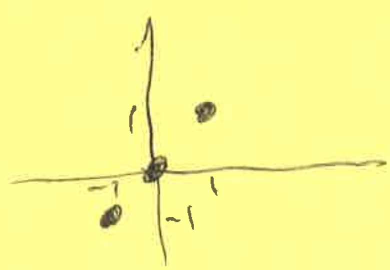
if all 0 (or minimal)

Consider: $P = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$



Need to move so mean of pts is [0;0]

mean: [2;2] $P_0 = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$



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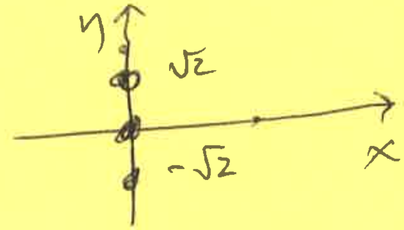
If rotate 0°, then

$$V = (-1)(-1) + (0)(0) + (1)(1) = 2$$

If rotate 45°, then

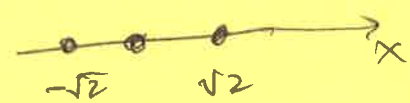


$$\begin{bmatrix} 0 & 0 & 0 \\ -\sqrt{2} & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & -\sqrt{2} \\ 0 & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$$

so, not bad, but uses dimension 2

If rotate -45°, then

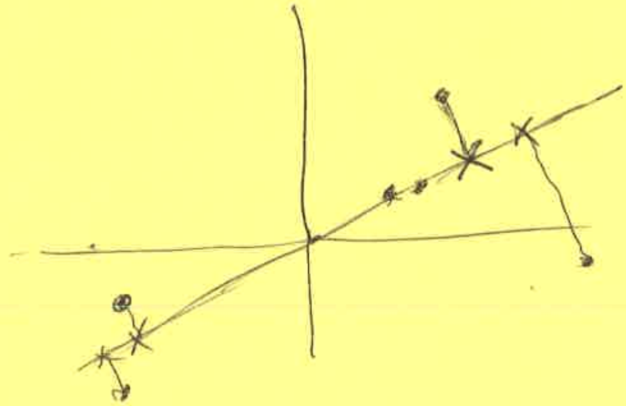


$$\begin{bmatrix} -\sqrt{2} & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 0 \\ \sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$$

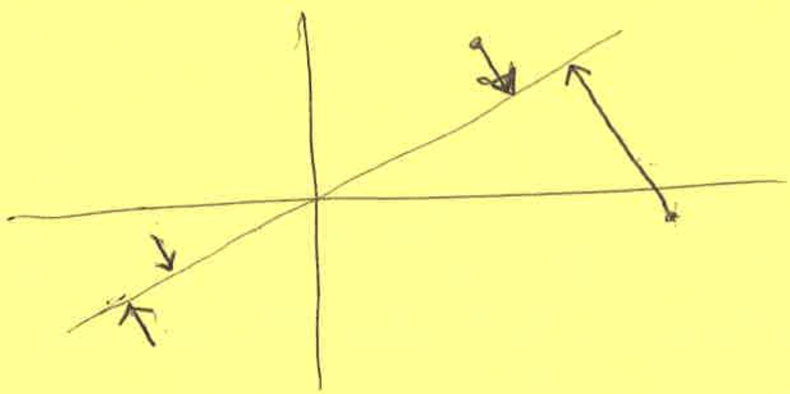
In 2D, we've done, since 2nd axis is orthogonal to first

But let's look at explanation from text

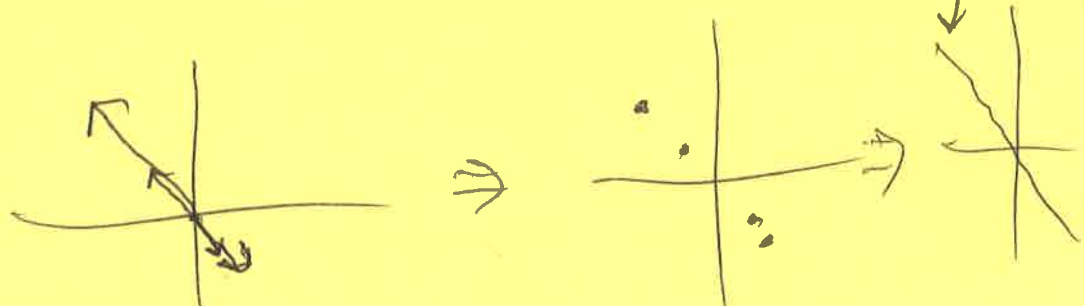
(1) Project points onto 1st axis (original frame)
+ points in ~~text~~



(2) subtract these from original points (get vector)



Plot these vectors as points



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Consider 3D example

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