Error Ellipses

Input: one sigma uncertainty in the x-direction: σx

one-sigma uncertainty in the y-direction: σy

correlation coefficient ρxy

center of the ellipse: (x,y)

aspect ratio of the plot: (Δy/vertical plot length)/(Δx/horizontal plot length)

1) Calculate the covariance between x and y:

$$\sigma xy = \rho xy \cdot \sigma x \cdot \sigma y$$

2) Construct the covariance matrix:

$$covmat = \begin{pmatrix} (\sigma x)^2 & \sigma xy \\ \sigma xy & (\sigma y)^2 \end{pmatrix}$$

3) Calculate the lengths of the ellipse axes, which are the square root of the eigenvalues of the covariance matrix:

$$eigval = eigenvalues(c)$$

4) Calculate the counter-clockwise rotation (θ) of the ellipse:

$$\theta = \frac{1}{2} \cdot \mathrm{Tan}^{-1}\left[\left(\frac{1}{aspectratio}\right) \cdot \left(\frac{2 \cdot \sigma xy}{(\sigma x)^2 - (\sigma y)^2}\right)\right]$$

5) To create a 95% confidence ellipse from the 1σ error ellipse, we must enlarge it by a factor of $scalefactor = 2.4477$.

6) Plot the ellipse:

7a) Plot an ellipse with semi-major and semi-monor axes parallel to the x- and y-axes of the graph, centered at (x,y). These axis lengths are the square roots of the eigenvalues. The larger eigenvalue belongs to the axis with the larger uncertainty:

If σx > σy semi-major axis length parallel to x = sqrt(maximum(eigenvalues))*scalefactor

semi-minor axis length parallel to y = sqrt(minimum(eigenvalues))*scalefactor

If σy > σx semi-minor axis length parallel to x = sqrt(minimum(eigenvalues))*scalefactor

semi-major axis length parallel to y = sqrt(maximum(eigenvalues))*scalefactor

7b) Rotate the ellipse θ radians counter-clockwise from its original orientation.