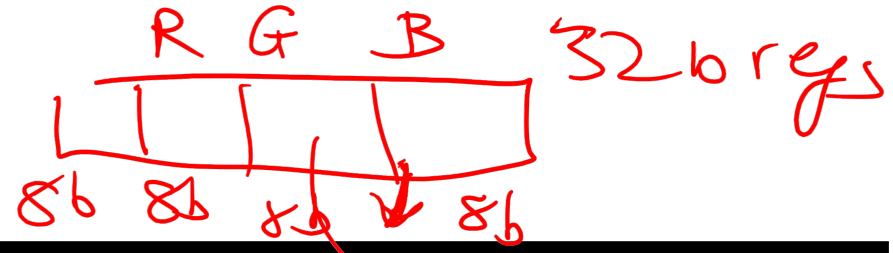
Lecture 12: Hardware for Arithmetic

- Today's topics:
  - Digital logic intro
  - Logic for common operations
  - Designing an ALU

Subword Parallelism

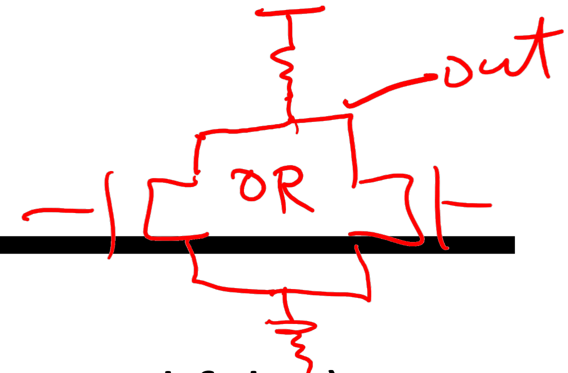


ALU
SP

SIMD
Vector

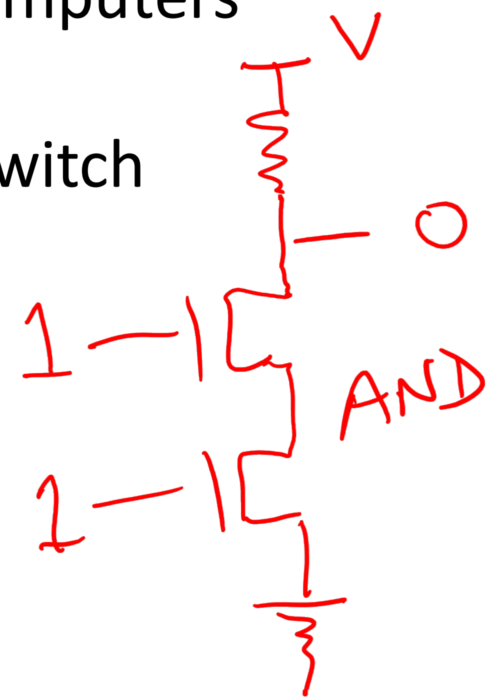
- ALUs are typically designed to perform 64-bit or 128-bit arithmetic
- Some data types are much smaller, e.g., bytes for pixel RGB values, half-words for audio samples
- Partitioning the carry-chains within the ALU can convert the 64-bit adder into 4 16-bit adders or 8 8-bit adders
- A single load can fetch multiple values, and a single add instruction can perform multiple parallel additions, referred to as subword parallelism

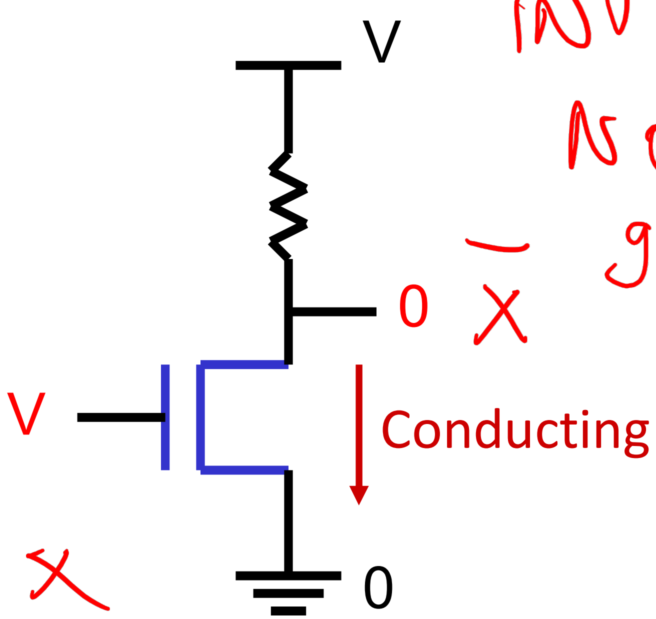
Digital Design Basics



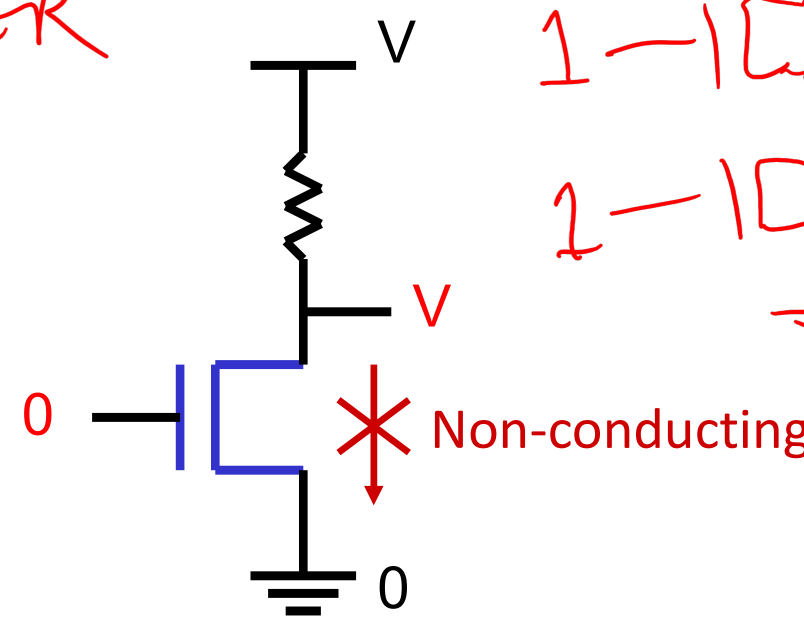
- Two voltage levels – high and low (1 and 0, true and false) Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch





INVERTER
NOT
gate



Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors

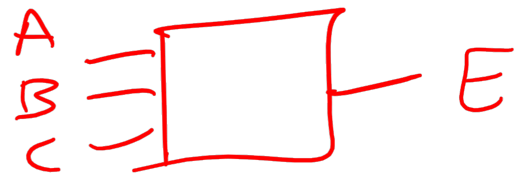
- A logic block is termed *combinational* if the output is only a function of the inputs

- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output

- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table



- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

Dec#	A	B	C	E
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Truth Table

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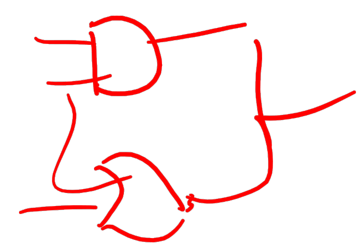


A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , $X = A + B$ ➔ X is true if at least one of A or B is true
  - AND : symbol . , $X = A . B$ ➔ X is true if both A and B are true
  - NOT : symbol $\bar{\ }$, $X = \bar{A}$ ➔ X is the inverted value of A

Boolean Algebra Rules

- Identity law : $A + 0 = A$; $A . 1 = A$
- Zero and One laws : $A + 1 = 1$; $A . 0 = 0$
- Inverse laws : $A . \overline{A} = 0$; $A + \overline{A} = 1$
- Commutative laws : $A + B = B + A$; $A . B = B . A$
- Associative laws : $A + (B + C) = (A + B) + C$

  $A . (B . C) = (A . B) . C$
- Distributive laws : $A . (B + C) = (A . B) + (A . C)$

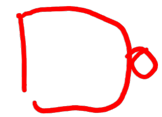
  $A + (B . C) = (A + B) . (A + C)$

DeMorgan's Laws

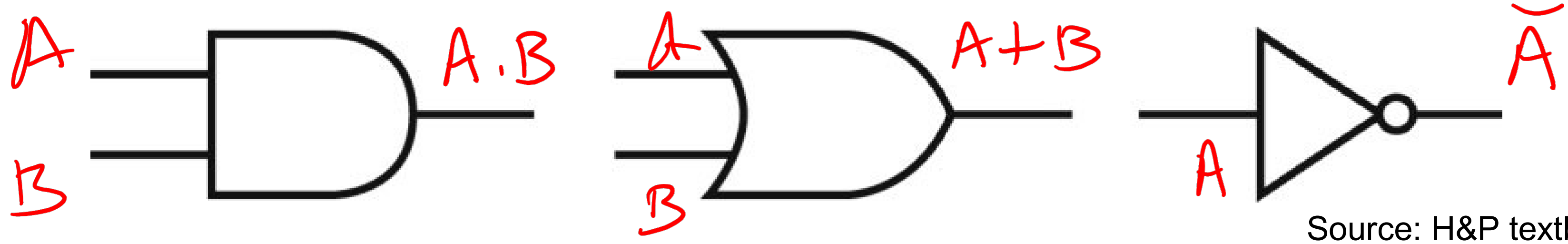
- $\overline{A + B} = \overline{A} \,.\, \overline{B}$
- $\overline{A \,.\, B} = \overline{A} + \overline{B}$
- Confirm that these are indeed true

Pictorial Representations

NAND



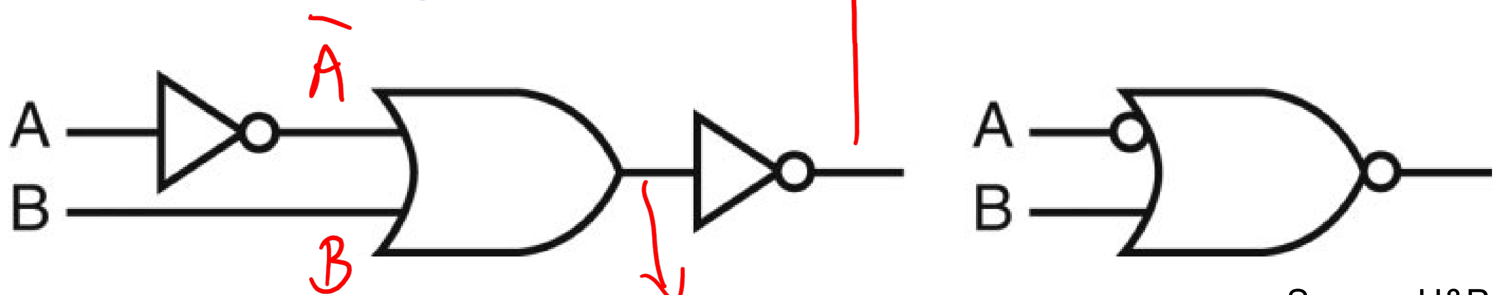
AND OR NOT



Source: H&P textbook

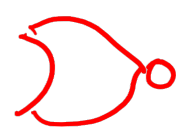
$$\overline{\bar{A}+B} = \bar{\bar{A}} \cdot \bar{B} = A \cdot \bar{B}$$

What logic function is this?



$\bar{A}+B$

Source: H&P textbook



NOR

Boolean Equation

- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

Multiple correct equations:

Two must be true, but all three cannot be true:

$E = ((A \cdot B) + (B \cdot C) + (A \cdot C)) \cdot \overline{(A \cdot B \cdot C)}$

Identify the three cases where it is true:

$E = (A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$

prod of sums:

Sum of Products

$(\bar{A}+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\quad) \cdot (\ ) \cdot (\ )$

- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$\bar{A} \cdot B \cdot C$

sum of prod

1

$(A \,.\, B \,.\, \overline{C}) + (A \,.\, C \,.\, \overline{B}) + (C \,.\, B \,.\, \overline{A})$

- Can also use "product of sums"
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

NAND and NOR

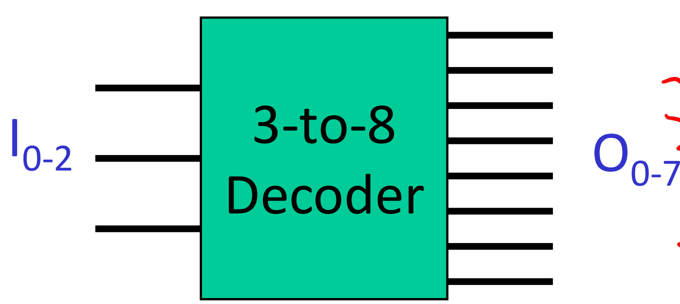
- NAND : NOT of AND : A nand B = $\overline{A \cdot B}$
- NOR : NOT of OR : A nor B = $\overline{A + B}$
- NAND and NOR are *universal gates*, i.e., they can be used to construct any complex logical function

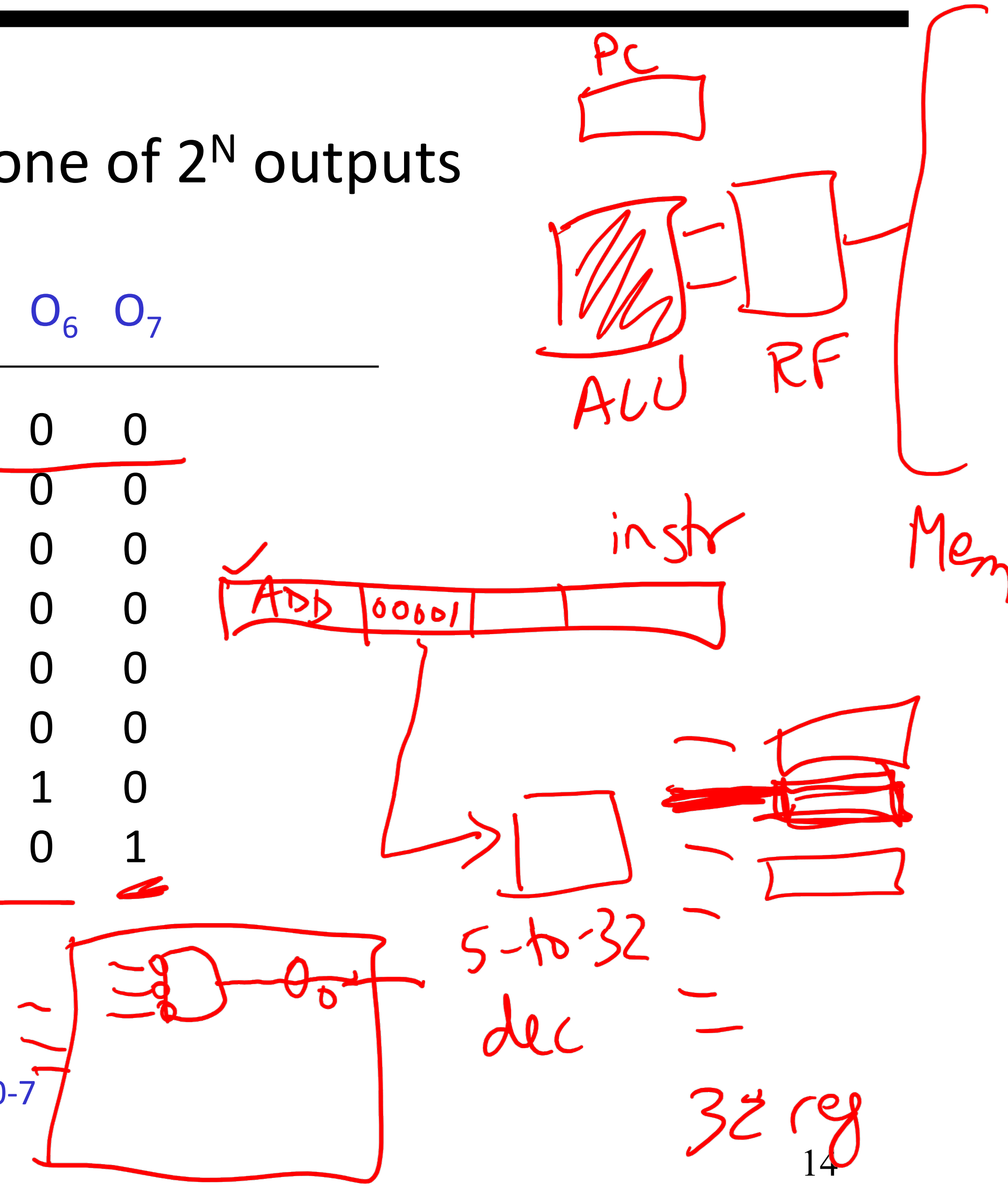
Common Logic Blocks – Decoder

$O_0 = \overline{I_0} \cdot \overline{I_1} \cdot \overline{I_2}$

Takes in N inputs and activates one of 2^N outputs

I_0	I_1	I_2	O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1





Common Logic Blocks – Multiplexor

PC program counter

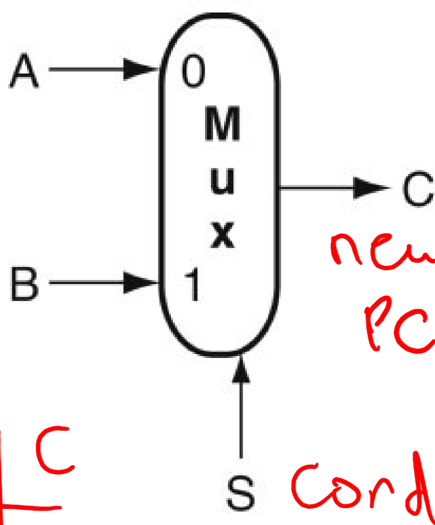
beq

code

- Multiplexor or selector: one of N inputs is reflected on the output depending on the value of the $\log_2 N$ selector bits

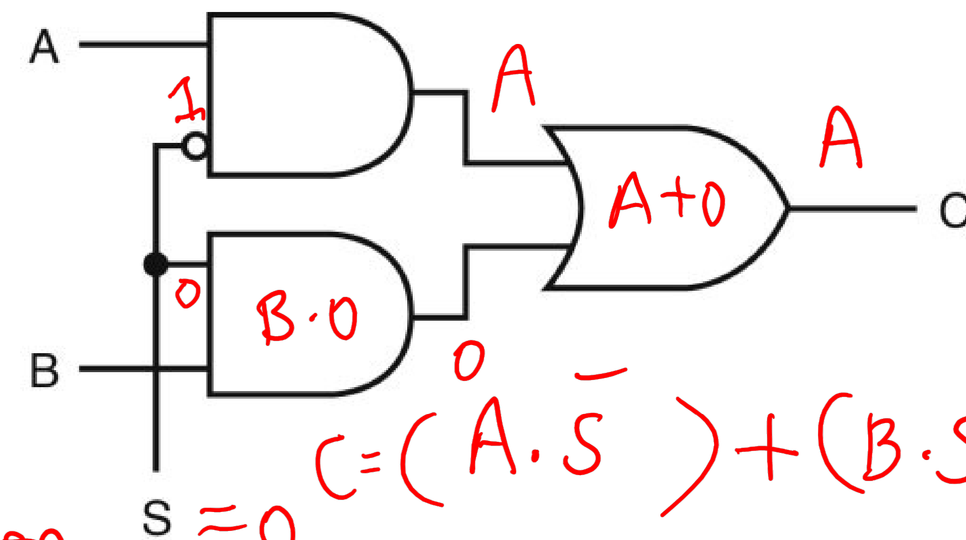
PC+4

PC+100



new PC

condition



A·1

A

A+0

B·0

S = 0

$$C = (A \cdot \bar{S}) + (B \cdot S)$$

Source: H&P textbook

A	B	S	C

2-input mux

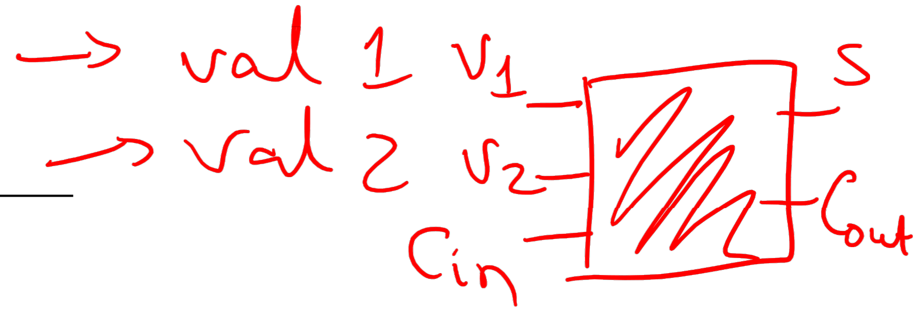
$\boxed{\text{SUB}}$

C = A if S = 0

C = B if S = 1

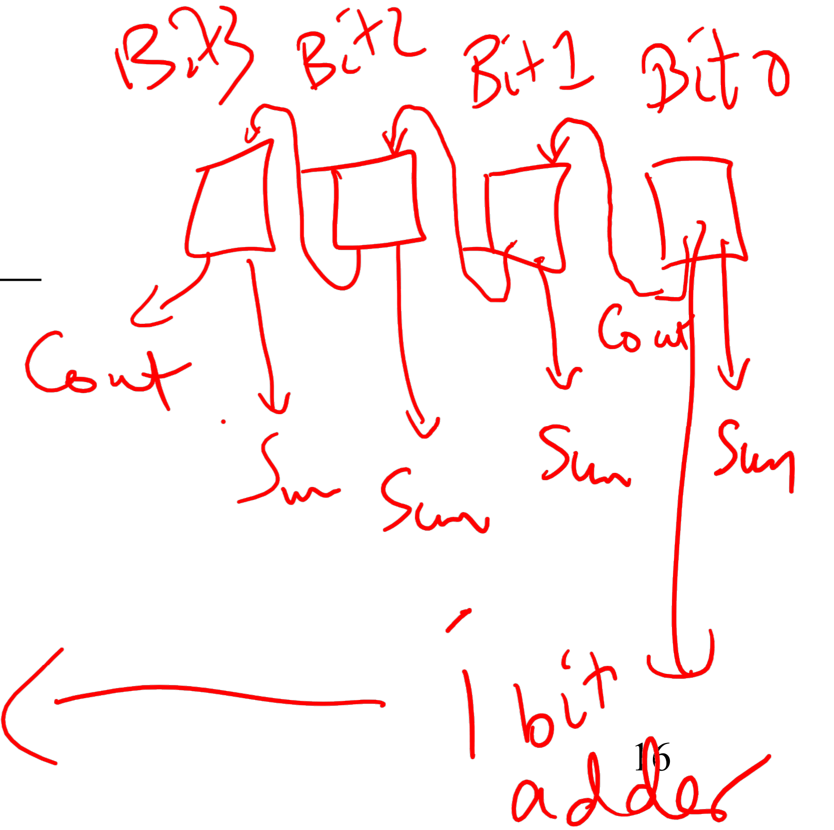
Adder Algorithm

	0	0	1	
	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1



Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Adder Algorithm

	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
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Truth Table for the above operations:

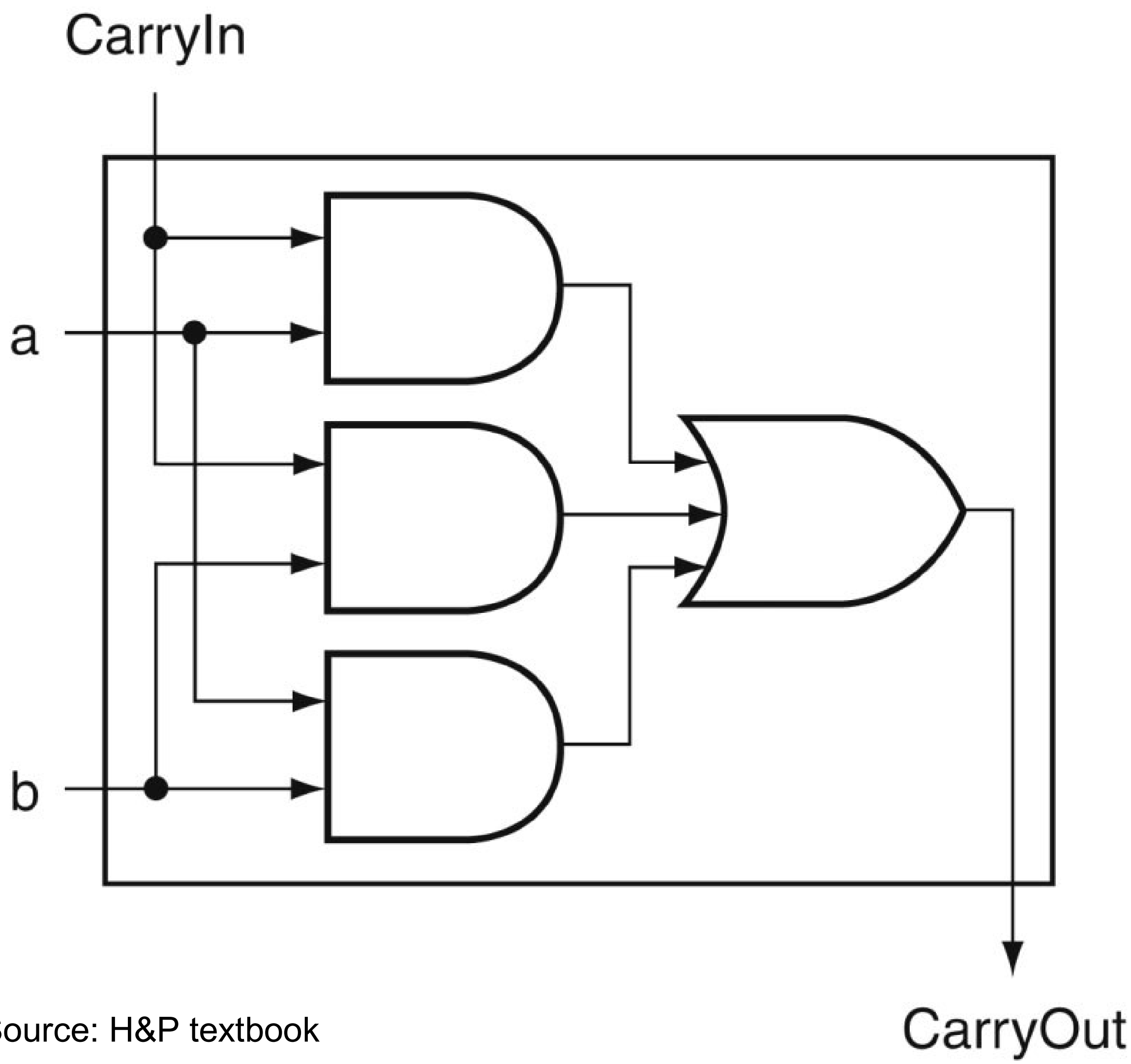
A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Equations:

Sum = $Cin \cdot \overline{A} \cdot \overline{B}$ +
$B \cdot \overline{Cin} \cdot \overline{A}$ +
$A \cdot \overline{Cin} \cdot \overline{B}$ +
$A \cdot B \cdot Cin$

Cout = $A \cdot B \cdot Cin$ +
$A \cdot B \cdot \overline{Cin}$ +
$A \cdot Cin \cdot \overline{B}$ +
$B \cdot Cin \cdot \overline{A}$
= $A \cdot B$ +
$A \cdot Cin$ +
$B \cdot Cin$

Carry Out Logic



Source: H&P textbook

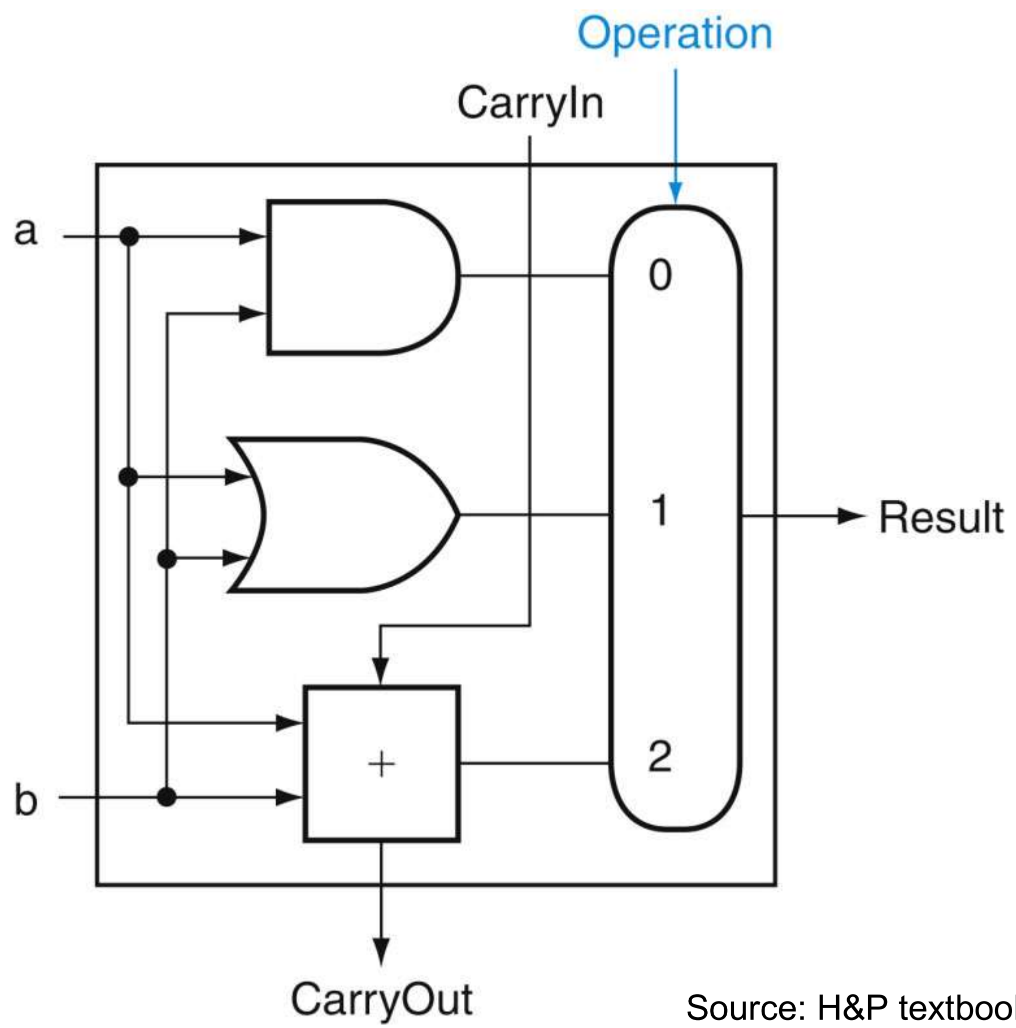
Equations:

$$\begin{aligned} Sum = {} & Cin \,.\, \overline{A} \,.\, \overline{B} \;+ \\ & B \,.\, \overline{Cin} \,.\, \overline{A} \;+ \\ & A \,.\, \overline{Cin} \,.\, \overline{B} \;+ \\ & A \,.\, B \,.\, Cin \end{aligned}$$

$$\begin{aligned} Cout = {} & A \,.\, B \,.\, Cin \;+ \\ & A \,.\, B \,.\, \overline{Cin} \;+ \\ & A \,.\, Cin \,.\, \overline{B} \;+ \\ & B \,.\, Cin \,.\, \overline{A} \\ = {} & A \,.\, B \;+ \\ & A \,.\, Cin \;+ \\ & B \,.\, Cin \end{aligned}$$

1-Bit ALU with Add, Or, And

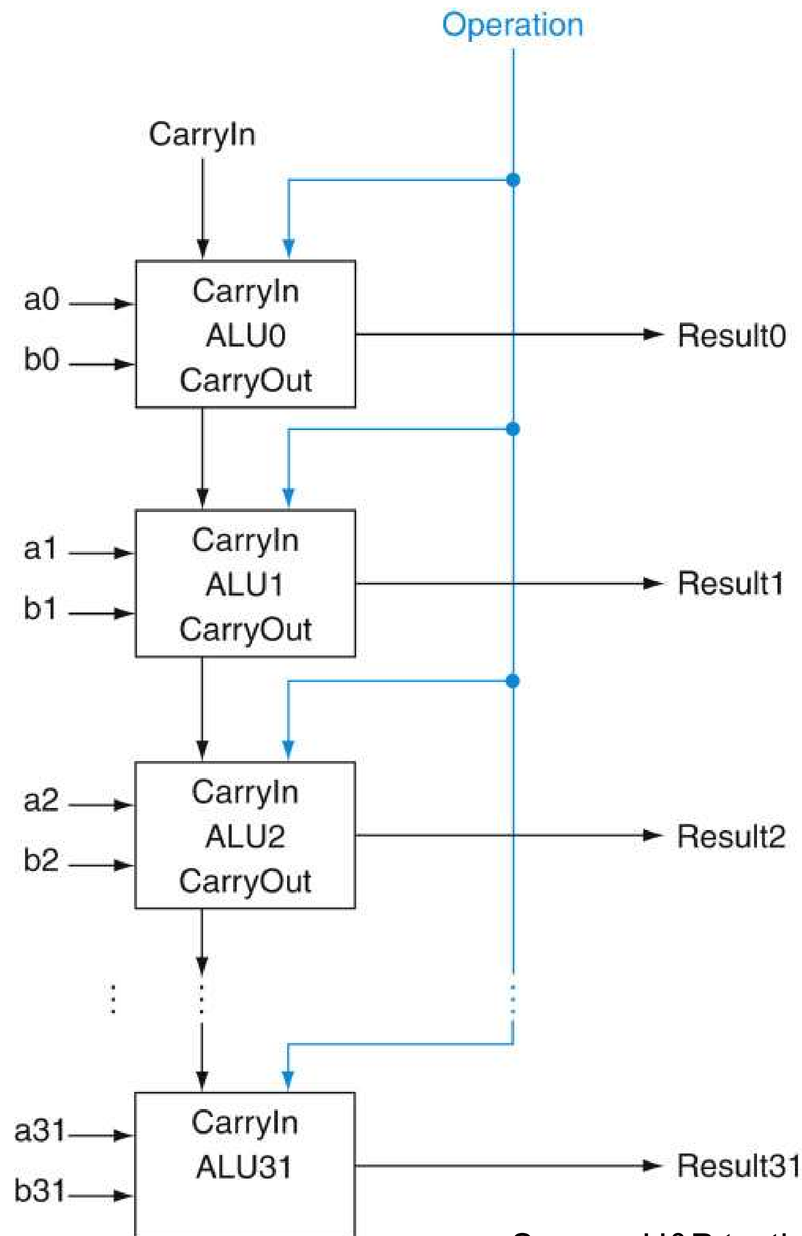
- Multiplexor selects between Add, Or, And operations



Source: H&P textbook

32-bit Ripple Carry Adder

1-bit ALUs are connected "in series" with the carry-out of 1 box going into the carry-in of the next box



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