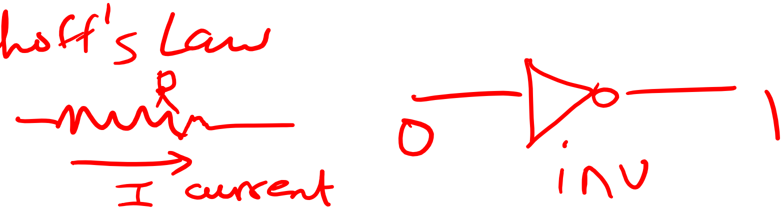
Lecture 12: Hardware for Arithmetic

- Today's topics:
  - Digital logic intro
  - Logic for common operations
  - Designing an ALU

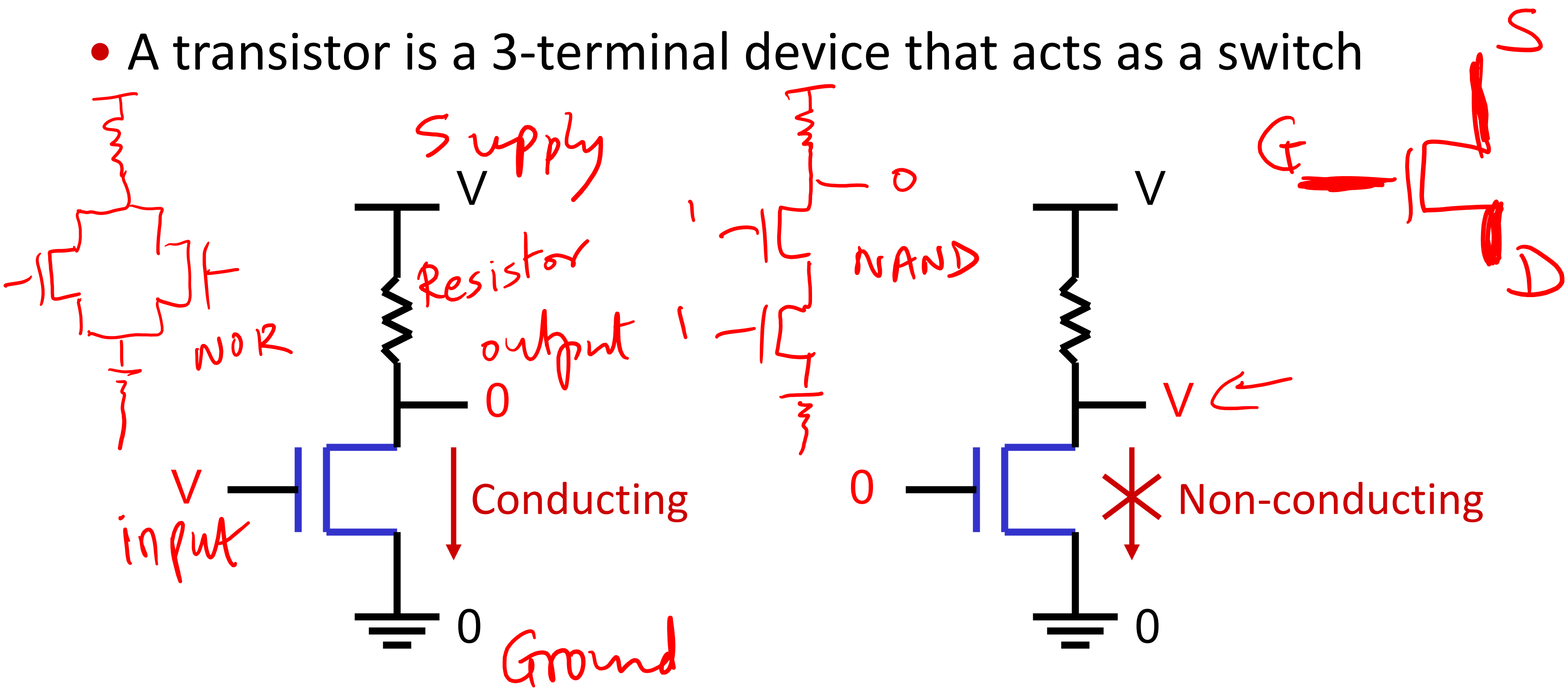
Digital Design Basics



V = IR

- Two voltage levels – high and low (1 and 0, true and false) Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch



Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors
- A logic block is termed *combinational* if the output is only a function of the inputs

FINITE STATE MACHINES

- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output
- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true — 1

False - 0

$2 \times 2 \times 2 = 8$

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	

Truth Table

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A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

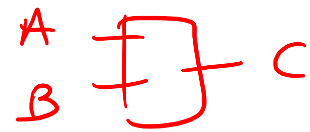
Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , $X = A + B$ ➔ X is true if at least one of A or B is true
  - AND : symbol **.** , $X = A \cdot B$ ➔ X is true if both A and B are true
  - NOT : symbol $\bar{\ }$, $X = \bar{A}$ ➔ X is the inverted value of A

$A' = \bar{A}$

Boolean Algebra Rules



$C = \overline{A+B}$

- Identity law : $A + 0 = A$; $A \cdot 1 = A$
- Zero and One laws : $A + 1 = 1$; $A \cdot 0 = 0$

ins of A

- Inverse laws : $A \cdot \overline{A} = 0$; $A + \overline{A} = 1$
- Commutative laws : $A + B = B + A$; $A \cdot B = B \cdot A$
- Associative laws : $A + (B + C) = (A + B) + C$

  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- Distributive laws : $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

  $A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's Laws

- $\overline{A + B} = \overline{A} \cdot \overline{B}$

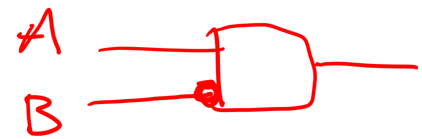
  NOR AND of inv

- $\overline{A \cdot B} = \overline{A} + \overline{B}$

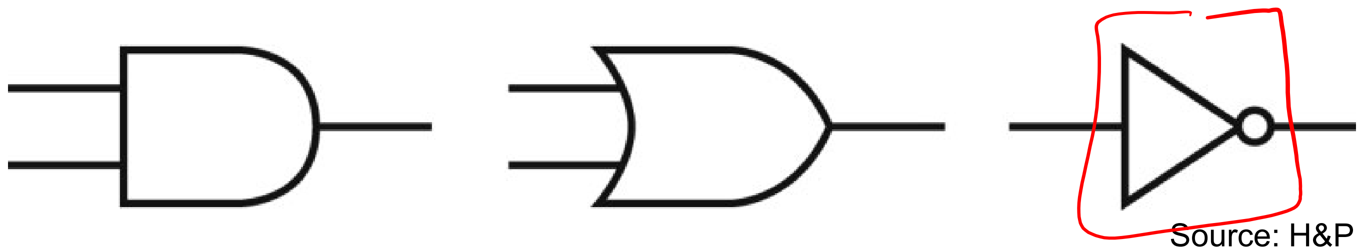
  NAND OR of inv

- Confirm that these are indeed true

Pictorial Representations

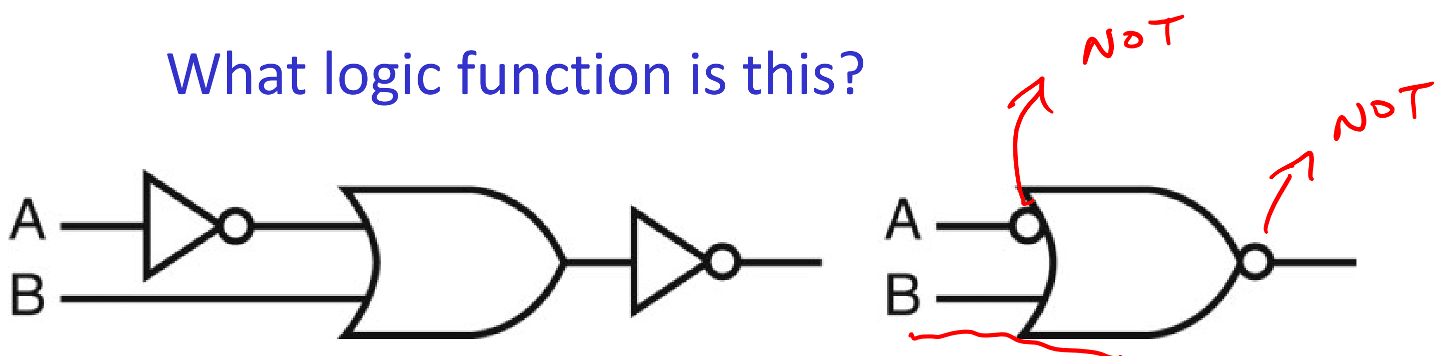


AND OR NOT



Source: H&P textbook

What logic function is this?



Source: H&P textbook

$A \cdot \bar{B} \leftarrow \bar{\bar{A}} \cdot \bar{B} \leftarrow \overline{\bar{A} + B}$

Boolean Equation

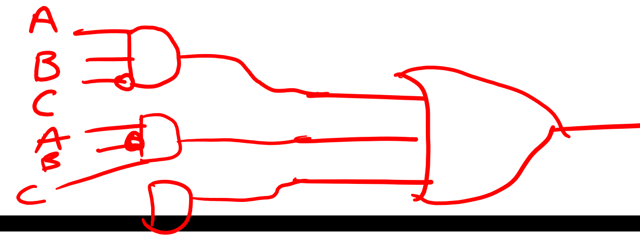
- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

Multiple correct equations:

Two must be true, but all three cannot be true:
$E = ((A \,.\, B) + (B \,.\, C) + (A \,.\, C)) \,.\, \overline{(A \,.\, B \,.\, C)}$

Identify the three cases where it is true:
$E = (A \,.\, B \,.\, \overline{C}) + (A \,.\, C \,.\, \overline{B}) + (C \,.\, B \,.\, \overline{A})$

Sum of Products



- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

Sum of Products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

1 1 1

$$(A . B . \overline{C}) + (A . C . \overline{B}) + (C . B . \overline{A})$$

- Can also use "product of sums"
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

Product of sums

$(A+B+C) \cdot (A+B+\overline{C}) \cdot (\quad)$

NAND and NOR

- NAND : NOT of AND : A nand B = $\overline{A . B}$
- NOR : NOT of OR : A nor B = $\overline{A + B}$
- NAND and NOR are *universal gates*, i.e., they can be used to construct any complex logical function

Common Logic Blocks – Decoder

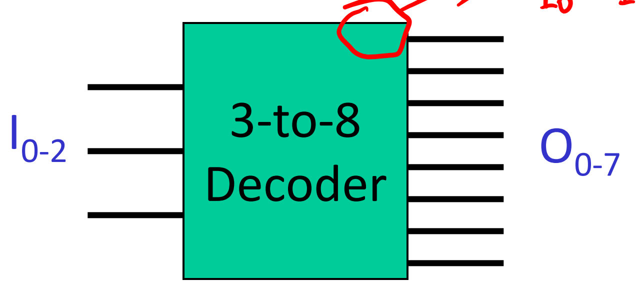
Takes in N inputs and activates one of 2^N outputs

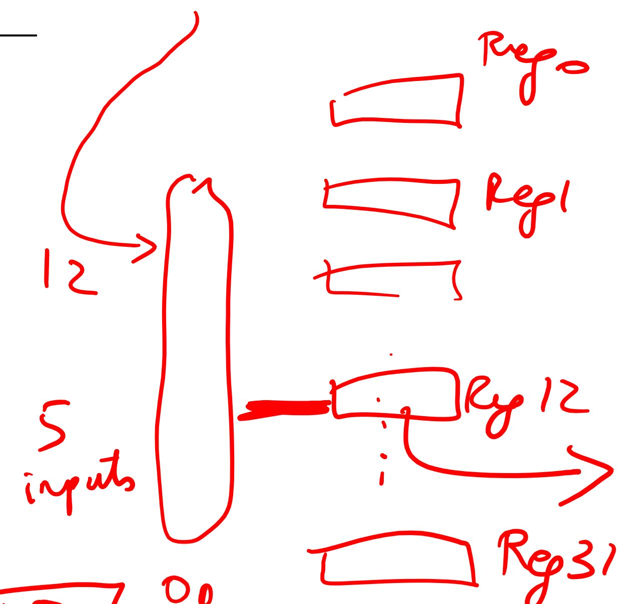
addi $t1, $t3, 0

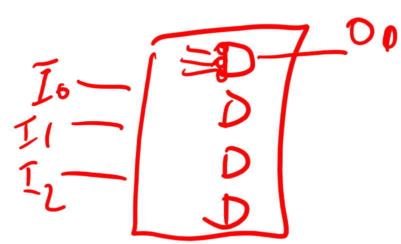
$12

I_0	I_1	I_2	O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

$\bar{I_0} \cdot \bar{I_1} \cdot \bar{I_2}$







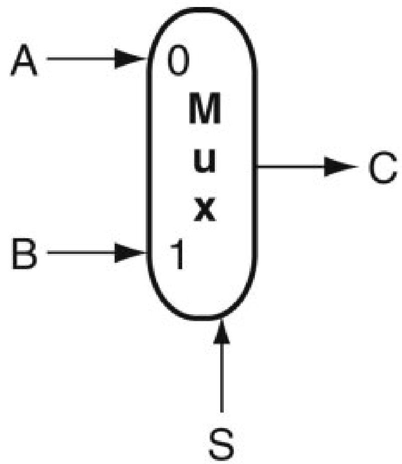
Common Logic Blocks – Multiplexor

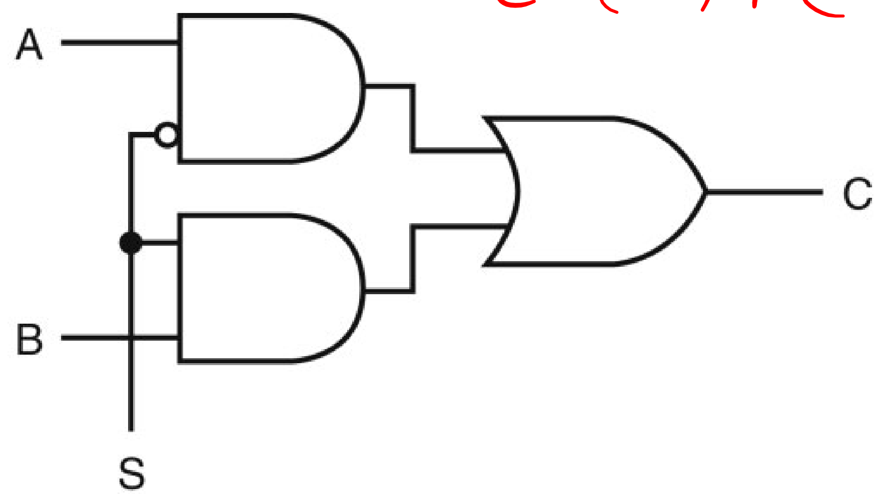
$S=1 \quad C = A \cdot 0 + B \cdot 1$
$= B$

$S=0 \quad C = A \cdot 1 + B \cdot 0$
$= A$

- Multiplexor or selector: one of N inputs is reflected on the output depending on the value of the $\log_2 N$ selector bits

$C = (A \cdot \bar{S}) + (B \cdot S)$



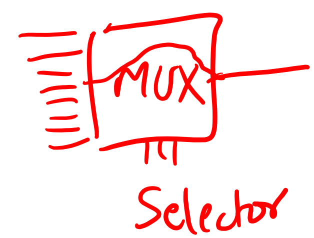


Source: H&P textbook

4-to-1
2 selector bits

2-input mux

2-to-1



Adder Algorithm

a	1	0	0	1
b	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1

a_i b_i C_{i-1}

Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Adder Algorithm

$A \cdot B \cdot C_{in} + A \cdot B \cdot \overline{C_{in}}$
$= A \cdot B \cdot (C_{in} + \overline{C_{in}})$
$= A \cdot B$

	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1

Equations:

Sum = $C_{in} \cdot \bar{A} \cdot \bar{B}$ + $B \cdot \overline{C_{in}} \cdot \bar{A}$ + $A \cdot \overline{C_{in}} \cdot \bar{B}$ + $A \cdot B \cdot C_{in}$

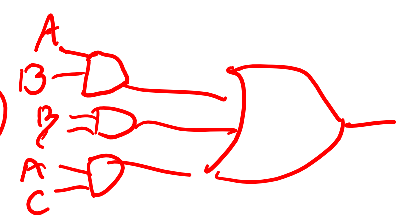
$\bar{A} \cdot \bar{B} \cdot C_{in}$

Truth Table for the above operations:

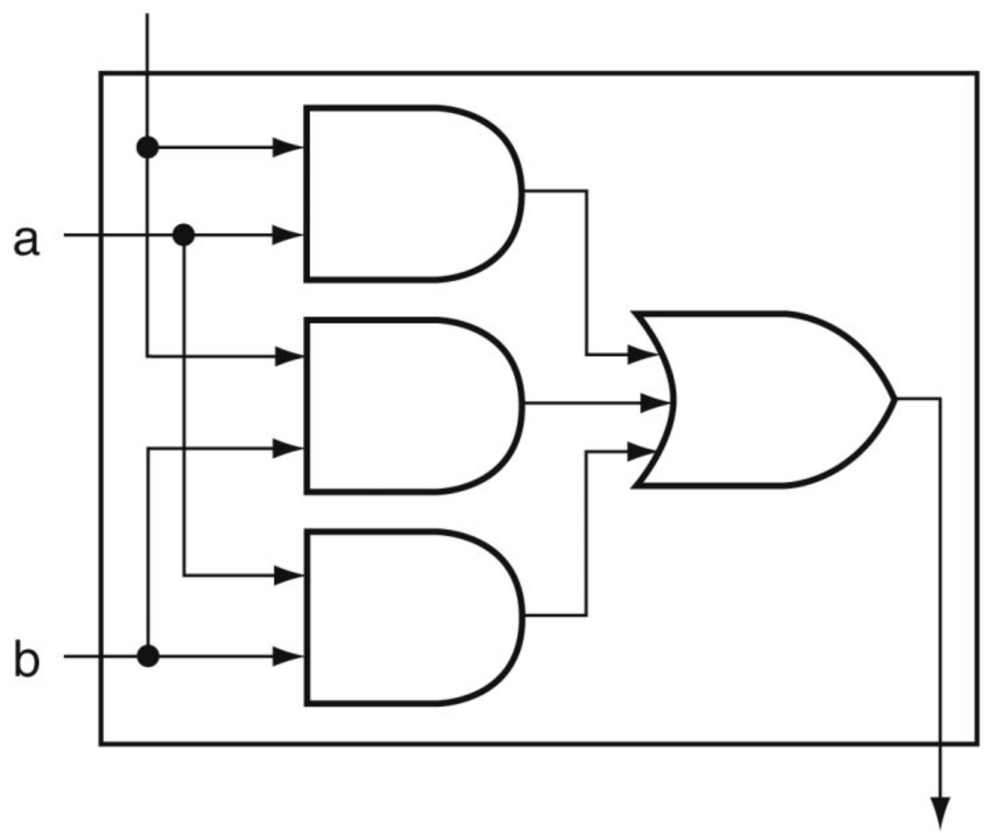
A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Cout = $A \cdot B \cdot C_{in}$ + $A \cdot B \cdot \overline{C_{in}}$ + $A \cdot C_{in} \cdot \bar{B}$ + $B \cdot C_{in} \cdot \bar{A}$

$= \boxed{A \cdot B + A \cdot C_{in} + B \cdot C_{in}}$



Carry Out Logic



Source: H&P textbook

Equations:

$$\text{Sum} = \text{Cin} \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{\text{Cin}} \cdot \overline{A} + A \cdot \overline{\text{Cin}} \cdot \overline{B} + A \cdot B \cdot \text{Cin}$$

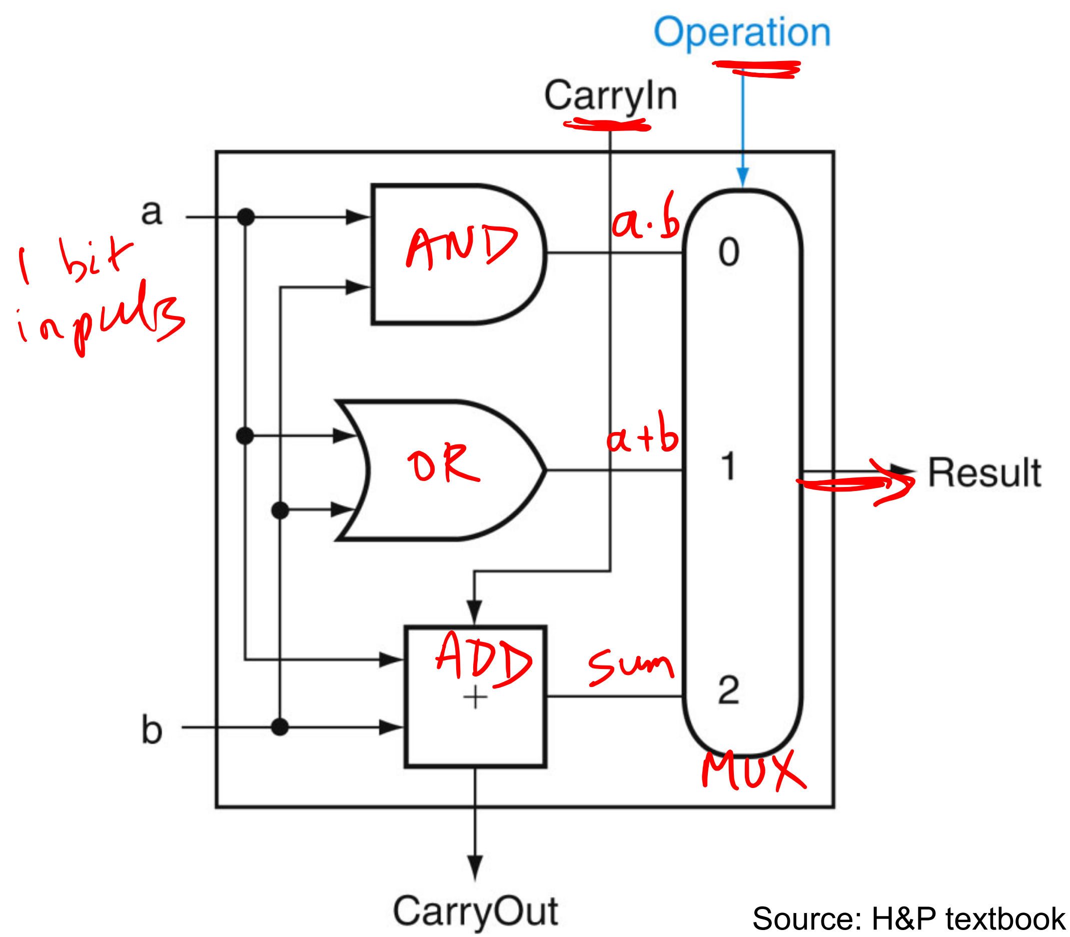
$$\text{Cout} = A \cdot B \cdot \text{Cin} + A \cdot B \cdot \overline{\text{Cin}} + A \cdot \text{Cin} \cdot \overline{B} + B \cdot \text{Cin} \cdot \overline{A}$$

$$= A \cdot B + A \cdot \text{Cin} + B \cdot \text{Cin}$$

Arith + logical unit

1-Bit ALU with Add, Or, And

- Multiplexor selects between Add, Or, And operations



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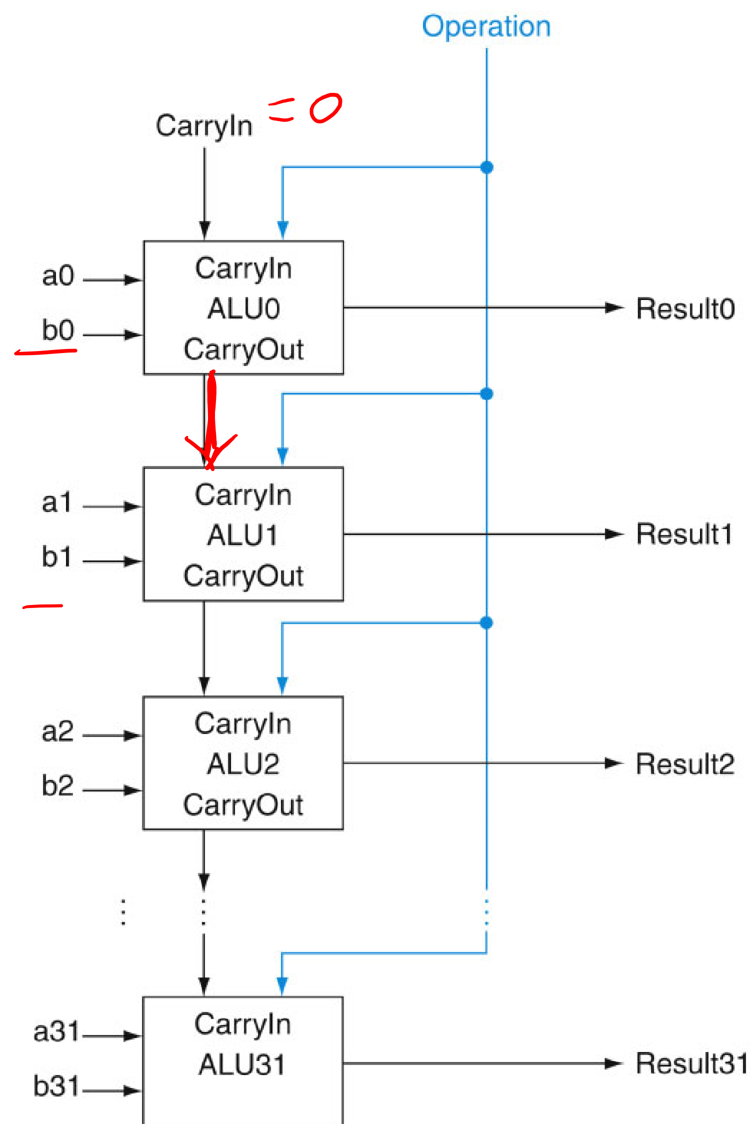
32-bit Ripple Carry Adder

1-bit ALUs are connected "in series" with the carry-out of 1 box going into the carry-in of the next box



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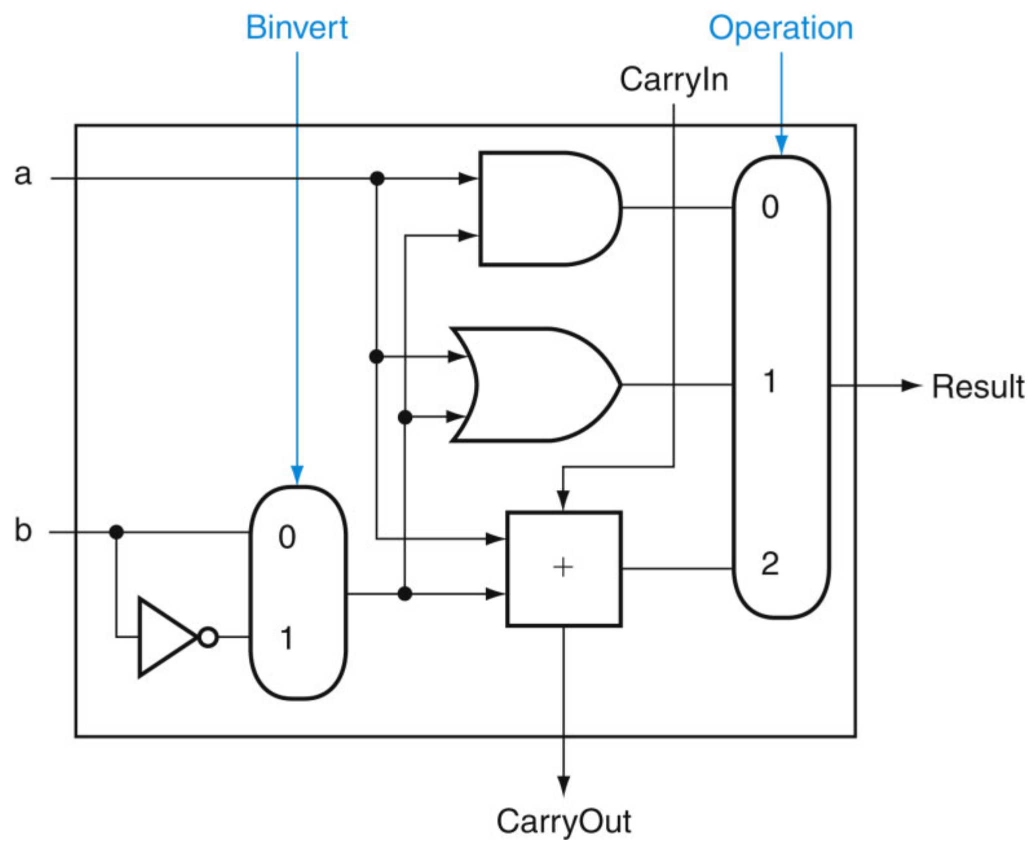
Incorporating Subtraction

Must invert bits of B and add a 1

- Include an inverter
- CarryIn for the first bit is 1
- The CarryIn signal (for the first bit) can be the same as the Binvert signal



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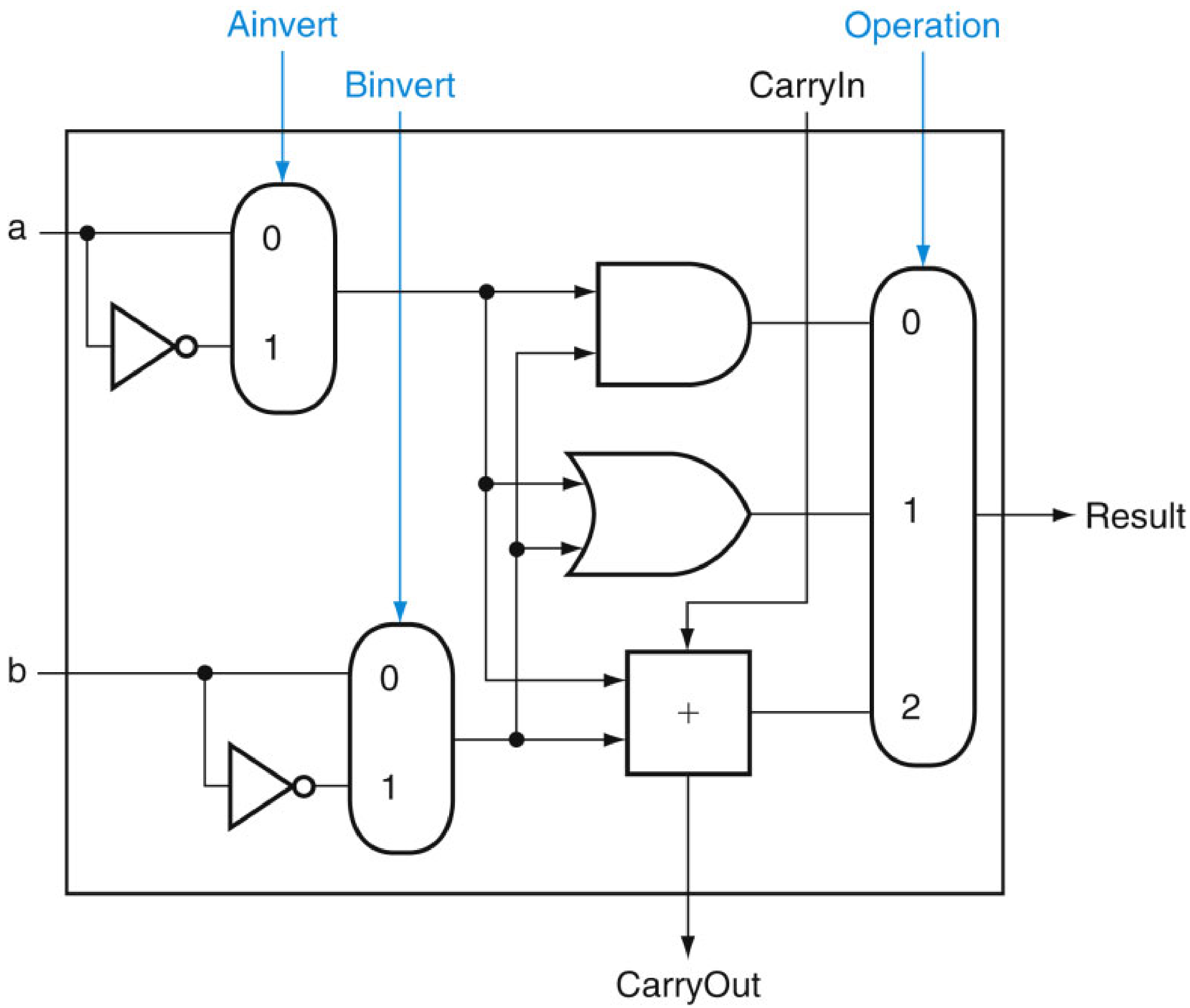
Incorporating NOR and NAND

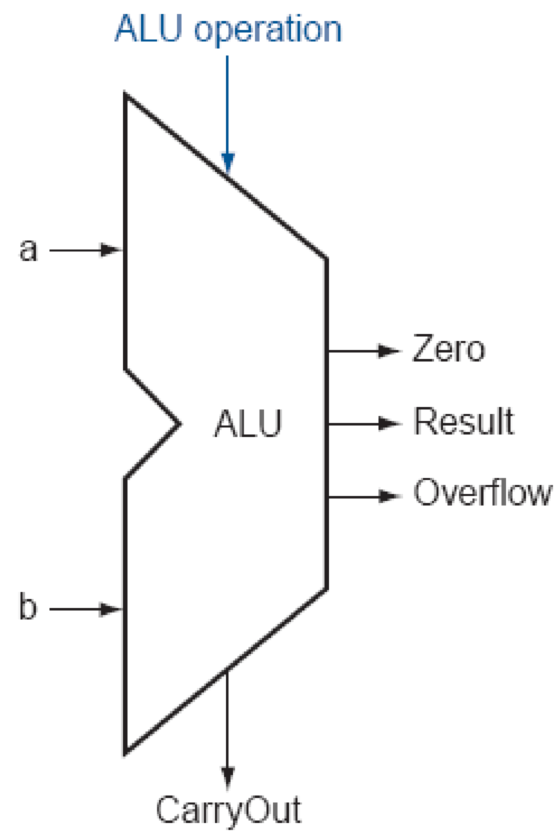


Source: H&P textbook

Control Lines

What are the values
of the control lines
and what operations
do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
NAND	1	1	01
NOR	1	1	00



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