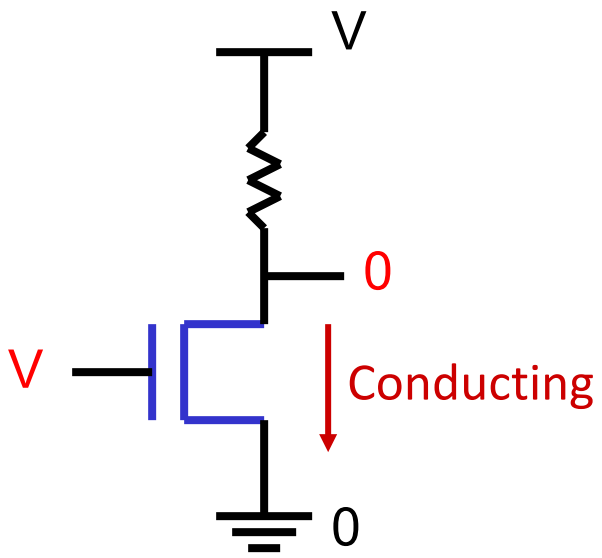
Lecture 12: Hardware for Arithmetic

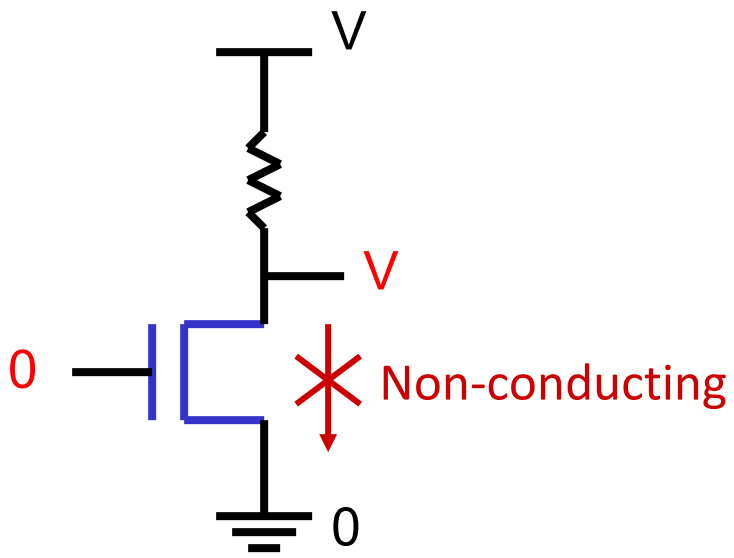
- Today's topics:
  - Digital logic intro
  - Logic for common operations
  - Designing an ALU

Digital Design Basics

- Two voltage levels – high and low (1 and 0, true and false)
  Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch





Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors
- A logic block is termed *combinational* if the output is only a function of the inputs
- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output
- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E

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A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , X = A + B ➔ X is true if at least one of A or B is true
  - AND : symbol **.** , X = A **.** B ➔ X is true if both A and B are true
  - NOT : symbol $\bar{}$, X = $\bar{A}$ ➔ X is the inverted value of A

Boolean Algebra Rules

- Identity law : $A + 0 = A \quad ; \quad A \cdot 1 = A$
- Zero and One laws : $A + 1 = 1 \; ; \; A \cdot 0 = 0$
- Inverse laws : $A \cdot \overline{A} = 0 \; ; \; A + \overline{A} = 1$
- Commutative laws : $A + B = B + A \quad ; \quad A \cdot B = B \cdot A$
- Associative laws : $A + (B + C) = (A + B) + C$

  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- Distributive laws : $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

  $A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's Laws

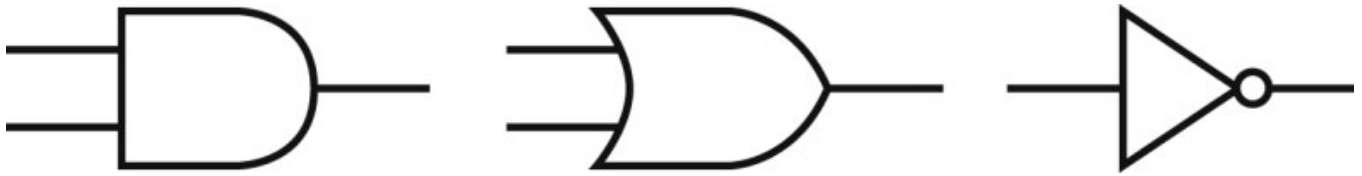
- $\overline{A + B} = \overline{A} \cdot \overline{B}$

- $\overline{A \cdot B} = \overline{A} + \overline{B}$

- Confirm that these are indeed true

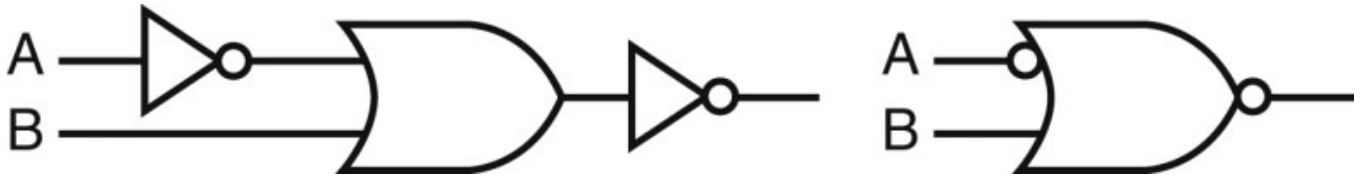
Pictorial Representations

AND OR NOT



Source: H&P textbook

What logic function is this?



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Boolean Equation

- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

Multiple correct equations:

Two must be true, but all three cannot be true:

$E = ((A \cdot B) + (B \cdot C) + (A \cdot C)) \cdot \overline{(A \cdot B \cdot C)}$

Identify the three cases where it is true:

$E = (A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$

Sum of Products

- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$(A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$

- Can also use “product of sums”
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

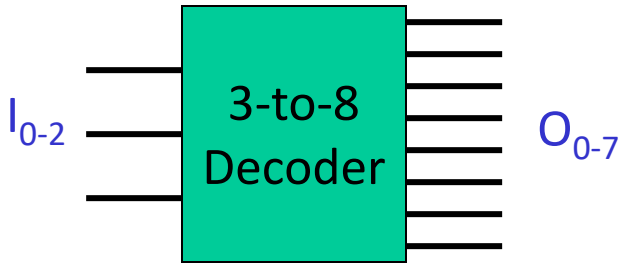
NAND and NOR

- NAND : NOT of AND : A nand B = $\overline{A \cdot B}$
- NOR : NOT of OR : A nor B = $\overline{A + B}$
- NAND and NOR are *universal gates*, i.e., they can be used to construct any complex logical function

Common Logic Blocks – Decoder

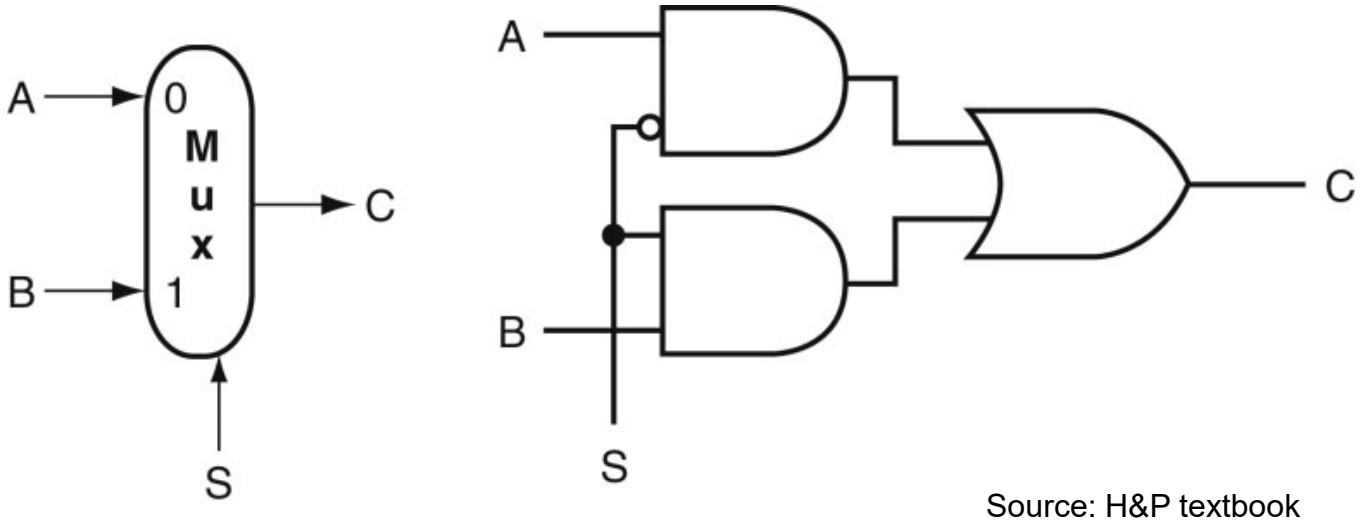
Takes in N inputs and activates one of 2^N outputs

I_0	I_1	I_2	O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



Common Logic Blocks – Multiplexor

- Multiplexor or selector: one of N inputs is reflected on the output depending on the value of the $\log_2 N$ selector bits



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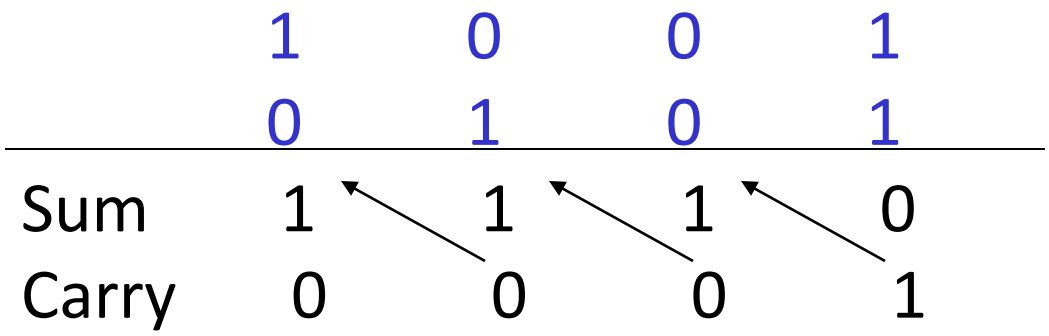
2-input mux

Adder Algorithm



Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Adder Algorithm

	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1

Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

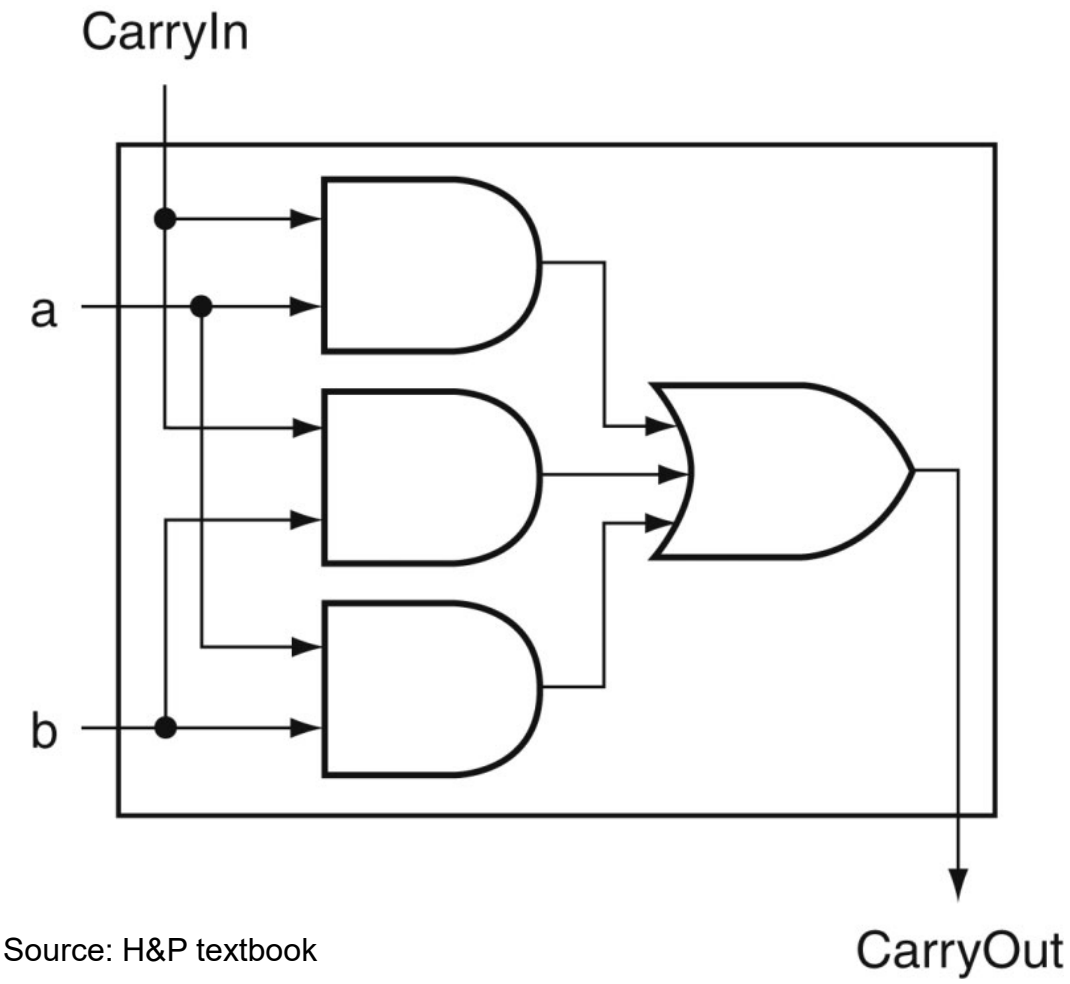
Equations:

$$Sum = Cin \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{Cin} \cdot \overline{A} + A \cdot \overline{Cin} \cdot \overline{B} + A \cdot B \cdot Cin$$

$$Cout = A \cdot B \cdot Cin + A \cdot B \cdot \overline{Cin} + A \cdot Cin \cdot \overline{B} + B \cdot Cin \cdot \overline{A}$$

$$= A \cdot B + A \cdot Cin + B \cdot Cin$$

Carry Out Logic



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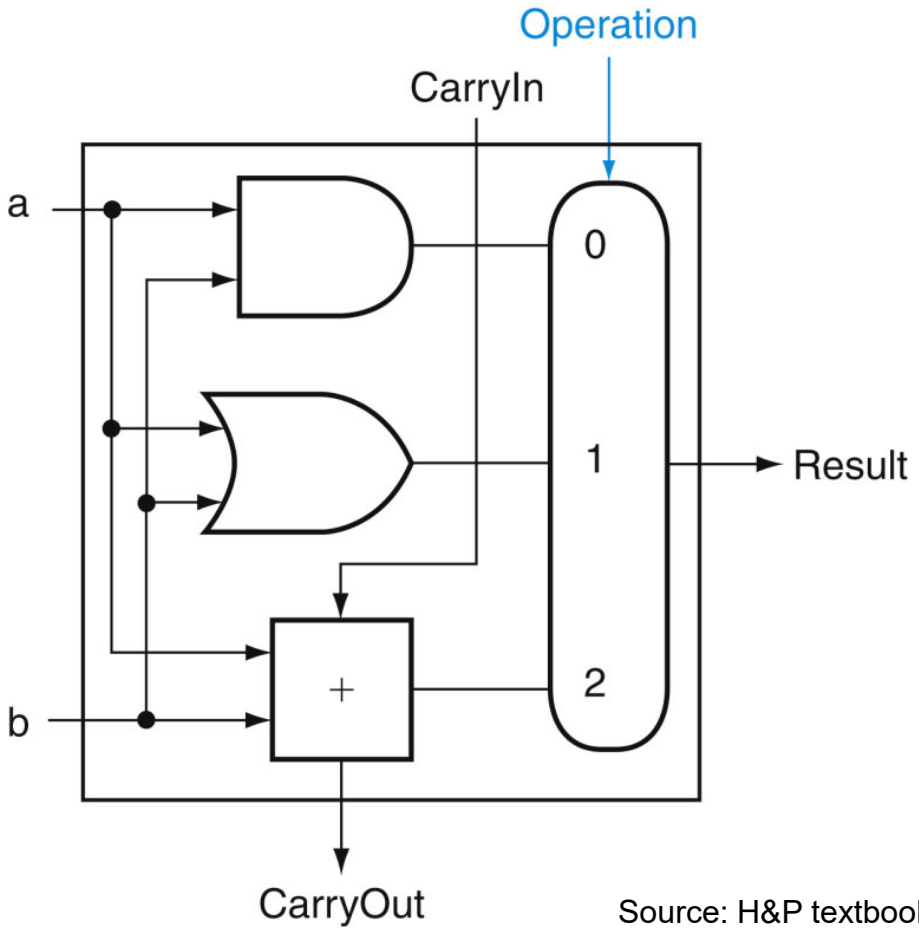
Equations:

$$\text{Sum} = Cin \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{Cin} \cdot \overline{A} + A \cdot \overline{Cin} \cdot \overline{B} + A \cdot B \cdot Cin$$

$$\text{Cout} = A \cdot B \cdot Cin + A \cdot B \cdot \overline{Cin} + A \cdot Cin \cdot \overline{B} + B \cdot Cin \cdot \overline{A}$$
$$= A \cdot B + A \cdot Cin + B \cdot Cin$$

1-Bit ALU with Add, Or, And

- Multiplexor selects between Add, Or, And operations



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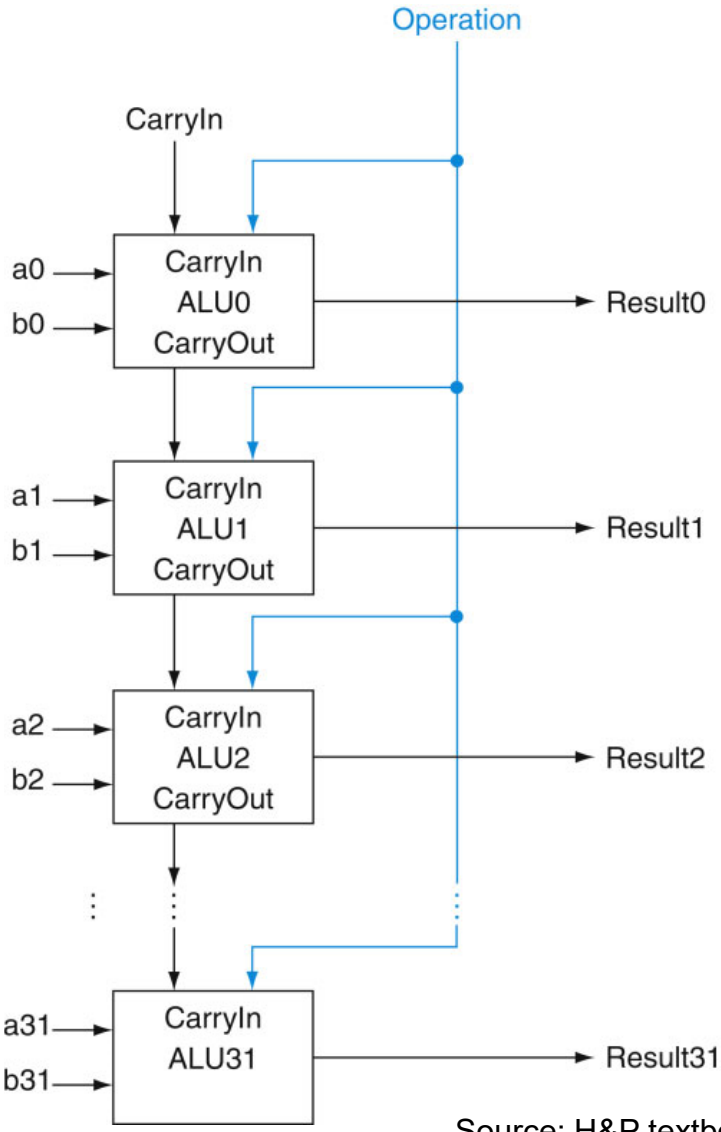
32-bit Ripple Carry Adder

1-bit ALUs are connected "in series" with the carry-out of 1 box going into the carry-in of the next box



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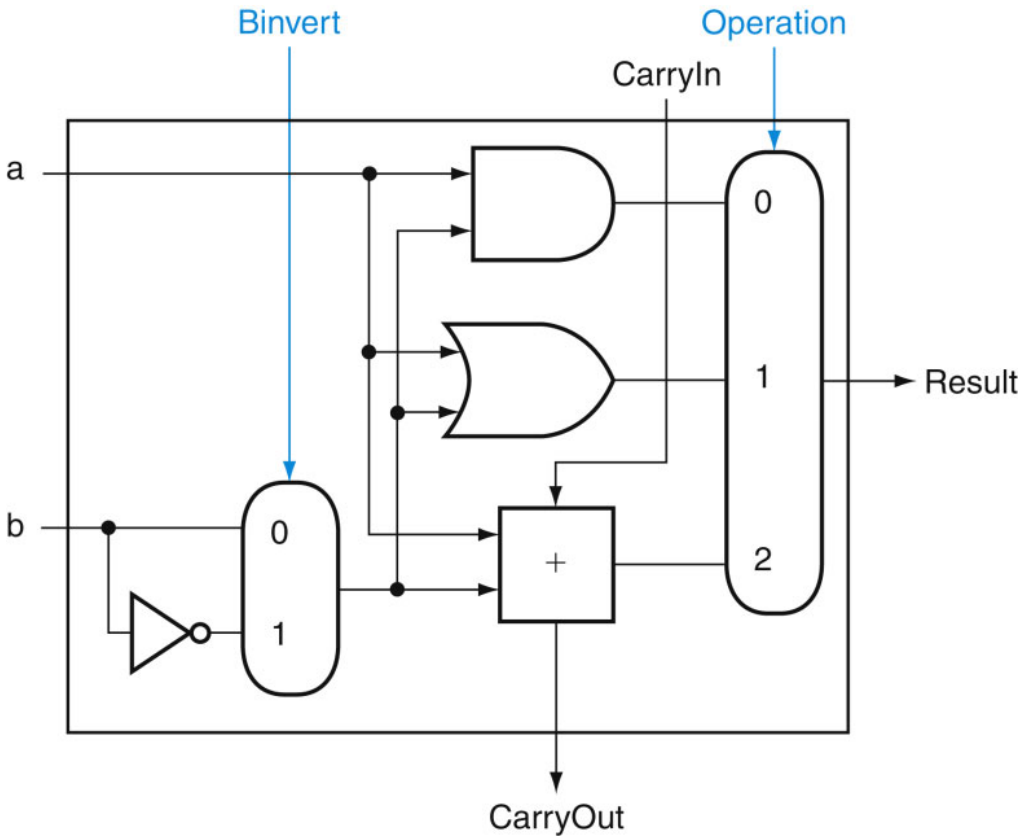
Incorporating Subtraction

Must invert bits of B and add a 1

- Include an inverter
- CarryIn for the first bit is 1
- The CarryIn signal (for the first bit) can be the same as the Binvert signal



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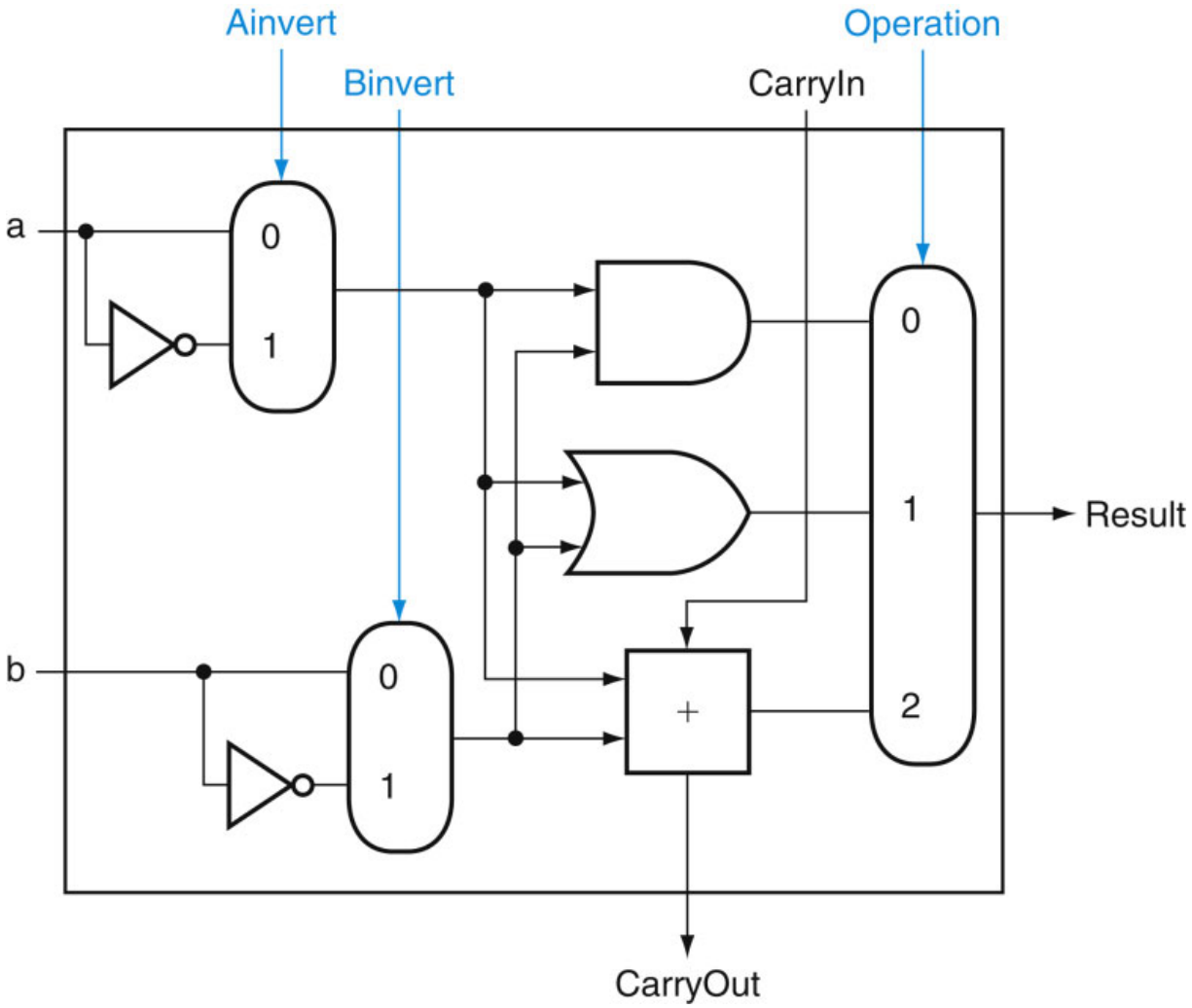
Incorporating NOR and NAND



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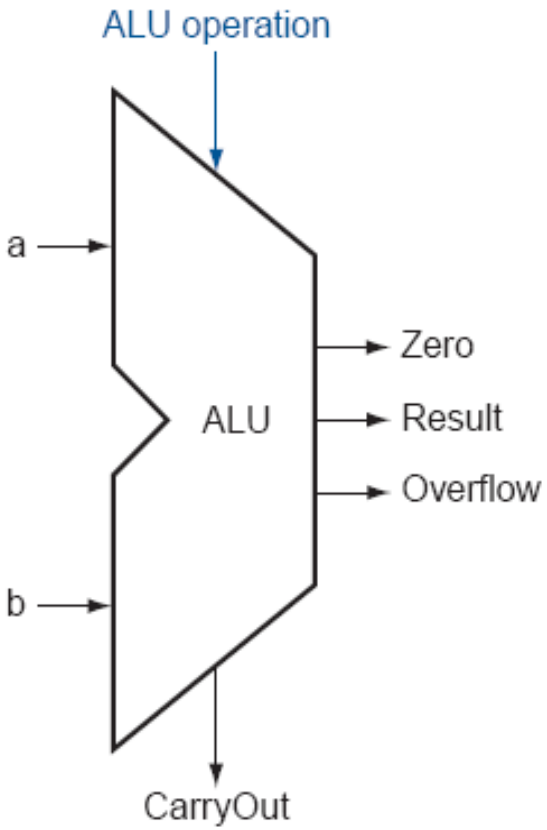
Control Lines

What are the values of the control lines and what operations do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
NAND	1	1	01
NOR	1	1	00



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