Lecture 11: Floating Point, Digital Design

- Today's topics:
  - FP formats, arithmetic
  - Intro to Boolean functions

Value inf — 2 special cases up top that use the reserved exponent field of 255 — 0 255 00…0

Value NAN — 0 255 xx….x

Highest value ~2×2^{127} — 0 254 11….1

Value 1 — 0 127 00…0

Exponent field < 127, i.e., after subtracting bias, they are negative exponents, representing numbers < 1

Smallest Norm ~2×2^{-126} — 0 0..01 00…0

Largest Denorm ~1×2^{-126} — 0 0..00 11…1

Smallest Denorm ~2^{-149} — 0 0..00 00…1

Special case with exponent field 0, used to represent denorms, that help us gradually approach 0

Value 0 — 0 00..0 00…0

Same rules as above, but the sign bit is 1

Same magnitudes as above, but negative numbers

Example 2

Final representation: $(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$

- Represent 36.90625_{ten} in single-precision format

36 / 2 = 18 rem 0	0.90625 x 2 = 1.81250
18 / 2 = 9 rem 0	0.8125 x 2 = 1.6250
9 / 2 = 4 rem 1	0.625 x 2 = 1.250
4 / 2 = 2 rem 0	0.25 x 2 = 0.50
2 / 2 = 1 rem 0	0.5 x 2 = 1.00
1 / 2 = 0 rem 1	0.0 x 2 = 0.0

36 is 100100

0.90625 is 0.1110100…0

Example 2

Final representation: $(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$

We've calculated that 36.90625_{ten} = 100100.1110100…0 in binary
Normalized form = $1.001001110100…0 \times 2^5$
(had to shift 5 places to get only one bit left of the point)

The sign bit is 0 (positive number)
The fraction field is 001001110100…0 (the 23 bits after the point)
The exponent field is 5 + 127 (have to add the bias) = 132,
which in binary is 10000100

The IEEE 754 format is 0 10000100 001001110100…..0
sign exponent 23 fraction bits

Examples

Final representation: $(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$

- Represent -0.75_{ten} in single and double-precision formats

Single: (1 + 8 + 23)

Double: (1 + 11 + 52)

Remember:

True exponent → (+127) → Exponent in register

Exponent in register → (-127) → True exponent

- What decimal number is represented by the following single-precision number?

1 1000 0001 01000…0000

Examples

Final representation: $(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$

- Represent -0.75_{ten} in single and double-precision formats

  Single: (1 + 8 + 23)
  1 0111 1110 1000…000

  Double: (1 + 11 + 52)
  1 0111 1111 110 1000…000

- What decimal number is represented by the following single-precision number?
  1 1000 0001 01000…0000
  -5.0

FP Addition

- Consider the following decimal example (can maintain only 4 decimal digits and 2 exponent digits)

$9.999 \times 10^{1} + 1.610 \times 10^{-1}$

Convert to the larger exponent:

$9.999 \times 10^{1} + 0.016 \times 10^{1}$

Add

10.015×10^{1}

Normalize

1.0015×10^{2}

Check for overflow/underflow

Round

1.002×10^{2}

Re-normalize

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Re-normalize

If we had more fraction bits, these errors would be minimized

FP Addition – Binary Example

- Consider the following binary example

$1.010 \times 2^{1} + 1.100 \times 2^{3}$

Convert to the larger exponent:

$0.0101 \times 2^{3} + 1.1000 \times 2^{3}$

Add

1.1101×2^{3}

Normalize

1.1101×2^{3}

Check for overflow/underflow

Round

Re-normalize

IEEE 754 format: 0 10000010 11010000000000000000000

FP Multiplication

- Similar steps:
  - Compute exponent (careful!)
  - Multiply significands (set the binary point correctly)
  - Normalize
  - Round (potentially re-normalize)
  - Assign sign

MIPS Instructions

- The usual add.s, add.d, sub, mul, div
- Comparison instructions: c.eq.s, c.neq.s, c.lt.s….
  These comparisons set an internal bit in hardware that is then inspected by branch instructions: bc1t, bc1f
- Separate register file \$f0 - \$f31 : a double-precision value is stored in (say) \$f4-\$f5 and is referred to by \$f4
- Load/store instructions (lwc1, swc1) must still use integer registers for address computation

Code Example

```
float  f2c (float fahr)
{
    return ((5.0/9.0) * (fahr – 32.0));
}
```

(argument fahr is stored in $f12)

```
lwc1   $f16, const5
lwc1   $f18, const9
div.s  $f16, $f16, $f18
lwc1   $f18, const32
sub.s  $f18, $f12, $f18
mul.s  $f0, $f16, $f18
jr     $ra
```

Fixed Point

- FP operations are much slower than integer ops
- Fixed point arithmetic uses integers, but assumes that every number is multiplied by the same factor
- Example: with a factor of 1/1000, the fixed-point representations for 1.46, 1.7198, and 5624 are respectively 1460, 1720, and 5624000
- More programming effort and possibly lower precision for higher performance

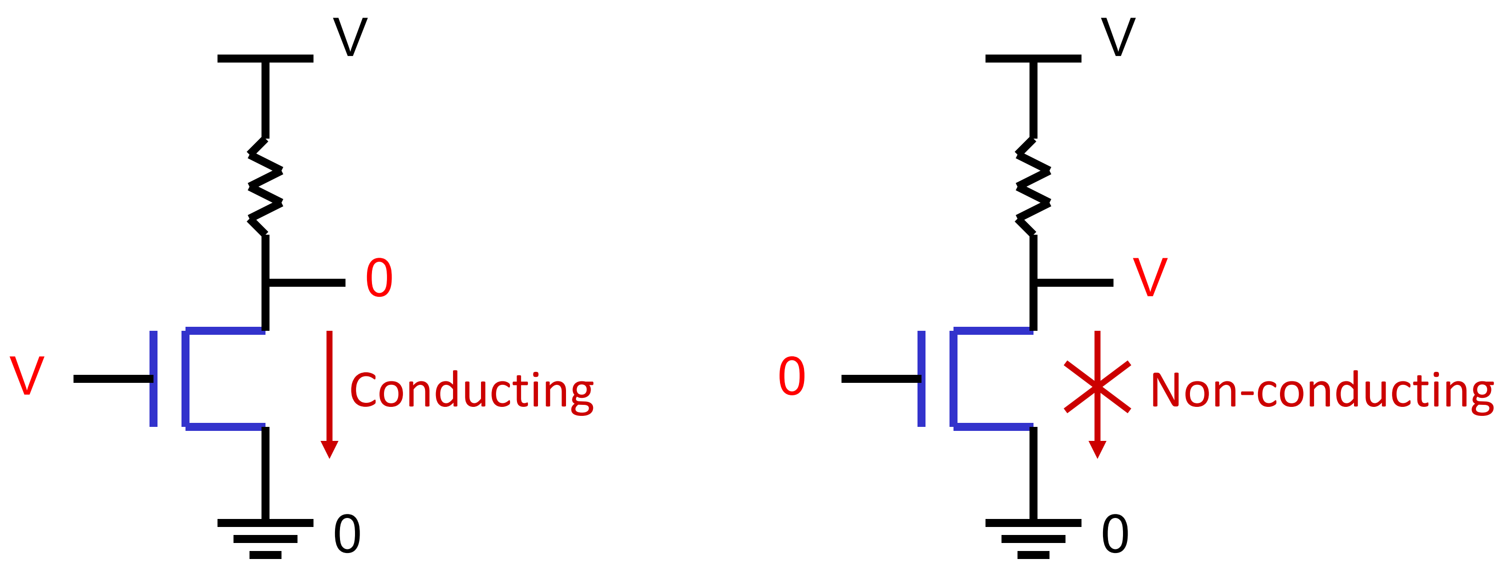
Subword Parallelism

- ALUs are typically designed to perform 64-bit or 128-bit arithmetic
- Some data types are much smaller, e.g., bytes for pixel RGB values, half-words for audio samples
- Partitioning the carry-chains within the ALU can convert the 64-bit adder into 4 16-bit adders or 8 8-bit adders
- A single load can fetch multiple values, and a single add instruction can perform multiple parallel additions, referred to as subword parallelism

Digital Design Basics

- Two voltage levels – high and low (1 and 0, true and false)
  Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch



Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors
- A logic block is termed *combinational* if the output is only a function of the inputs
- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output
- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E

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- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , X = A + B → X is true if at least one of A or B is true
  - AND : symbol **.** , X = A **.** B → X is true if both A and B are true
  - NOT : symbol $\overline{}$, $X = \overline{A}$ → X is the inverted value of A

Boolean Algebra Rules

- Identity law : $A + 0 = A$; $A \cdot 1 = A$
- Zero and One laws : $A + 1 = 1$; $A \cdot 0 = 0$
- Inverse laws : $A \cdot \overline{A} = 0$; $A + \overline{A} = 1$
- Commutative laws : $A + B = B + A$; $A \cdot B = B \cdot A$
- Associative laws : $A + (B + C) = (A + B) + C$
  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- Distributive laws : $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
  $A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's Laws

- $\overline{A + B} = \overline{A} \cdot \overline{B}$

- $\overline{A \cdot B} = \overline{A} + \overline{B}$

- Confirm that these are indeed true