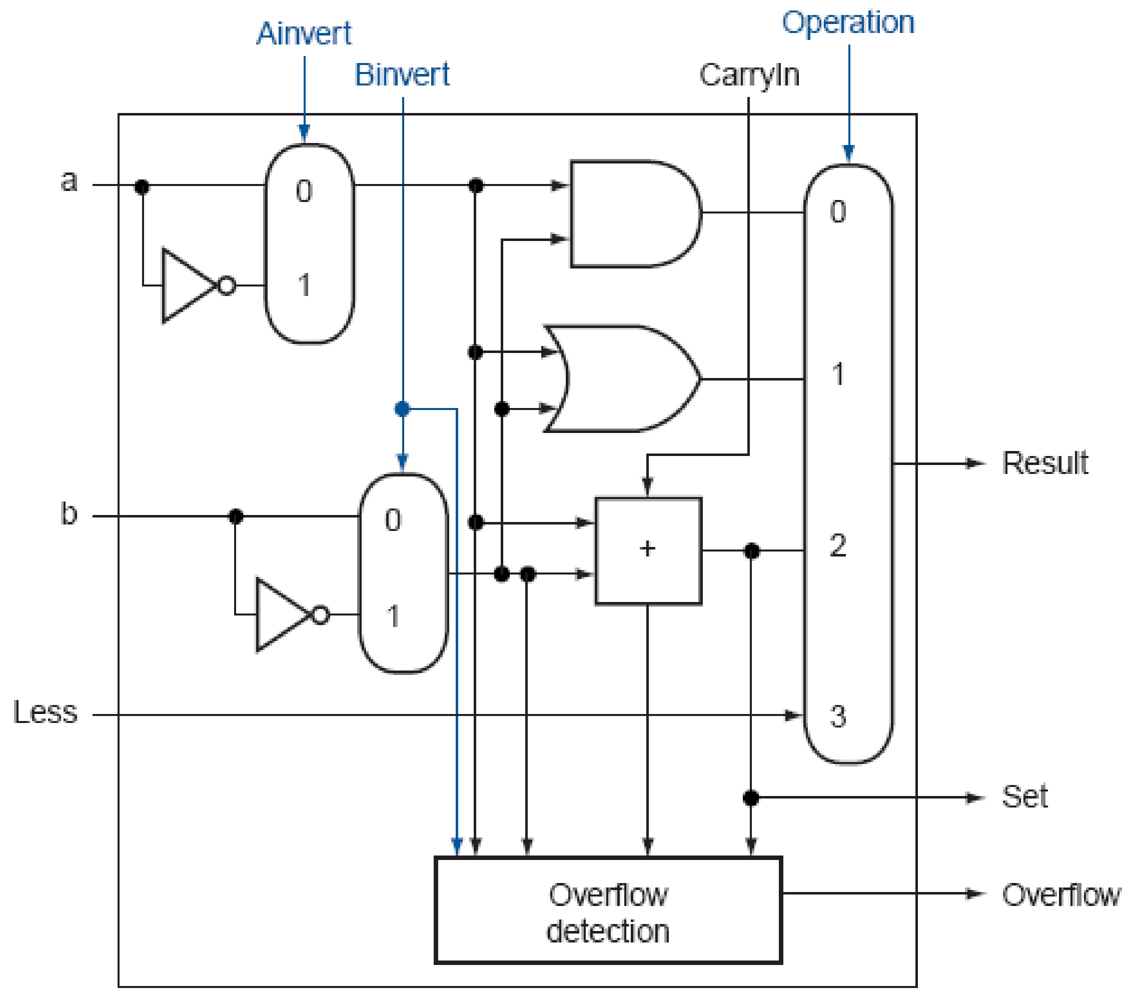
Lecture 13: Adders, Sequential Circuits

- Today's topics:
  - Carry-lookahead adder
  - Clocks, latches, sequential circuits

Incorporating slt

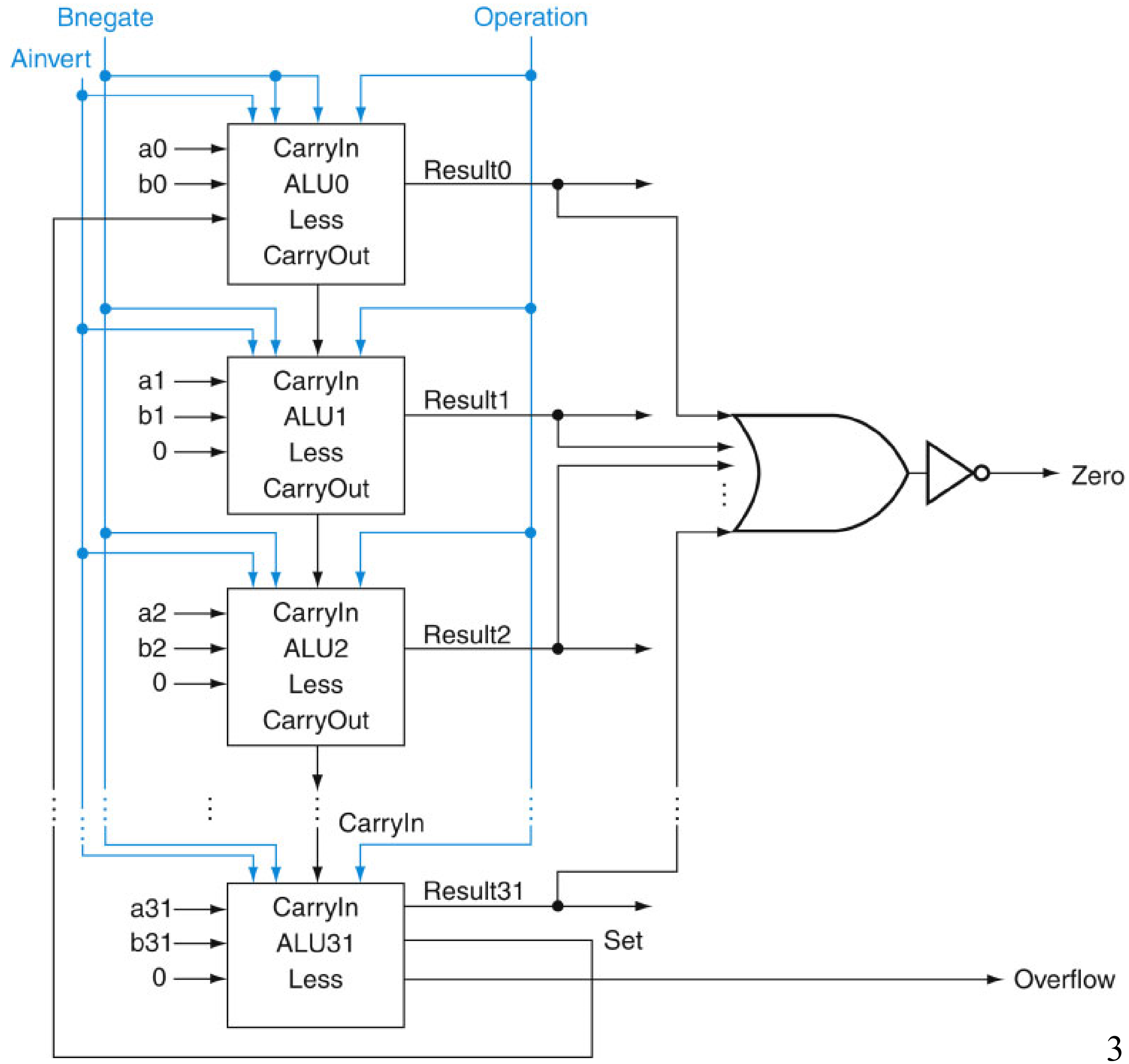
- Perform a – b and check the sign
- New signal (Less) that is zero for ALU boxes 1-31
- The 31st box has a unit to detect overflow and sign – the sign bit serves as the Less signal for the 0th box



Source: H&P textbook

Incorporating beq

- Perform a – b and confirm that the result is all zero's



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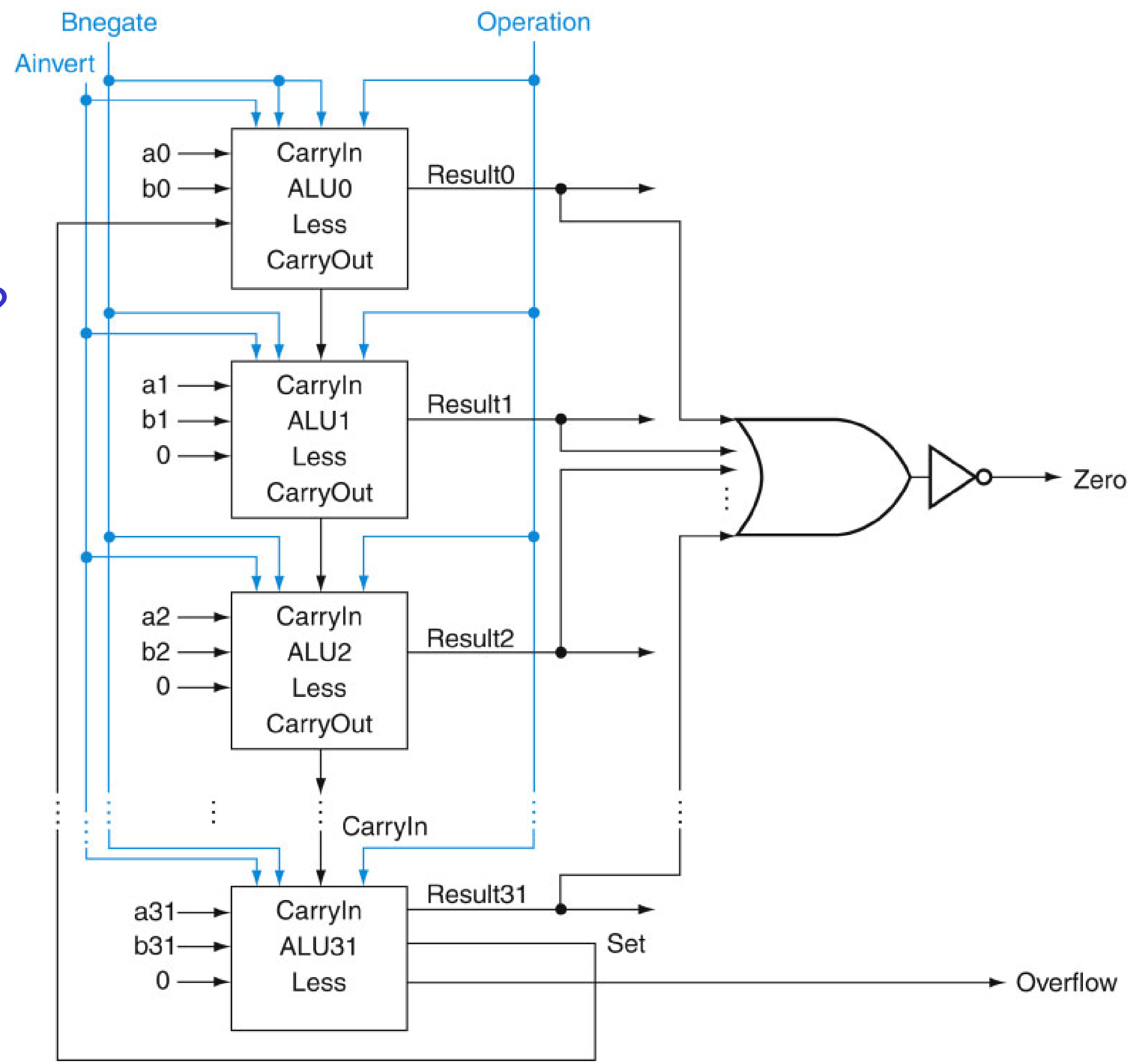
Control Lines

What are the values of the control lines and what operations do they correspond to?

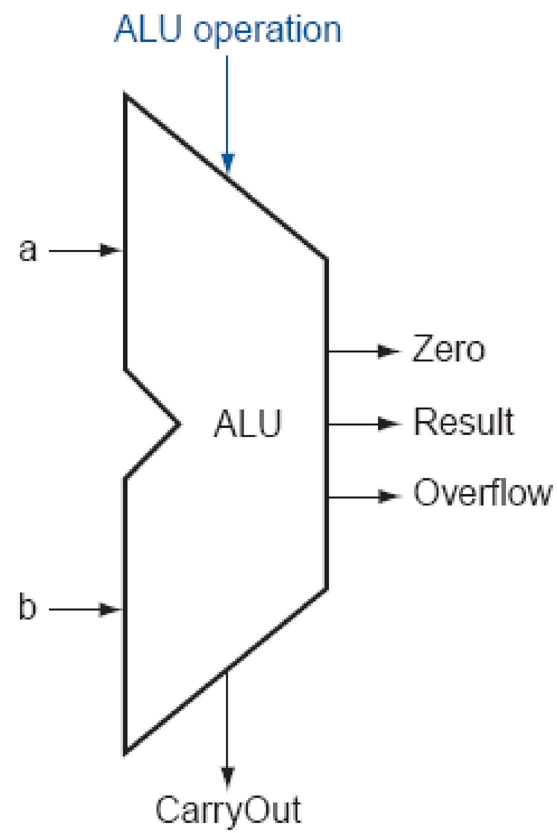


Source: H&P textbook

Control Lines

What are the values of the control lines and what operations do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
SLT	0	1	11
NOR	1	1	00
NAND	1	1	01



Source: H&P textbook

Speed of Ripple Carry

- The carry propagates thru every 1-bit box: each 1-bit box sequentially implements AND and OR – total delay is the time to go through 64 gates!

- We've already seen that any logic equation can be expressed as the sum of products – so it should be possible to compute the result by going through only 2 gates!

- Caveat: need many parallel gates and each gate may have a very large number of inputs – it is difficult to efficiently build such large gates, so we'll find a compromise:
    - moderate number of gates
    - moderate number of inputs to each gate
    - moderate number of sequential gates traversed

Computing CarryOut

CarryIn1 = b0.CarryIn0 + a0.CarryIn0 + a0.b0
CarryIn2 = b1.CarryIn1 + a1.CarryIn1 + a1.b1
= b1.b0.c0 + b1.a0.c0 + b1.a0.b0 +
a1.b0.c0 + a1.a0.c0 + a1.a0.b0 + a1.b1
…
CarryIn32 = a really large sum of really large products

- Potentially fast implementation as the result is computed by going thru just 2 levels of logic – unfortunately, each gate is enormous and slow

Generate and Propagate

Equation re-phrased:

$C_{i+1} = a_i.b_i + a_i.C_i + b_i.C_i$

$= (a_i.b_i) + (a_i + b_i).C_i$

Stated verbally, the current pair of bits will *generate* a carry if they are both 1 and the current pair of bits will *propagate* a carry if either is 1

Generate signal = $a_i.b_i$

Propagate signal = $a_i + b_i$

Therefore, $C_{i+1} = G_i + P_i . C_i$

Generate and Propagate

$$c1 = g0 + p0.c0$$

$$c2 = g1 + p1.c1$$

$$= g1 + p1.g0 + p1.p0.c0$$

$$c3 = g2 + p2.g1 + p2.p1.g0 + p2.p1.p0.c0$$

$$c4 = g3 + p3.g2 + p3.p2.g1 + p3.p2.p1.g0 + p3.p2.p1.p0.c0$$

Either,

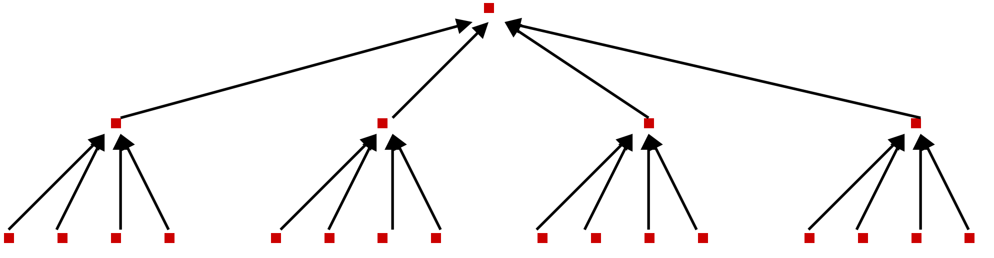
- a carry was just generated, or
- a carry was generated in the last step and was propagated, or
- a carry was generated two steps back and was propagated by both the next two stages, or
- a carry was generated N steps back and was propagated by every single one of the N next stages

Divide and Conquer

- The equations on the previous slide are still difficult to implement as logic functions – for the 32^{nd} bit, we must AND every single propagate bit to determine what becomes of c0 (among other things)

- Hence, the bits are broken into groups (of 4) and each group computes its group-generate and group-propagate

- For example, to add 32 numbers, you can partition the task as a tree



P and G for 4-bit Blocks

- Compute P0 and G0 (super-propagate and super-generate) for the first group of 4 bits (and similarly for other groups of 4 bits)
  P0 = p0.p1.p2.p3
  G0 = g3 + g2.p3 + g1.p2.p3 + g0.p1.p2.p3

- Carry out of the first group of 4 bits is
  C1 = G0 + P0.c0
  C2 = G1 + P1.G0 + P1.P0.c0
  C3 = G2 + (P2.G1) + (P2.P1.G0) + (P2.P1.P0.c0)
  C4 = G3 + (P3.G2) + (P3.P2.G1) + (P3.P2.P1.G0) + (P3.P2.P1.P0.c0)

- By having a tree of sub-computations, each AND, OR gate has few inputs and logic signals have to travel through a modest set of gates (equal to the height of the tree)

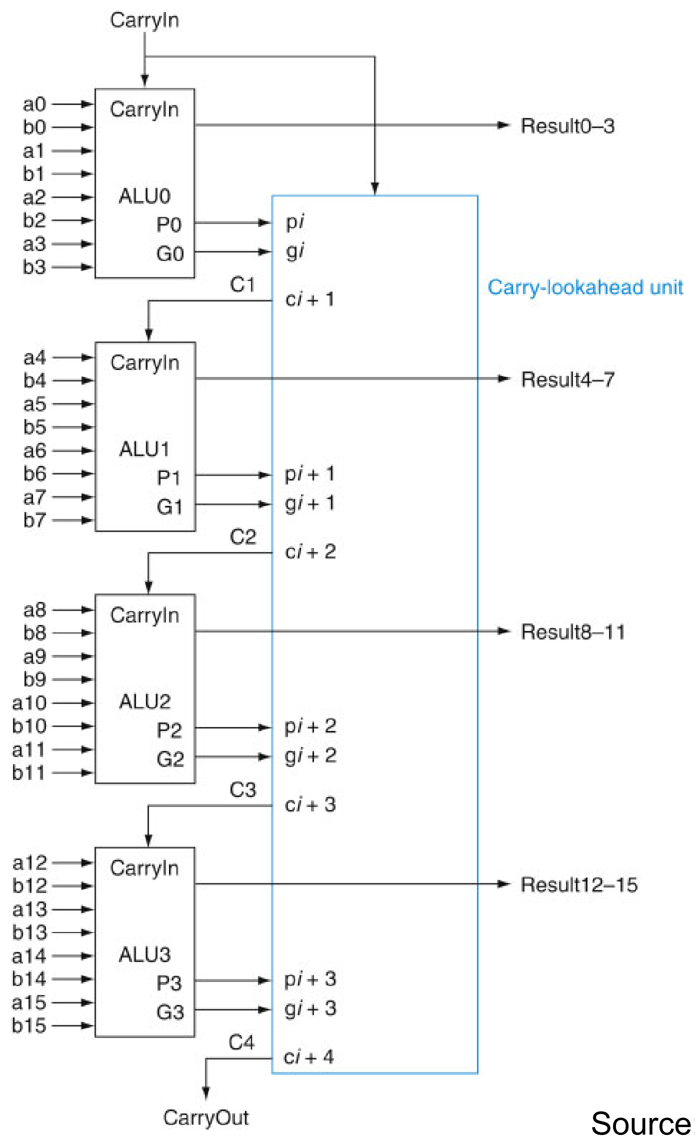
Example

Add	A	0001	1010	0011	0011
	B	1110	0101	1110	1011
	g	0000	0000	0010	0011
	p	1111	1111	1111	1011
	P	1	1	1	0
	G	0	0	1	0

C4 = 1

Carry Look-Ahead Adder

- 16-bit Ripple-carry takes 32 steps
- This design takes how many steps?

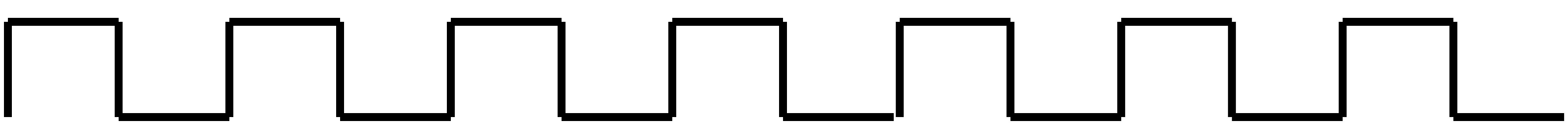


Source: H&P textbook

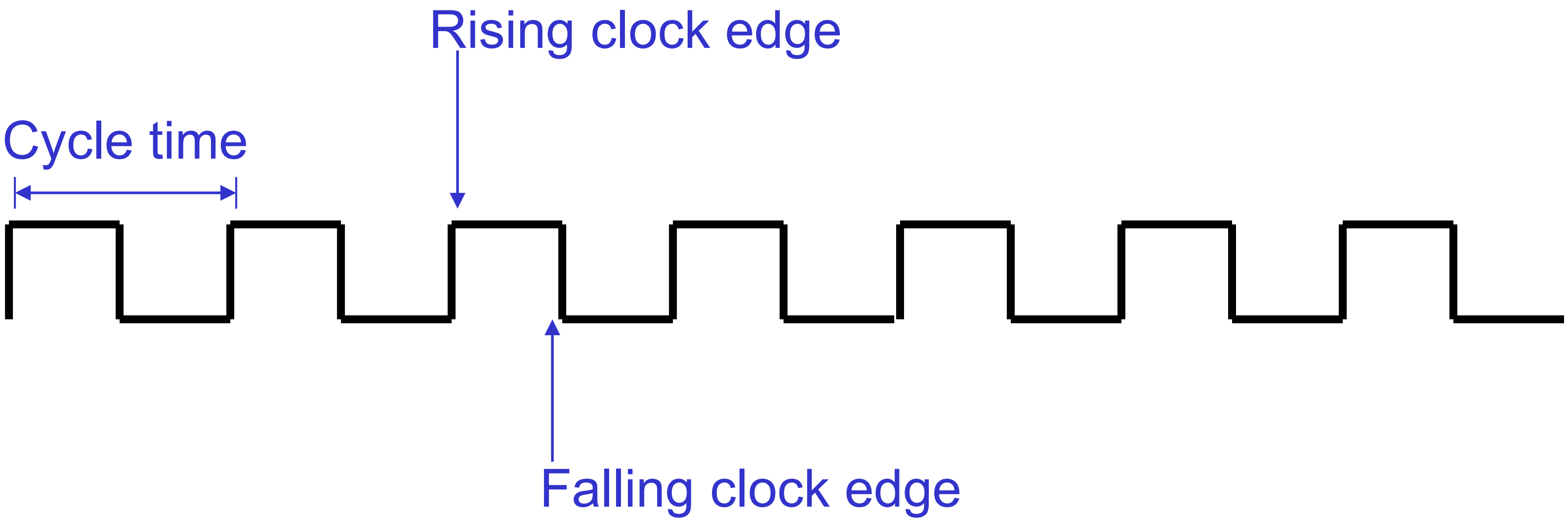
Clocks

- A microprocessor is composed of many different circuits that are operating simultaneously – if each circuit X takes in inputs at time TI_X, takes time TE_X to execute the logic, and produces outputs at time TO_X, imagine the complications in co-ordinating the tasks of every circuit

- A major school of thought (used in most processors built today): all circuits on the chip share a clock signal (a square wave) that tells every circuit when to accept inputs, how much time they have to execute the logic, and when they must produce outputs



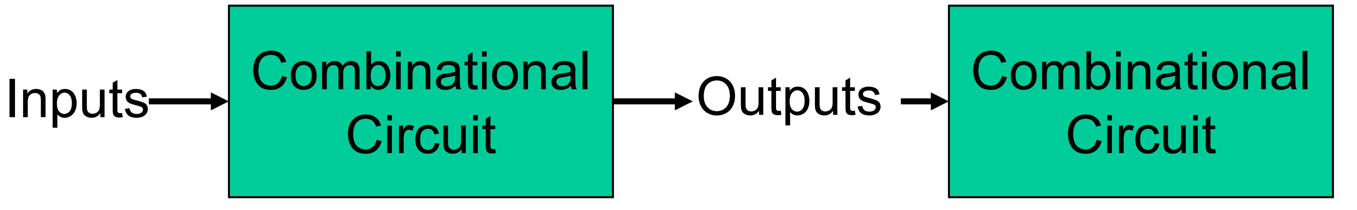
Clock Terminology



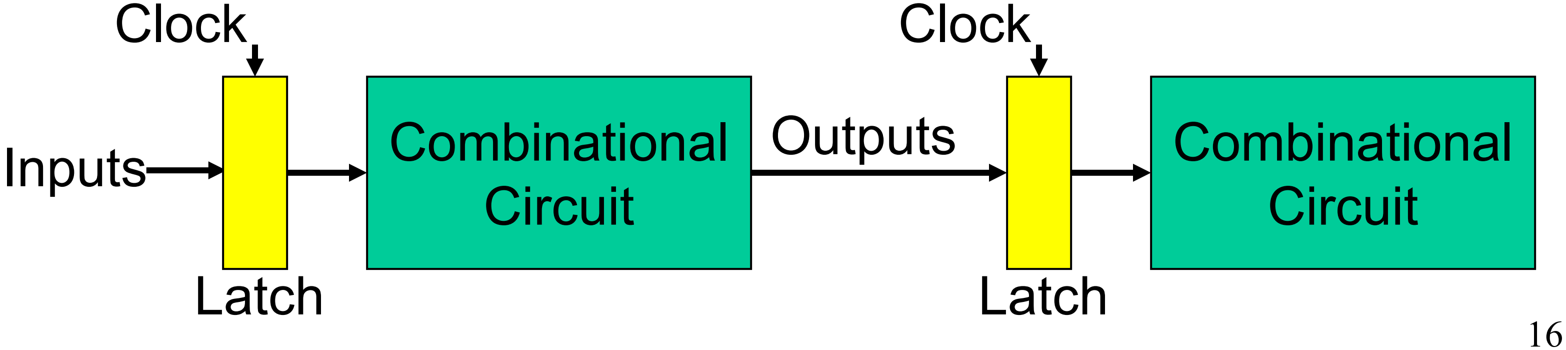
$$4 \text{ GHz} = \text{clock speed} = \frac{1}{\text{cycle time}} = \frac{1}{250 \text{ ps}}.$$

Sequential Circuits

- Until now, circuits were combinational – when inputs change, the outputs change after a while (time = logic delay thru circuit)

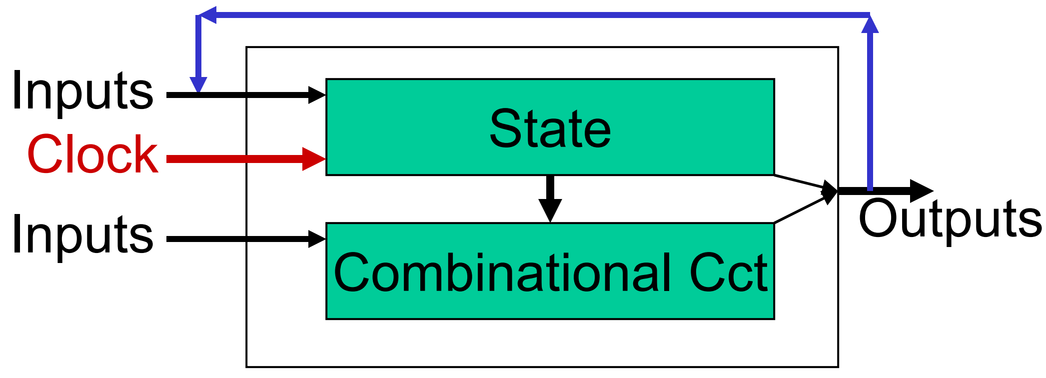


- We want the clock to act like a start and stop signal – a “latch” is a storage device that separates these circuits – it ensures that the inputs to the circuit do not change during a clock cycle



Sequential Circuits

- Sequential circuit: consists of combinational circuit and a storage element



- At the start of the clock cycle, the rising edge causes the “state” storage to store some input values

- This state will not change for an entire cycle (until next rising edge)

- The combinational circuit has some time to accept the value of “state” and “inputs” and produce “outputs”

- Some of the outputs (for example, the value of next “state”) may feed back (but through the latch so they’re only seen in the next cycle)