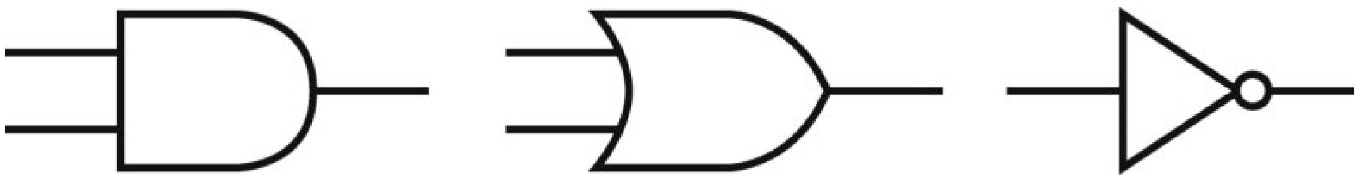
Lecture 11: Hardware for Arithmetic

- Today's topics:
  - Logic for common operations
  - Designing an ALU

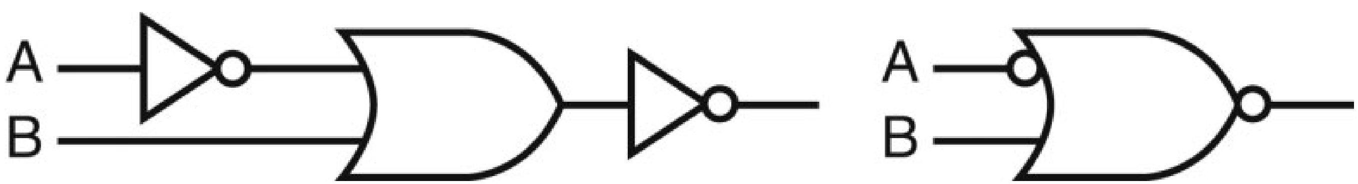
Pictorial Representations

AND OR NOT



Source: H&P textbook

What logic function is this?



Source: H&P textbook

Boolean Equation

- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

  Multiple correct equations:

  Two must be true, but all three cannot be true:

  $E = ((A \,.\, B) + (B \,.\, C) + (A \,.\, C)) \,.\, (\overline{A \,.\, B \,.\, C})$

  Identify the three cases where it is true:

  $E = (A \,.\, B \,.\, \overline{C}) + (A \,.\, C \,.\, \overline{B}) + (C \,.\, B \,.\, \overline{A})$

Sum of Products

- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$(A \,.\, B \,.\, \overline{C}) + (A \,.\, C \,.\, \overline{B}) + (C \,.\, B \,.\, \overline{A})$$

- Can also use “product of sums”
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

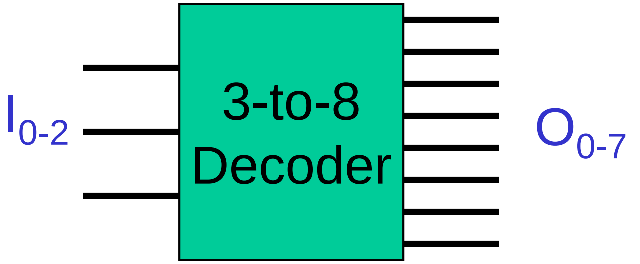
NAND and NOR

- NAND : NOT of AND : A nand B = $\overline{A . B}$
- NOR : NOT of OR : A nor B = $\overline{A + B}$
- NAND and NOR are *universal gates*, i.e., they can be used to construct any complex logical function

Common Logic Blocks – Decoder

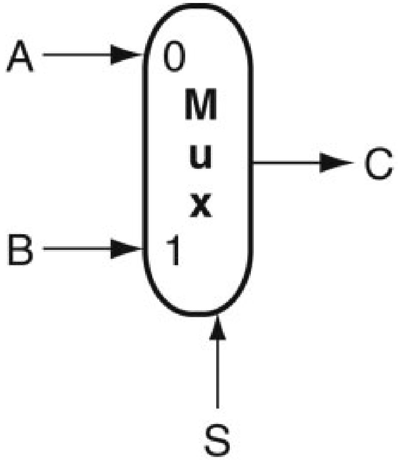
Takes in N inputs and activates one of 2^N outputs

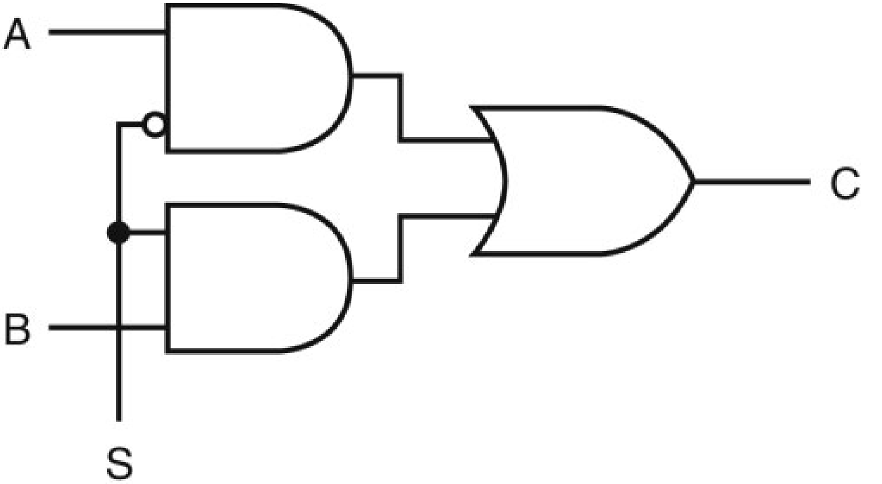
I_0	I_1	I_2	O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



Common Logic Blocks – Multiplexor

- Multiplexor or selector: one of N inputs is reflected on the output depending on the value of the $\log_2 N$ selector bits





Source: H&P textbook

2-input mux

Adder Algorithm

	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1

Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Adder Algorithm

	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1

Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

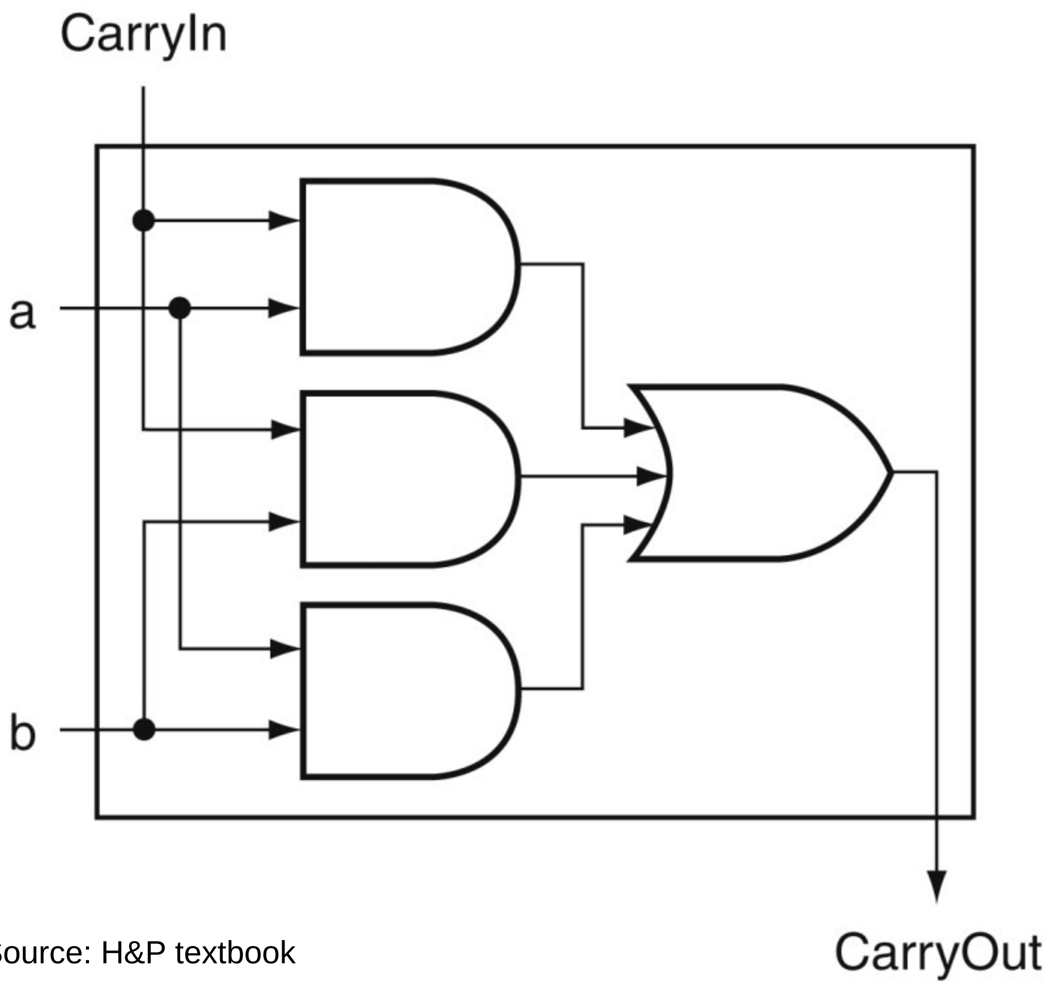
Equations:

$$Sum = Cin \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{Cin} \cdot \overline{A} + A \cdot \overline{Cin} \cdot \overline{B} + A \cdot B \cdot Cin$$

$$Cout = A \cdot B \cdot Cin + A \cdot B \cdot \overline{Cin} + A \cdot Cin \cdot \overline{B} + B \cdot Cin \cdot \overline{A}$$

$$= A \cdot B + A \cdot Cin + B \cdot Cin$$

Carry Out Logic



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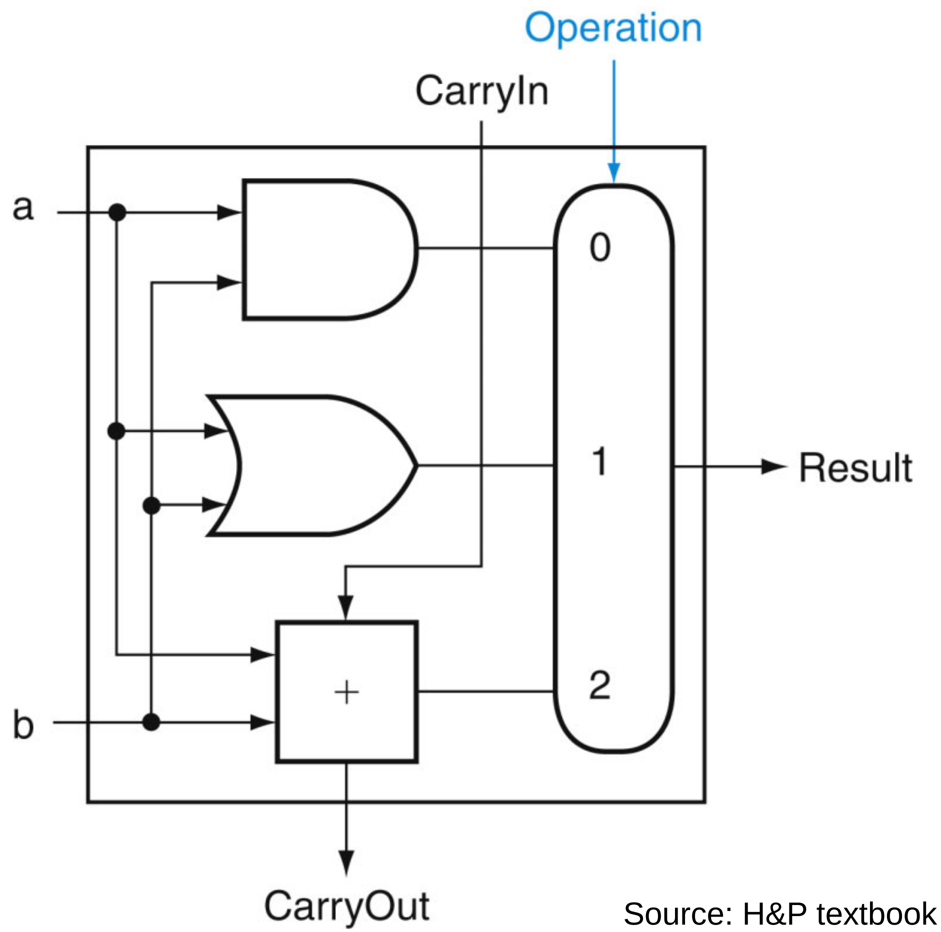
Equations:

$Sum = Cin \cdot \overline{A} \cdot \overline{B} +$
$B \cdot \overline{Cin} \cdot \overline{A} +$
$A \cdot \overline{Cin} \cdot \overline{B} +$
$A \cdot B \cdot Cin$

$Cout = A \cdot B \cdot Cin +$
$A \cdot B \cdot \overline{Cin} +$
$A \cdot Cin \cdot \overline{B} +$
$B \cdot Cin \cdot \overline{A}$
$= A \cdot B +$
$A \cdot Cin +$
$B \cdot Cin$

1-Bit ALU with Add, Or, And

- Multiplexor selects between Add, Or, And operations



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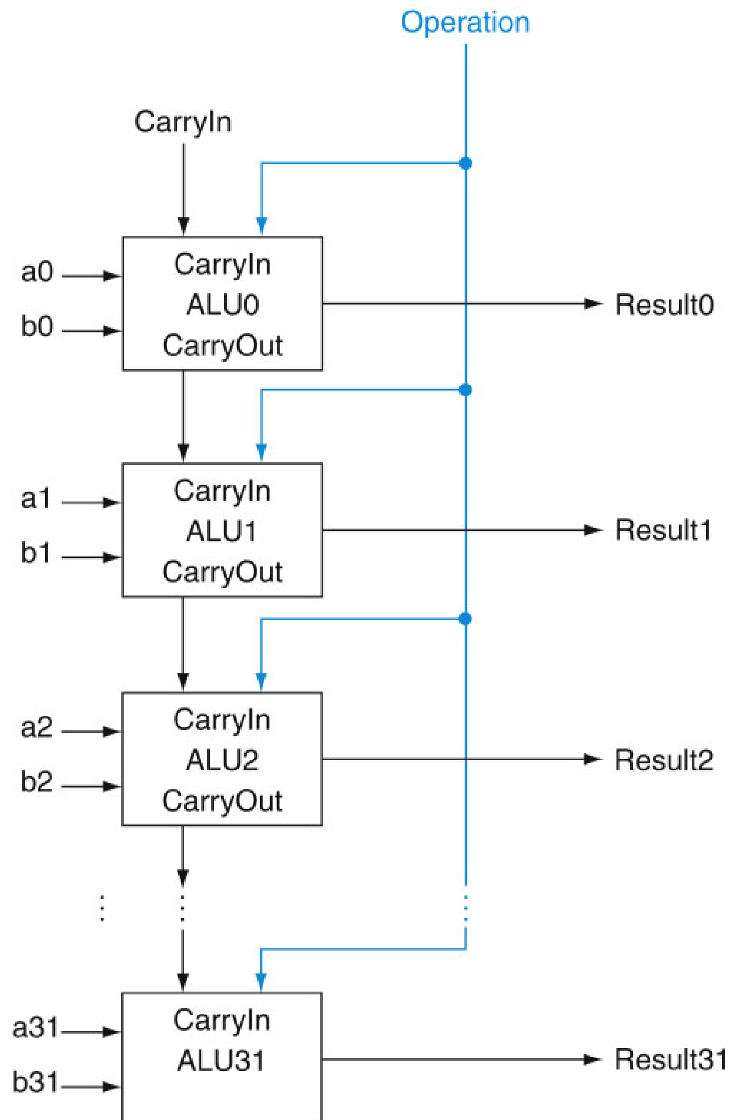
32-bit Ripple Carry Adder

1-bit ALUs are connected "in series" with the carry-out of 1 box going into the carry-in of the next box



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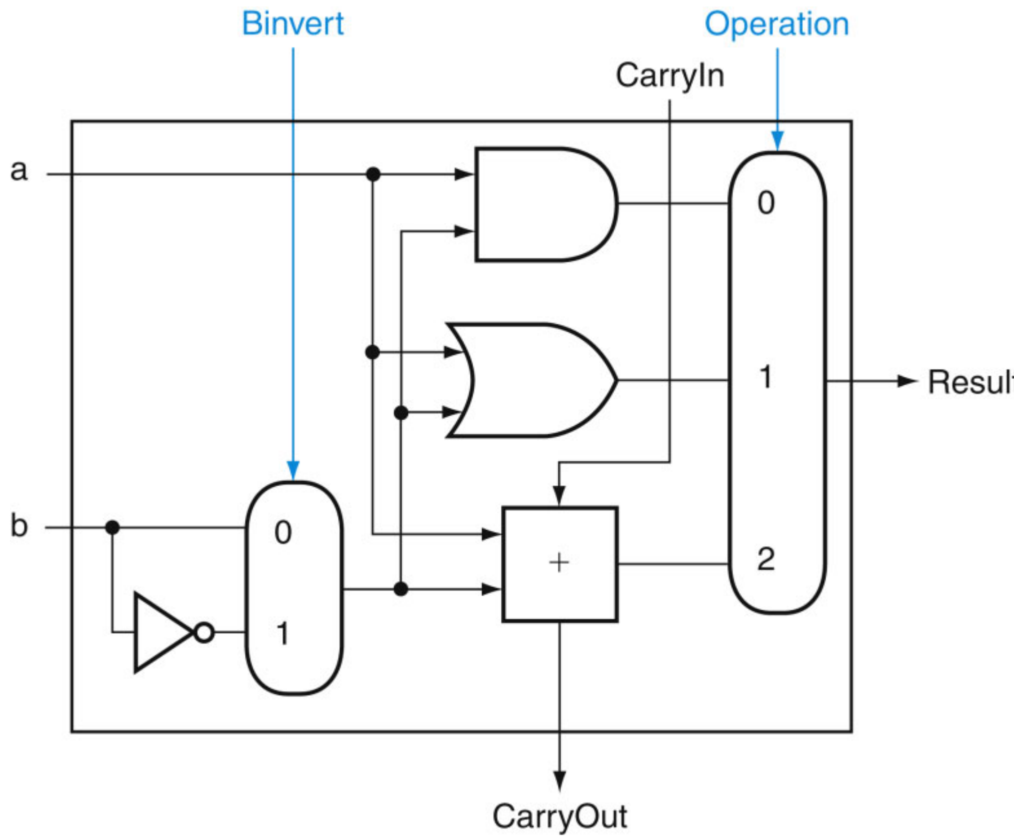
Incorporating Subtraction

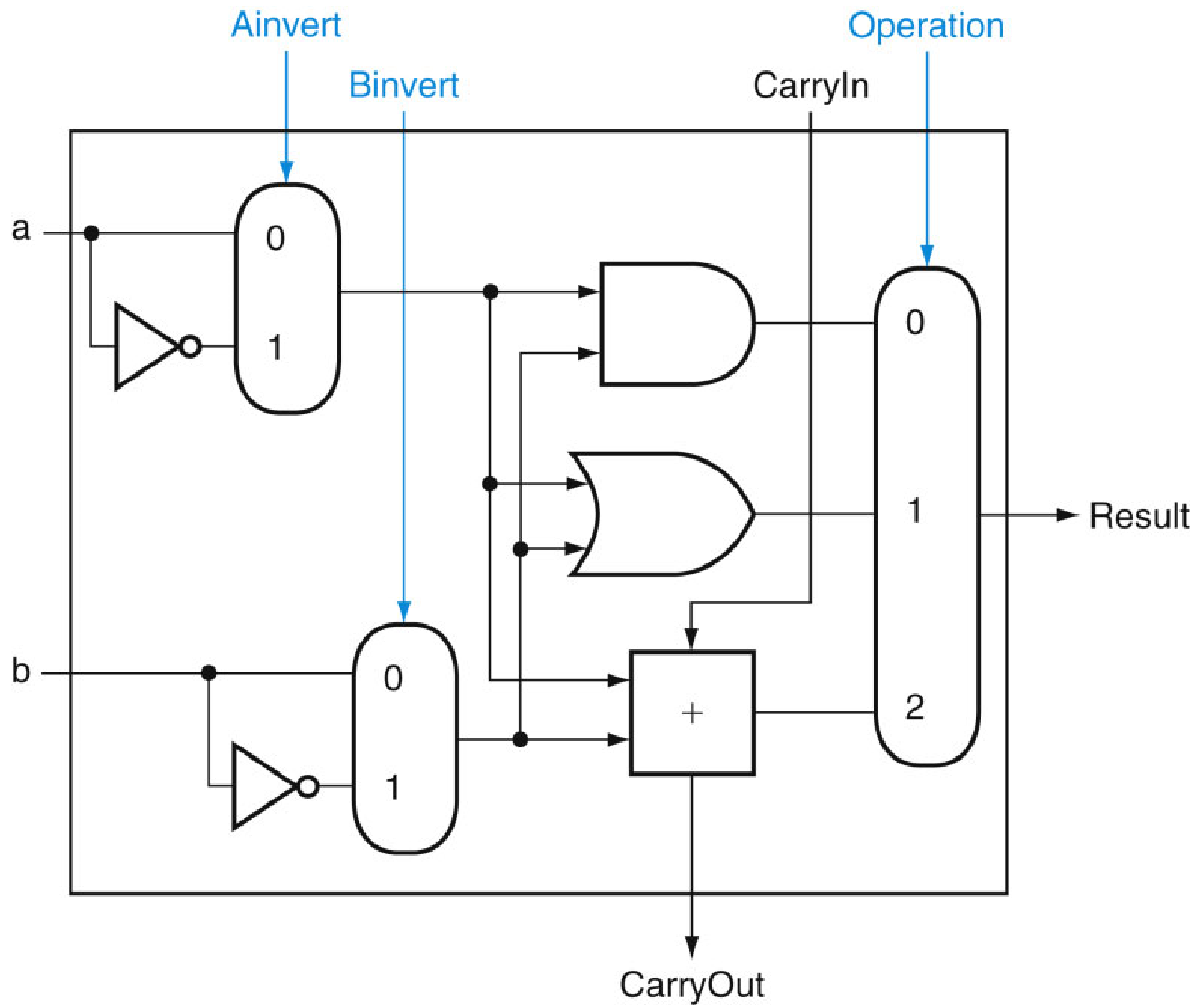
Must invert bits of B and add a 1

- Include an inverter
- CarryIn for the first bit is 1
- The CarryIn signal (for the first bit) can be the same as the Binvert signal



Source: H&P textbook

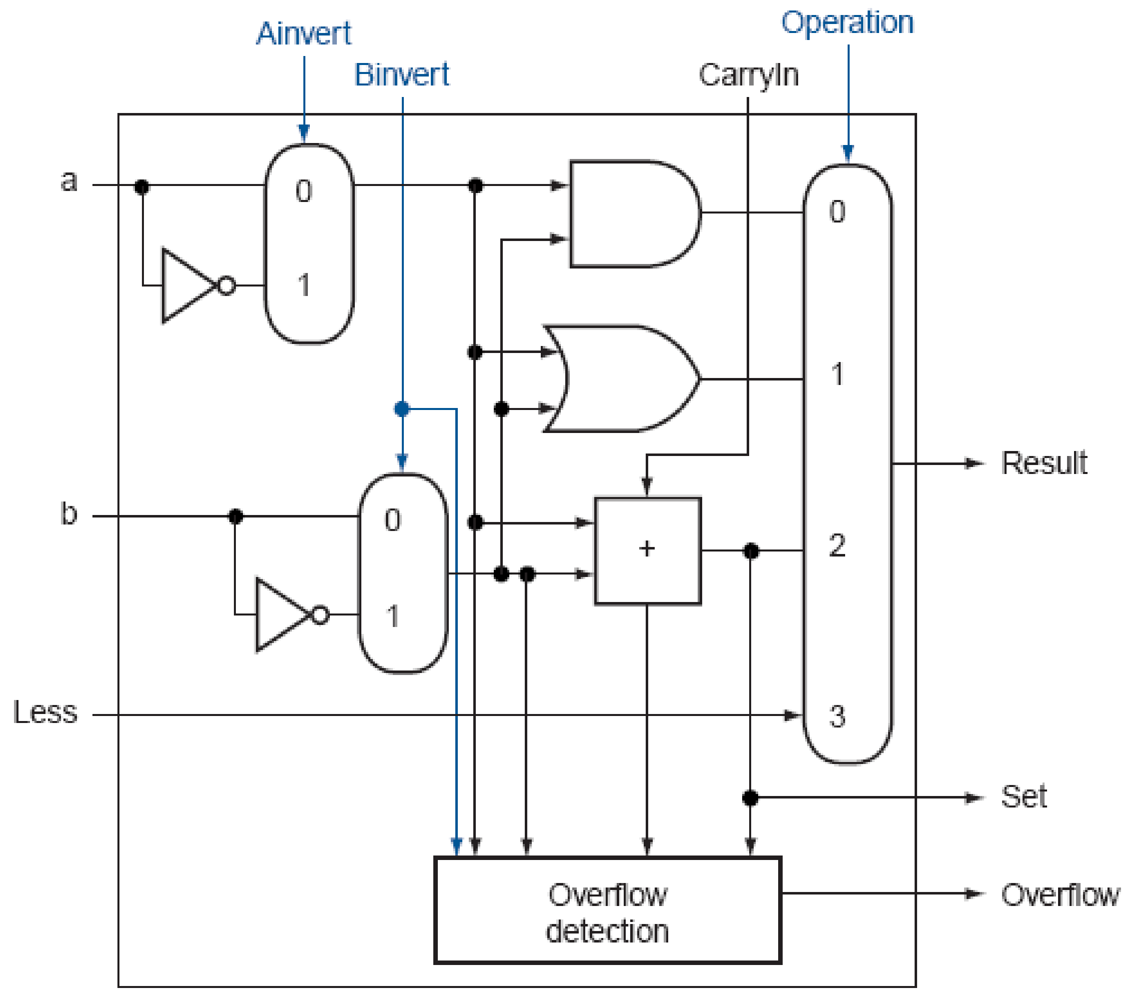
Incorporating NOR and NAND



Source: H&P textbook

Incorporating slt

- Perform a – b and check the sign
- New signal (Less) that is zero for ALU boxes 1-31
- The 31st box has a unit to detect overflow and sign – the sign bit serves as the Less signal for the 0th box



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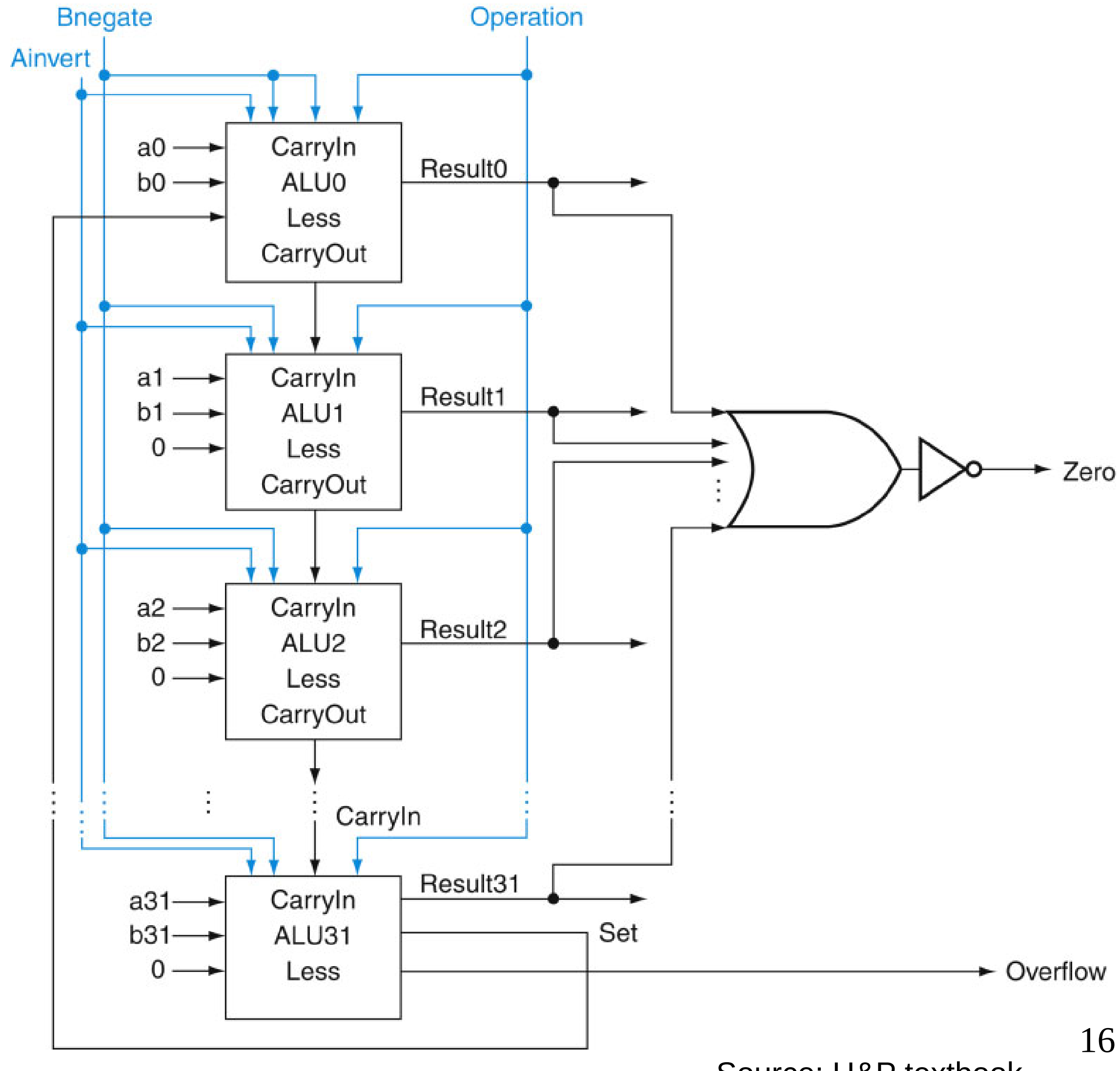
Incorporating beq

- Perform a – b and confirm that the result is all zero's



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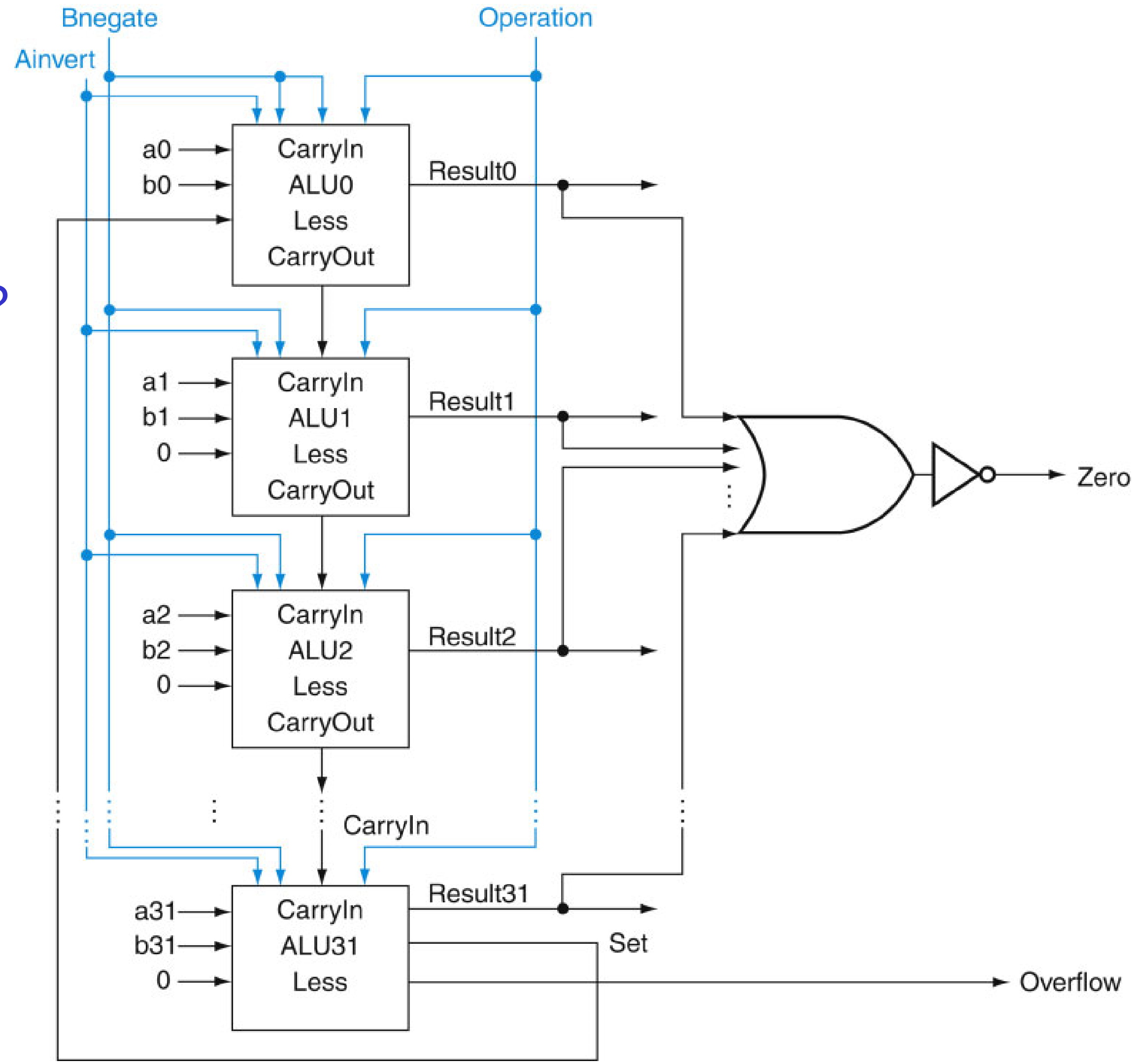
Control Lines

What are the values of the control lines and what operations do they correspond to?



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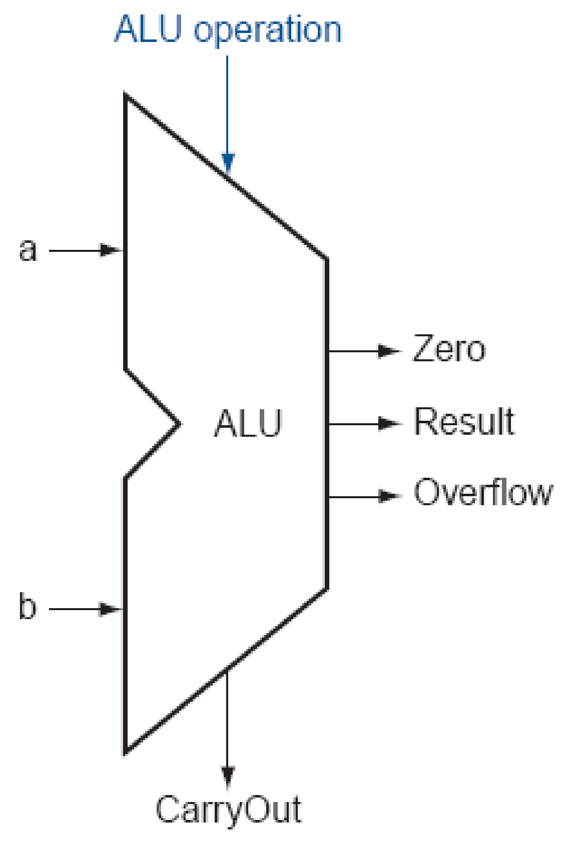
Control Lines

What are the values of the control lines and what operations do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
SLT	0	1	11
NOR	1	1	00



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Title

- Bullet