

Lecture 14: Recap

- Today's topics:
 - Recap for mid-term
 - No electronic devices in midterm

Modern Trends

- Historical contributions to performance:
 - Better processes (faster devices) ~20%
 - Better circuits/pipelines ~15%
 - Better organization/architecture ~15%

Today, annual improvement is closer to 20%; this is primarily because of slowly increasing transistor count and more cores.

Need multi-thread parallelism and accelerators to boost performance every year.

Performance Measures

- Performance = $1 / \text{execution time}$
- Speedup = ratio of performance
- Performance improvement = speedup - 1
- Execution time = clock cycle time x CPI x number of instrs

Program takes 100 seconds on ProcA and 150 seconds on ProcB

Speedup of A over B = $150/100 = 1.5$

Performance improvement of A over B = $1.5 - 1 = 0.5 = 50\%$

Speedup of B over A = $100/150 = 0.66$ (speedup less than 1 means performance went down)

Performance improvement of B over A = $0.66 - 1 = -0.33 = -33\%$
or Performance degradation of B, relative to A = 33%

If multiple programs are executed, the execution times are combined into a single number using AM, weighted AM, or GM

Performance Equations

CPU execution time = CPU clock cycles x Clock cycle time

CPU clock cycles = number of instrs x avg clock cycles
per instruction (CPI)

Substituting in previous equation,

Execution time = clock cycle time x number of instrs x avg CPI

If a 2 GHz processor graduates an instruction every third cycle,
how many instructions are there in a program that runs for
10 seconds?

Power Consumption

- Dyn power \propto activity x capacitance x voltage² x frequency
- Capacitance per transistor and voltage are decreasing, but number of transistors and frequency are increasing at a faster rate
- Leakage power is also rising and will soon match dynamic power
- Power consumption is already around 100W in some high-performance processors today

Basic MIPS Instructions

- lw \$t1, 16(\$t2)
- add \$t3, \$t1, \$t2
- addi \$t3, \$t3, 16
- sw \$t3, 16(\$t2)
- beq \$t1, \$t2, 16
- blt is implemented as slt and bne
- j 64
- jr \$t1
- sll \$t1, \$t1, 2

Convert to assembly:
while (save[i] == k)
 i += 1;

i and k are in \$s3 and \$s5 and
base of array save[] is in \$s6

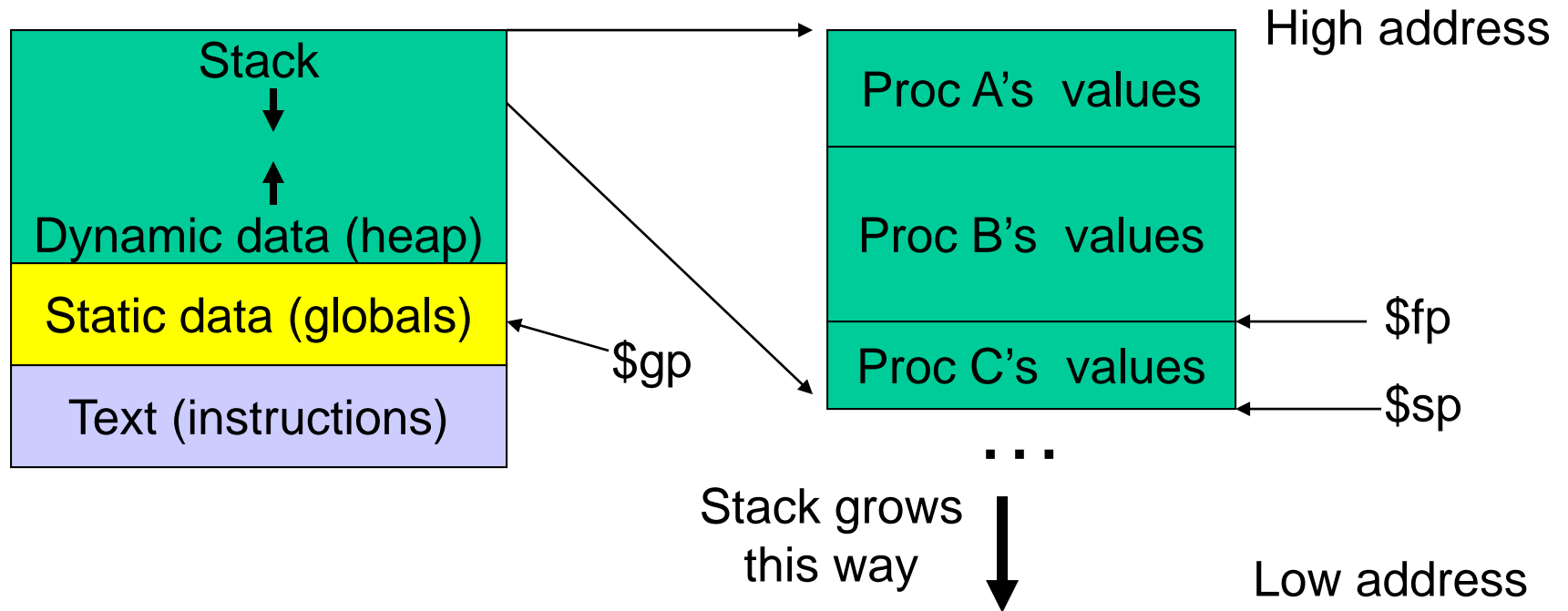
```
Loop: sll    $t1, $s3, 2
      add    $t1, $t1, $s6
      lw     $t0, 0($t1)
      bne    $t0, $s5, Exit
      addi   $s3, $s3, 1
      j      Loop
```

Exit:

Registers

- The 32 MIPS registers are partitioned as follows:
 - Register 0 : \$zero always stores the constant 0
 - Regs 2-3 : \$v0, \$v1 return values of a procedure
 - Regs 4-7 : \$a0-\$a3 input arguments to a procedure
 - Regs 8-15 : \$t0-\$t7 temporaries
 - Regs 16-23: \$s0-\$s7 variables
 - Regs 24-25: \$t8-\$t9 more temporaries
 - Reg 28 : \$gp global pointer
 - Reg 29 : \$sp stack pointer
 - Reg 30 : \$fp frame pointer
 - Reg 31 : \$ra return address

Memory Organization



Procedure Calls/Returns

```
procA (int i)
{
    int j;
    j = ...;
    call procB(j);
    ... = j;
}
```

```
procB (int j)
{
    int k;
    ... = j;
    k = ...;
    return k;
}
```

```
procA:
    $s0 = ... # value of j
    $t0 = ... # some tempval
    $a0 = $s0 # the argument
    ...
    jal procB
    ...
    ... = $v0
```

```
procB:
    $t0 = ... # some tempval
    ... = $a0 # using the argument
    $s0 = ... # value of k
    $v0 = $s0;
    jr $ra
```

Saves and Restores

- Caller saves:
 - \$ra, \$a0, \$t0, \$fp (if reqd)
- Callee saves:
 - \$s0

- As every element is saved on stack, the stack pointer is decremented

```
procA:  
    $s0 = ... # value of j  
    $t0 = ... # some tempval  
    $a0 = $s0 # the argument  
    ...  
    jal procB  
    ...  
    ... = $v0
```

```
procB:  
    $t0 = ... # some tempval  
    ... = $a0 # using the argument  
    $s0 = ... # value of k  
    $v0 = $s0;  
    jr $ra
```

Example 2

```
int fact (int n)
{
    if (n < 1) return (1);
    else return (n * fact(n-1));
}
```

Notes:

The caller saves \$a0 and \$ra in its stack space.

Temps are never saved.

```
fact:
    addi    $sp, $sp, -8
    sw      $ra, 4($sp)
    sw      $a0, 0($sp)
    slti    $t0, $a0, 1
    beq     $t0, $zero, L1
    addi    $v0, $zero, 1
    addi    $sp, $sp, 8
    jr      $ra
L1:
    addi    $a0, $a0, -1
    jal     fact
    lw      $a0, 0($sp)
    lw      $ra, 4($sp)
    addi    $sp, $sp, 8
    mul     $v0, $a0, $v0
    jr      $ra
```

Recap – Numeric Representations

- Decimal $35_{10} = 3 \times 10^1 + 5 \times 10^0$
- Binary $00100011_2 = 1 \times 2^5 + 1 \times 2^1 + 1 \times 2^0$
- Hexadecimal (compact representation)
 $0x23$ or $23_{\text{hex}} = 2 \times 16^1 + 3 \times 16^0$

0-15 (decimal) \rightarrow 0-9, a-f (hex)

Dec	Binary	Hex	Dec	Binary	Hex	Dec	Binary	Hex	Dec	Binary	Hex
0	0000	00	4	0100	04	8	1000	08	12	1100	0c
1	0001	01	5	0101	05	9	1001	09	13	1101	0d
2	0010	02	6	0110	06	10	1010	0a	14	1110	0e
3	0011	03	7	0111	07	11	1011	0b	15	1111	0f

2's Complement

```

0000 0000 0000 0000 0000 0000 0000 0000two = 0ten
0000 0000 0000 0000 0000 0000 0000 0001two = 1ten
...
0111 1111 1111 1111 1111 1111 1111 1111two = 231-1

1000 0000 0000 0000 0000 0000 0000 0000two = -231
1000 0000 0000 0000 0000 0000 0000 0001two = -(231 - 1)
1000 0000 0000 0000 0000 0000 0000 0010two = -(231 - 2)
...
1111 1111 1111 1111 1111 1111 1111 1110two = -2
1111 1111 1111 1111 1111 1111 1111 1111two = -1

```

Note that the sum of a number x and its inverted representation x' always equals a string of 1s (-1).

$$x + x' = -1$$

$x' + 1 = -x$... hence, can compute the negative of a number by

$-x = x' + 1$ inverting all bits and adding 1

This format can directly undergo addition without any conversions!

Each number represents the quantity

$$x_{31} -2^{31} + x_{30} 2^{30} + x_{29} 2^{29} + \dots + x_1 2^1 + x_0 2^0$$

Multiplication Example

Multiplicand
Multiplier

1000_{ten}
x 1001_{ten}

1000
0000
0000
1000

Product

1001000_{ten}

In every step

- multiplicand is shifted
- next bit of multiplier is examined (also a shifting step)
- if this bit is 1, shifted multiplicand is added to the product

Division

			1001_{ten}	Quotient
Divisor	1000_{ten}		1001010_{ten}	Dividend
			-1000	
			10	
			101	
			1010	
			-1000	
			10_{ten}	Remainder

At every step,

- shift divisor right and compare it with current dividend
- if divisor is larger, shift 0 as the next bit of the quotient
- if divisor is smaller, subtract to get new dividend and shift 1 as the next bit of the quotient

Division

Divisor		<div> <div>1001_{ten}</div> <div>1001010_{ten}</div> </div>		Quotient
1000 _{ten}				Dividend
0001001010		0001001010		0000001010
100000000000 →		0001000000 →		0000100000 →
Quo: 0		000001		000001001

At every step,

- shift divisor right and compare it with current dividend
- if divisor is larger, shift 0 as the next bit of the quotient
- if divisor is smaller, subtract to get new dividend and shift 1 as the next bit of the quotient

Divide Example

- Divide 7_{ten} ($0000\ 0111_{\text{two}}$) by 2_{ten} (0010_{two})

Iter	Step	Quot	Divisor	Remainder
0	Initial values	0000	0010 0000	0000 0111
1	Rem = Rem – Div	0000	0010 0000	1110 0111
	Rem < 0 → +Div, shift 0 into Q	0000	0010 0000	0000 0111
	Shift Div right	0000	0001 0000	0000 0111
2	Same steps as 1	0000	0001 0000	1111 0111
		0000	0001 0000	0000 0111
		0000	0000 1000	0000 0111
3	Same steps as 1	0000	0000 0100	0000 0111
4	Rem = Rem – Div	0000	0000 0100	0000 0011
	Rem >= 0 → shift 1 into Q	0001	0000 0100	0000 0011
	Shift Div right	0001	0000 0010	0000 0011
5	Same steps as 4	0011	0000 0001	0000 0001

Binary FP Numbers

- 20.45 decimal = ? Binary
- 20 decimal = 10100 binary
- $0.45 \times 2 = 0.9$ (not greater than 1, first bit after binary point is 0)
 $0.90 \times 2 = 1.8$ (greater than 1, second bit is 1, subtract 1 from 1.8)
 $0.80 \times 2 = 1.6$ (greater than 1, third bit is 1, subtract 1 from 1.6)
 $0.60 \times 2 = 1.2$ (greater than 1, fourth bit is 1, subtract 1 from 1.2)
 $0.20 \times 2 = 0.4$ (less than 1, fifth bit is 0)
 $0.40 \times 2 = 0.8$ (less than 1, sixth bit is 0)
 $0.80 \times 2 = 1.6$ (greater than 1, seventh bit is 1, subtract 1 from 1.6)
... and the pattern repeats

10100.011100110011001100...

Normalized form = $1.0100011100110011... \times 2^4$

IEEE 754 Format

Final representation: $(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$

- Represent -0.75_{ten} in single and double-precision formats

Single: (1 + 8 + 23)

1 0111 1110 1000...000

Double: (1 + 11 + 52)

1 0111 1111 110 1000...000

- What decimal number is represented by the following single-precision number?

1 1000 0001 01000...0000

-5.0

FP Addition – Binary Example

- Consider the following binary example

$$1.010 \times 2^1 + 1.100 \times 2^3$$

Convert to the larger exponent:

$$0.0101 \times 2^3 + 1.1000 \times 2^3$$

Add

$$1.1101 \times 2^3$$

Normalize

$$1.1101 \times 2^3$$

Check for overflow/underflow

Round

Re-normalize

IEEE 754 format: 0 10000010 110100000000000000000000

Boolean Algebra

- $\overline{A + B} = \overline{A} \cdot \overline{B}$

- $\overline{A \cdot B} = \overline{A} + \overline{B}$

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Any truth table can be expressed as a sum of products

$$(A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$$

- Can also use “product of sums”
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

Adder Implementations

- Ripple-Carry adder – each 1-bit adder feeds its carry-out to next stage – simple design, but we must wait for the carry to propagate thru all bits
- Carry-Lookahead adder – each bit can be represented by an equation that only involves input bits (a_i, b_i) and initial carry-in (c_0) -- this is a complex equation, so it's broken into sub-parts

For bits a_i, b_i , and c_i , a carry is generated if $a_i \cdot b_i = 1$ and a carry is propagated if $a_i + b_i = 1$

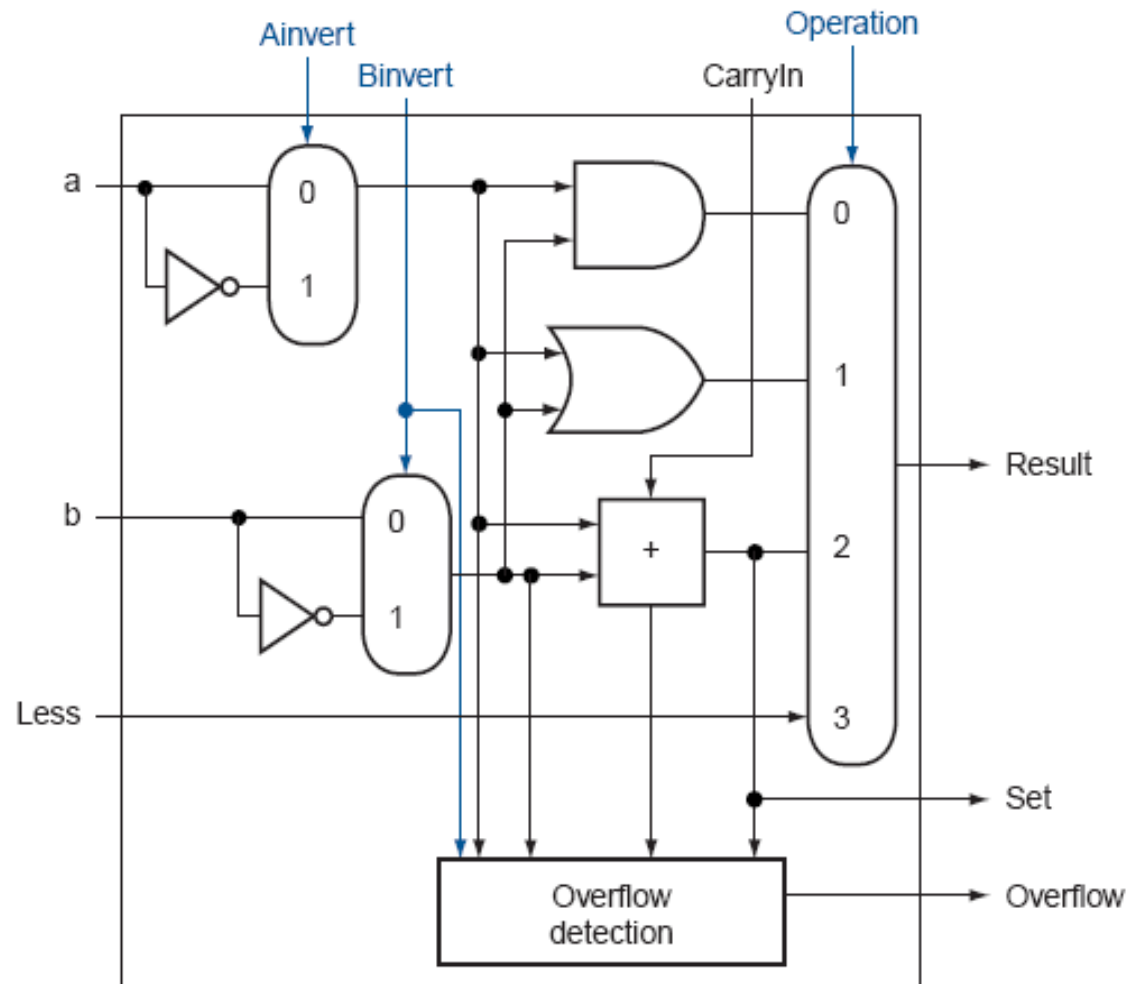
$$C_{i+1} = g_i + p_i \cdot C_i$$

Similarly, compute these values for a block of 4 bits, then for a block of 16 bits, then for a block of 64 bits....Finally, the carry-out for the 64th bit is represented by an equation such as this:

$$C_4 = G_3 + G_2 \cdot P_3 + G_1 \cdot P_2 \cdot P_3 + G_0 \cdot P_1 \cdot P_2 \cdot P_3 + C_0 \cdot P_0 \cdot P_1 \cdot P_2 \cdot P_3$$

Each of the sub-terms is also a similar expression

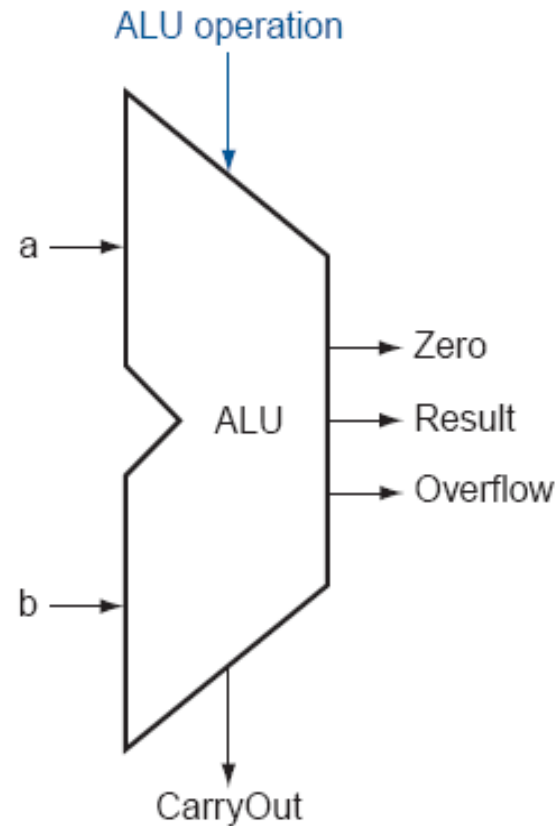
32-bit ALU



Control Lines

What are the values of the control lines and what operations do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
SLT	0	1	11
NOR	1	1	00



State Transition Table

- Problem description: A traffic light with only green and red; either the North-South road has green or the East-West road has green (both can't be red); there are detectors on the roads to indicate if a car is on the road; the lights are updated every 30 seconds; a light must change only if a car is waiting on the other road

State Transition Table:

CurrState	InputEW	InputNS	NextState=Output
N	0	0	N
N	0	1	N
N	1	0	E
N	1	1	E
E	0	0	E
E	0	1	N
E	1	0	E
E	1	1	N

Midterm Question

Thermostat can be OFF, HEAT, COOL

Two input sensors

Ext temp within 5 degrees of desired temp → OFF

Int temp within 1 degree of desired temp → no change

Int temp < desired temp -1 → HEAT

Int temp > desired temp +1 → COOL

Title
