Lecture 10: Floating Point, Digital Design

- Today's topics:
  - FP arithmetic
  - Intro to Boolean functions

Examples

Final representation: $(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$

- Represent -0.75_{ten} in single and double-precision formats

  Single: (1 + 8 + 23)

  Double: (1 + 11 + 52)

- What decimal number is represented by the following single-precision number?

  1 1000 0001 01000…0000

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  1 0111 1110 1000…000

  Double: (1 + 11 + 52)
  1 0111 1111 110 1000…000

- What decimal number is represented by the following single-precision number?
  1 1000 0001 01000…0000
  -5.0

FP Addition

- Consider the following decimal example (can maintain only 4 decimal digits and 2 exponent digits)

$9.999 \times 10^{1} \quad + \quad 1.610 \times 10^{-1}$

Convert to the larger exponent:

$9.999 \times 10^{1} \quad + \quad 0.016 \times 10^{1}$

Add

10.015×10^{1}

Normalize

1.0015×10^{2}

Check for overflow/underflow

Round

1.002×10^{2}

Re-normalize

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If we had more fraction bits, these errors would be minimized

FP Multiplication

- Similar steps:
  - Compute exponent (careful!)
  - Multiply significands (set the binary point correctly)
  - Normalize
  - Round (potentially re-normalize)
  - Assign sign

MIPS Instructions

- The usual add.s, add.d, sub, mul, div
- Comparison instructions: c.eq.s, c.neq.s, c.lt.s….
  These comparisons set an internal bit in hardware that
  is then inspected by branch instructions: bc1t, bc1f
- Separate register file $f0 - $f31 : a double-precision
  value is stored in (say) $f4-$f5 and is referred to by $f4
- Load/store instructions (lwc1, swc1) must still use
  integer registers for address computation

Code Example

```
float  f2c (float fahr)
{
    return ((5.0/9.0) * (fahr – 32.0));
}
```

(argument fahr is stored in $f12)

```
lwc1   $f16, const5
lwc1   $f18, const9
div.s  $f16, $f16, $f18
lwc1   $f18, const32
sub.s  $f18, $f12, $f18
mul.s  $f0, $f16, $f18
jr     $ra
```

Fixed Point

- FP operations are much slower than integer ops
- Fixed point arithmetic uses integers, but assumes that every number is multiplied by the same factor
- Example: with a factor of 1/1000, the fixed-point representations for 1.46, 1.7198, and 5624 are respectively 1460, 1720, and 5624000
- More programming effort and possibly lower precision for higher performance

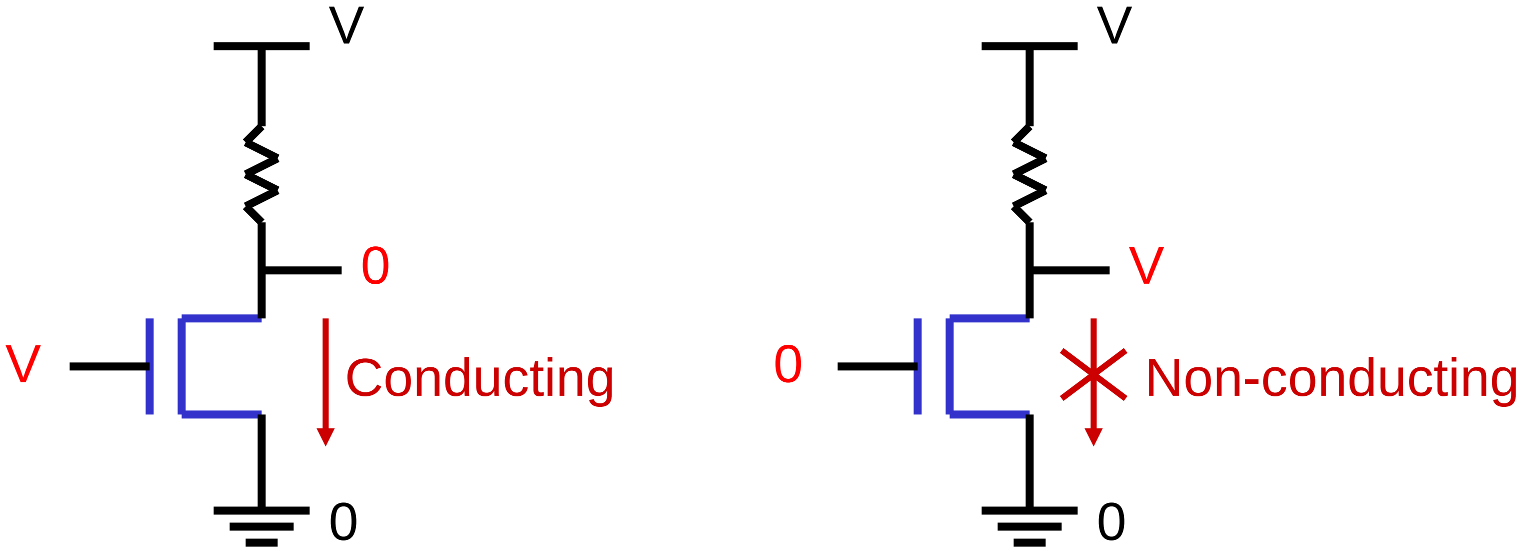
Subword Parallelism

- ALUs are typically designed to perform 64-bit or 128-bit arithmetic
- Some data types are much smaller, e.g., bytes for pixel RGB values, half-words for audio samples
- Partitioning the carry-chains within the ALU can convert the 64-bit adder into 4 16-bit adders or 8 8-bit adders
- A single load can fetch multiple values, and a single add instruction can perform multiple parallel additions, referred to as subword parallelism

Digital Design Basics

- Two voltage levels – high and low (1 and 0, true and false)
  Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch



Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors
- A logic block is termed *combinational* if the output is only a function of the inputs
- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output
- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E

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A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , X = A + B ➔ X is true if at least one of A or B is true
  - AND : symbol **.** , X = A **.** B ➔ X is true if both A and B are true
  - NOT : symbol $\overline{\ \ }$, X = $\overline{A}$ ➔ X is the inverted value of A

Boolean Algebra Rules

- Identity law : $A + 0 = A$; $A \cdot 1 = A$
- Zero and One laws : $A + 1 = 1$; $A \cdot 0 = 0$
- Inverse laws : $A \cdot \overline{A} = 0$; $A + \overline{A} = 1$
- Commutative laws : $A + B = B + A$; $A \cdot B = B \cdot A$
- Associative laws : $A + (B + C) = (A + B) + C$
  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- Distributive laws : $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
  $A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's Laws

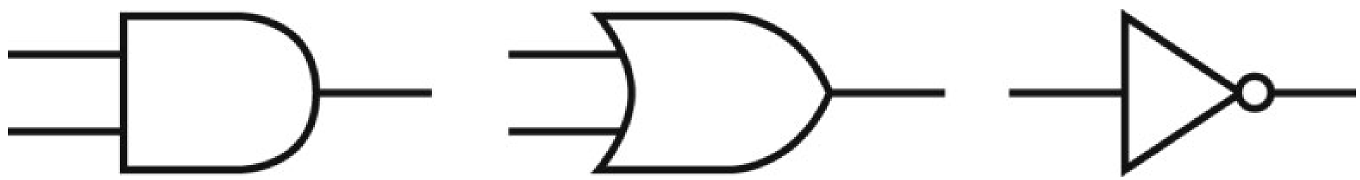
- $\overline{A + B} = \overline{A} \, . \, \overline{B}$

- $\overline{A \, . \, B} = \overline{A} + \overline{B}$

- Confirm that these are indeed true

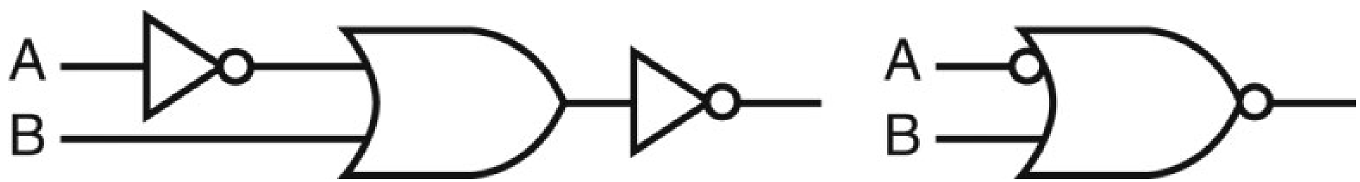
Pictorial Representations

AND OR NOT



Source: H&P textbook

What logic function is this?



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Boolean Equation

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  Multiple correct equations:

  Two must be true, but all three cannot be true:

  $E = ((A . B) + (B . C) + (A . C)) . \overline{(A . B . C)}$

  Identify the three cases where it is true:

  $E = (A . B . \overline{C}) + (A . C . \overline{B}) + (C . B . \overline{A})$

Sum of Products

- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$(A \,.\, B \,.\, \overline{C}) + (A \,.\, C \,.\, \overline{B}) + (C \,.\, B \,.\, \overline{A})$

- Can also use "product of sums"
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

Title

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