Lecture 7: MARS, Computer Arithmetic

- Today's topics:
  - MARS intro
  - Numerical representations
  - Addition and subtraction

MARS Intro

- Directives, labels, global pointers, system calls

Example Print Routine

```
.data
   str:     .asciiz   “the answer is ”
.text
   li       $v0, 4              # load immediate; 4 is the code for print_string
   la       $a0, str            #  the print_string syscall expects the string
                                #  address as the argument; la is the instruction
                                #  to load the address of the operand (str)
   syscall                      #  SPIM will now invoke syscall-4
   li       $v0, 1              #  syscall-1 corresponds to print_int
   li       $a0, 5              #  print_int expects the integer as its argument
   syscall                      #  SPIM will now invoke syscall-1
```

Example

- Write an assembly program to prompt the user for two numbers and print the sum of the two numbers

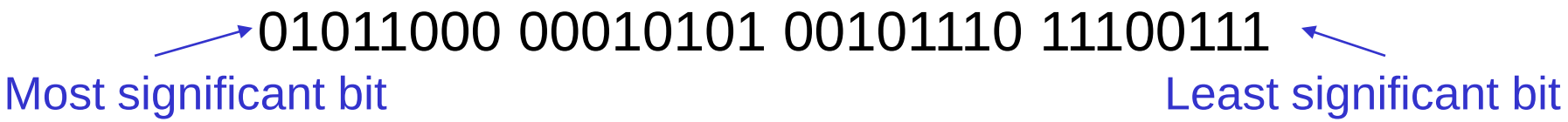
Example

```
.text
   .globl main
   main:
      li   $v0, 4
      la  $a0, str1
      syscall
      li   $v0, 5
      syscall
      add  $t0, $v0, $zero
      li   $v0, 5
      syscall
      add  $t1, $v0, $zero
      li   $v0, 4
      la  $a0, str2
      syscall
      li    $v0, 1
      add  $a0, $t1, $t0
      syscall
```

```
.data
   str1:  .asciiz  “Enter 2 numbers:”
   str2:  .asciiz  “The sum is ”
```

Binary Representation

- The binary number



represents the quantity

$0 \times 2^{31} + 1 \times 2^{30} + 0 \times 2^{29} + \ldots + 1 \times 2^{0}$

- A 32-bit word can represent 2^{32} numbers between 0 and $2^{32}-1$

  … this is known as the unsigned representation as we're assuming that numbers are always positive

ASCII Vs. Binary

- Does it make more sense to represent a decimal number in ASCII?
- Hardware to implement arithmetic would be difficult
- What are the storage needs? How many bits does it take to represent the decimal number 1,000,000,000 in ASCII and in binary?

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  In binary: 30 bits (2^{30} > 1 billion)

  In ASCII: 10 characters, 8 bits per char = 80 bits

Negative Numbers

32 bits can only represent 2^{32} numbers – if we wish to also represent negative numbers, we can represent 2^{31} positive numbers (incl zero) and 2^{31} negative numbers

$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = 0_{ten}$
$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001_{two} = 1_{ten}$
…
$0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{two} = 2^{31}-1$

$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = -2^{31}$
$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001_{two} = -(2^{31} – 1)$
$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0010_{two} = -(2^{31} – 2)$
…
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1110_{two} = -2$
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{two} = -1$

2's Complement

$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = 0_{ten}$
$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001_{two} = 1_{ten}$
…
$0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{two} = 2^{31}-1$

$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = -2^{31}$
$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001_{two} = -(2^{31} - 1)$
$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0010_{two} = -(2^{31} - 2)$
…
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1110_{two} = -2$
$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{two} = -1$

Why is this representation favorable?
Consider the sum of 1 and -2 …. we get -1
Consider the sum of 2 and -1 …. we get +1

This format can directly undergo addition without any conversions!

Each number represents the quantity
$x_{31}\ -2^{31} + x_{30}\ 2^{30} + x_{29}\ 2^{29} + \ldots + x_1\ 2^1 + x_0\ 2^0$

2's Complement

$0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{two} = 0_{ten}$
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…
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Note that the sum of a number x and its inverted representation x' always equals a string of 1s (-1).

$x + x' = -1$
$x' + 1 = -x$ … hence, can compute the negative of a number by
$-x = x' + 1$ inverting all bits and adding 1

Similarly, the sum of x and –x gives us all zeroes, with a carry of 1
In reality, $x + (-x) = 2^n$ … hence the name 2's complement

Example

- Compute the 32-bit 2's complement representations for the following decimal numbers:

  5, -5, -6

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- Compute the 32-bit 2's complement representations for the following decimal numbers:

  5, -5, -6

5: 0000 0000 0000 0000 0000 0000 0000 0101
-5: 1111 1111 1111 1111 1111 1111 1111 1011
-6: 1111 1111 1111 1111 1111 1111 1111 1010

Given -5, verify that negating and adding 1 yields the number 5

Signed / Unsigned

- The hardware recognizes two formats:

  unsigned (corresponding to the C declaration unsigned int)
  -- all numbers are positive, a 1 in the most significant bit just means it is a really large number

  signed (C declaration is signed int or just int)
  -- numbers can be +/- , a 1 in the MSB means the number is negative

This distinction enables us to represent twice as many numbers when we're sure that we don't need negatives

MIPS Instructions

Consider a comparison instruction:

```
    slt   $t0, $t1, $zero
```

and $t1 contains the 32-bit number 1111 01…01

What gets stored in $t0?

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Consider a comparison instruction:

slt $t0, $t1, $zero

and $t1 contains the 32-bit number 1111 01…01

What gets stored in $t0?

The result depends on whether $t1 is a signed or unsigned number – the compiler/programmer must track this and accordingly use either slt or sltu

slt $t0, $t1, $zero stores 1 in $t0

sltu $t0, $t1, $zero stores 0 in $t0

Sign Extension

- Occasionally, 16-bit signed numbers must be converted into 32-bit signed numbers – for example, when doing an add with an immediate operand

- The conversion is simple: take the most significant bit and use it to fill up the additional bits on the left – known as sign extension

So 2_{10} goes from 0000 0000 0000 0010 to
0000 0000 0000 0000 0000 0000 0000 0010

and -2_{10} goes from 1111 1111 1111 1110 to
1111 1111 1111 1111 1111 1111 1111 1110

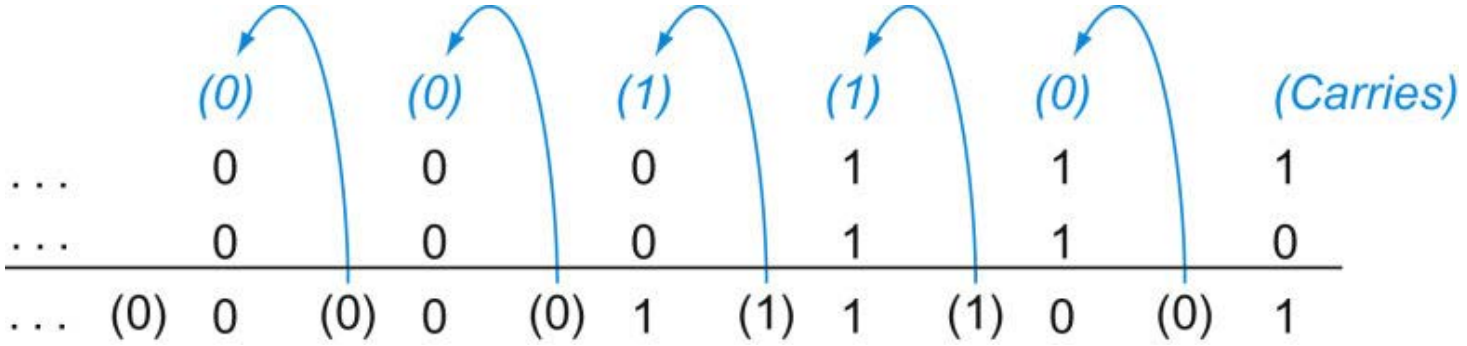
Alternative Representations

- The following two (intuitive) representations were discarded because they required additional conversion steps before arithmetic could be performed on the numbers
  - sign-and-magnitude: the most significant bit represents +/- and the remaining bits express the magnitude
  - one's complement: -x is represented by inverting all the bits of x

  Both representations above suffer from two zeroes

Addition and Subtraction

- Addition is similar to decimal arithmetic
- For subtraction, simply add the negative number – hence, subtract A-B involves negating B's bits, adding 1 and A



Source: H&P textbook

Overflows

- For an unsigned number, overflow happens when the last carry (1) cannot be accommodated

- For a signed number, overflow happens when the most significant bit is not the same as every bit to its left
  - when the sum of two positive numbers is a negative result
  - when the sum of two negative numbers is a positive result
  - The sum of a positive and negative number will never overflow

- MIPS allows addu and subu instructions that work with unsigned integers and never flag an overflow – to detect the overflow, other instructions will have to be executed

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