Lecture 11: Digital Design

- Today's topics:
  - Evaluating a system
  - Intro to boolean functions

Example

Execution time = clock cycle time x number of instrs x avg CPI

Which of the following two systems is better?

- A program is converted into 4 billion MIPS instructions by a compiler ; the MIPS processor is implemented such that each instruction completes in an average of 1.5 cycles and the clock speed is 1 GHz

- The same program is converted into 2 billion x86 instructions; the x86 processor is implemented such that each instruction completes in an average of 6 cycles and the clock speed is 1.5 GHz

Benchmark Suites

- Measuring performance components is difficult for most users: average CPI requires simulation/hardware counters, instruction count requires profiling tools/hardware counters, OS interference is hard to quantify, etc.

- Each vendor announces a SPEC rating for their system
  - a measure of execution time for a fixed collection of programs
  - is a function of a specific CPU, memory system, IO system, operating system, compiler
  - enables easy comparison of different systems

The key is coming up with a collection of relevant programs

SPEC CPU

- SPEC: System Performance Evaluation Corporation, an industry consortium that creates a collection of relevant programs
- The 2006 version includes 12 integer and 17 floating-point applications
- The SPEC rating specifies how much faster a system is, compared to a baseline machine – a system with SPEC rating 600 is 1.5 times faster than a system with SPEC rating 400
- Note that this rating incorporates the behavior of all 29 programs – this may not necessarily predict performance for your favorite program!

Deriving a Single Performance Number

How is the performance of 29 different apps compressed into a single performance number?

- SPEC uses geometric mean (GM) – the execution time of each program is multiplied and the N^{th} root is derived

- Another popular metric is arithmetic mean (AM) – the average of each program's execution time

- Weighted arithmetic mean – the execution times of some programs are weighted to balance priorities

Amdahl's Law

- Architecture design is very bottleneck-driven – make the common case fast, do not waste resources on a component that has little impact on overall performance/power

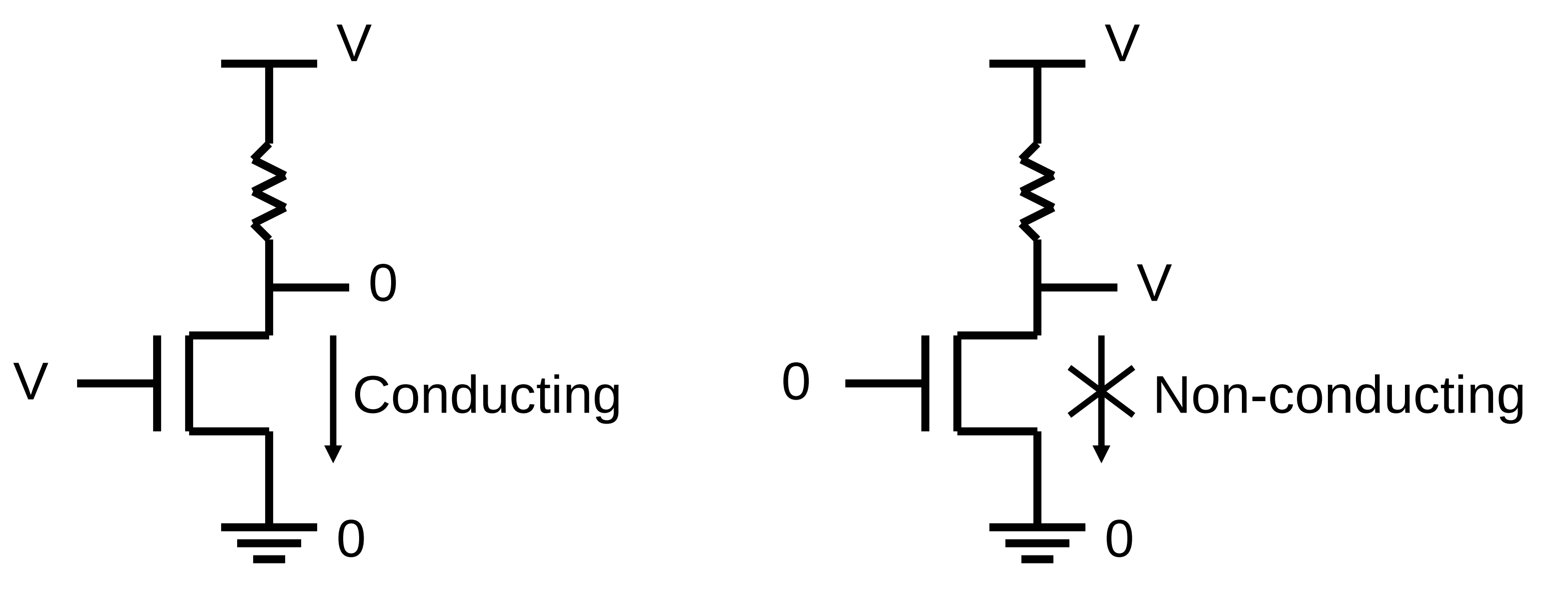
- Amdahl's Law: performance improvements through an enhancement is limited by the fraction of time the enhancement comes into play

- Example: a web server spends 40% of time in the CPU and 60% of time doing I/O – a new processor that is ten times faster results in a 36% reduction in execution time (speedup of 1.56) – Amdahl's Law states that maximum execution time reduction is 40% (max speedup of 1.66)

Digital Design Basics

- Two voltage levels – high and low (1 and 0, true and false)
  Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch



Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors

- A logic block is termed *combinational* if the output is only a function of the inputs

- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output

- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs

- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , X = A + B ➔ X is true if at least one of A or B is true
  - AND : symbol **.** , X = A **.** B ➔ X is true if both A and B are true
  - NOT : symbol $\overline{\ }$, X = $\overline{A}$ ➔ X is the inverted value of A

Boolean Algebra Rules

- Identity law : $A + 0 = A$; $A . 1 = A$
- Zero and One laws : $A + 1 = 1$; $A . 0 = 0$
- Inverse laws : $A . \overline{A} = 0$; $A + \overline{A} = 1$
- Commutative laws : $A + B = B + A$; $A . B = B . A$
- Associative laws : $A + (B + C) = (A + B) + C$
  $A . (B . C) = (A . B) . C$
- Distributive laws : $A . (B + C) = (A . B) + (A . C)$
  $A + (B . C) = (A + B) . (A + C)$

DeMorgan's Laws

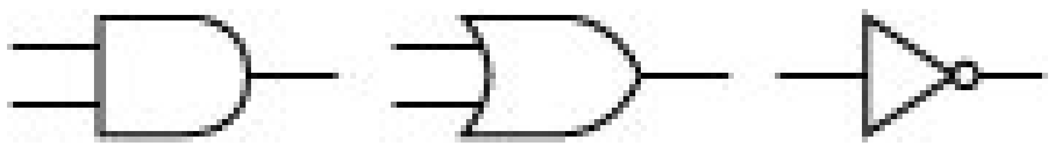
- $\overline{A + B} = \overline{A} \,.\, \overline{B}$

- $\overline{A \,.\, B} = \overline{A} + \overline{B}$

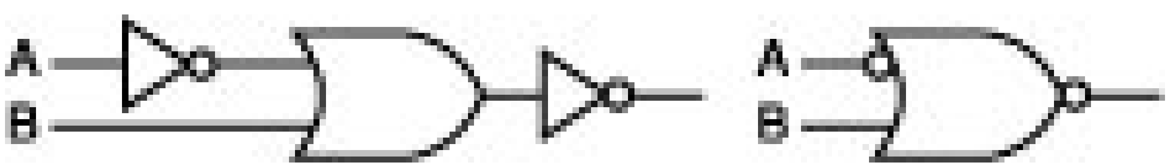
- Confirm that these are indeed true

Pictorial Representations

AND OR NOT



What logic function is this?



Boolean Equation

- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

Boolean Equation

- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

  Multiple correct equations:

  Two must be true, but all three cannot be true:
  $E = ((A \cdot B) + (B \cdot C) + (A \cdot C)) \cdot (\overline{A \cdot B \cdot C})$

  Identify the three cases where it is true:
  $E = (A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$

Sum of Products

- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$(A \,.\, B \,.\, \overline{C}) + (A \,.\, C \,.\, \overline{B}) + (C \,.\, B \,.\, \overline{A})$

- Can also use "product of sums"
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

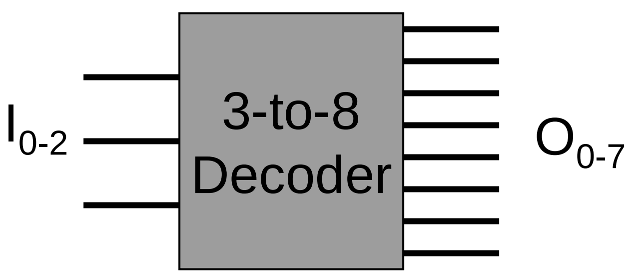
NAND and NOR

- NAND : NOT of AND : A nand B = $\overline{A . B}$
- NOR : NOT of OR : A nor B = $\overline{A + B}$
- NAND and NOR are *universal gates*, i.e., they can be used to construct any complex logical function

Common Logic Blocks – Decoder

Takes in N inputs and activates one of 2^N outputs

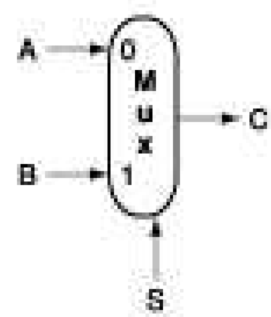
I_0	I_1	I_2	O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

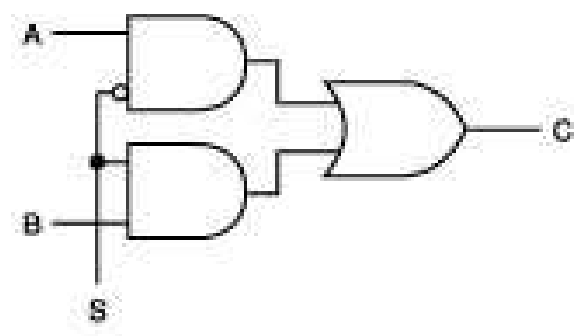


Common Logic Blocks – Multiplexor

- Multiplexor or selector: one of N inputs is reflected on the output depending on the value of the $log_2 N$ selector bits

2-input mux





Title

- Bullet