CS 6530: Advanced Database Systems Fall 2022

Lecture 22
Learned Indexes

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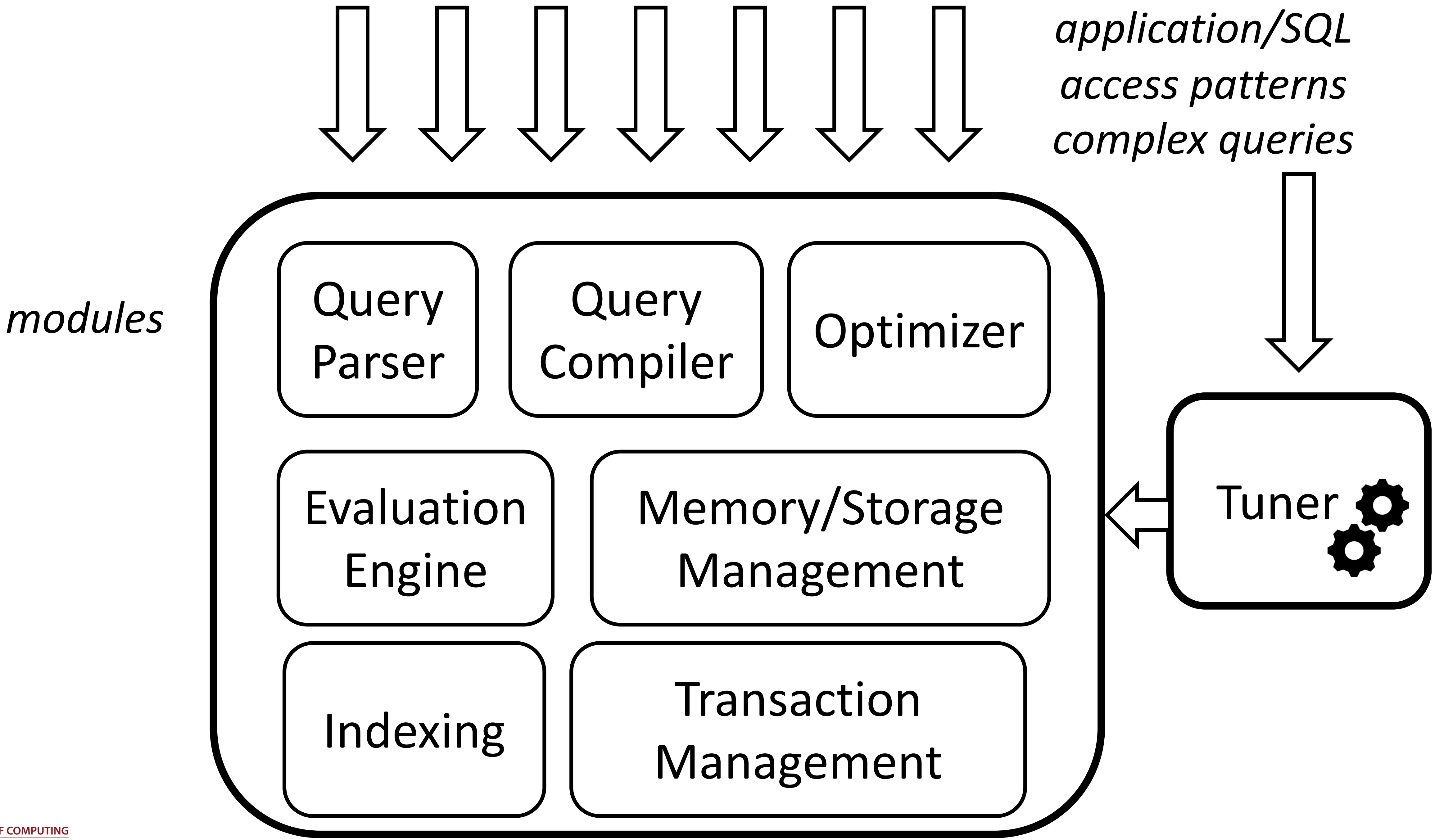
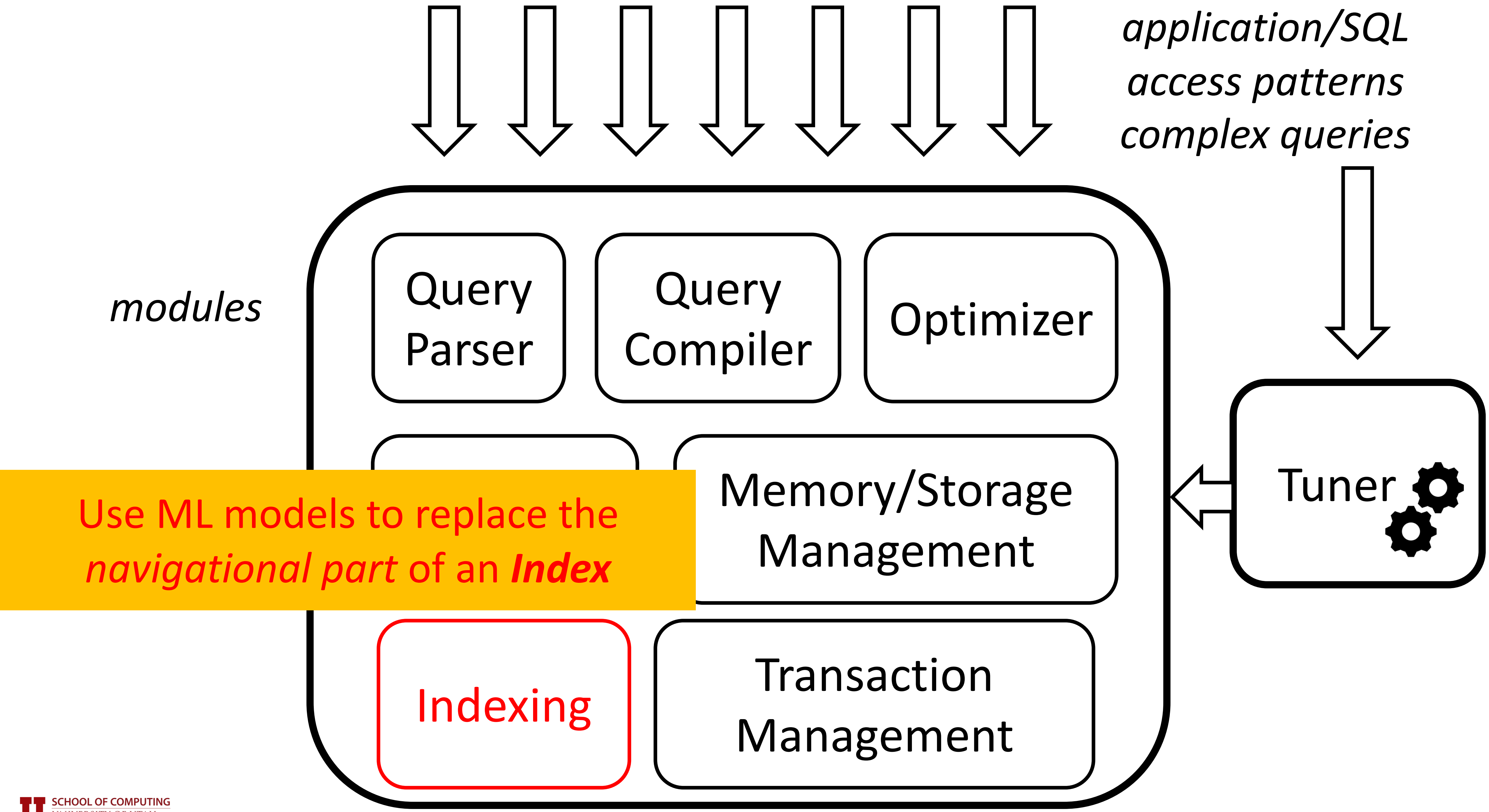
Acknowledgement: Slides taken from Prof. Manos Athanassoulis, BU

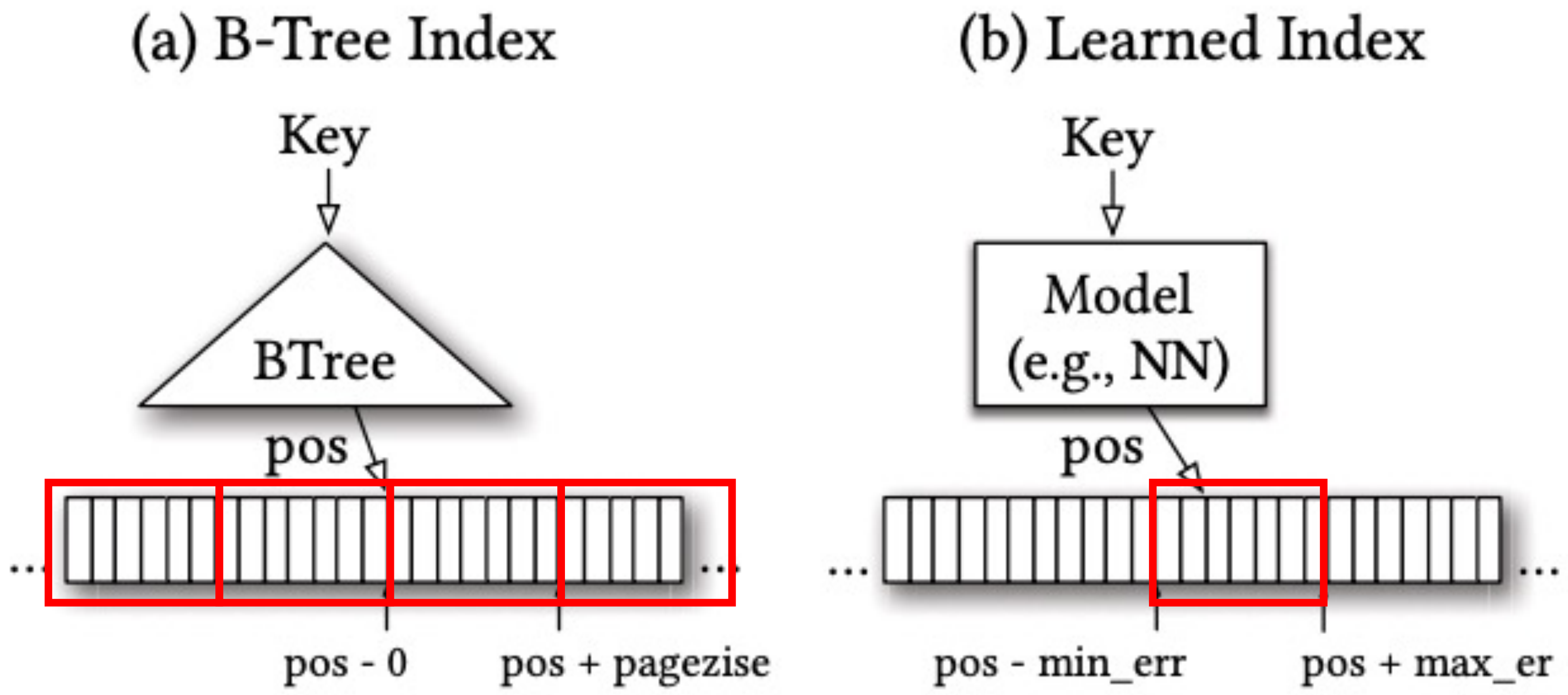
Some reminders…

- Please fill course evaluations!!
- Final paper report due on December 1st.
  - Please write reports on paper based on the current topics.
- Final quiz due on December 6th.
- Project presentations slots are up.
  - Prepare your final presentations and reports according to the guidelines.
- Final project reports due on December 8th.

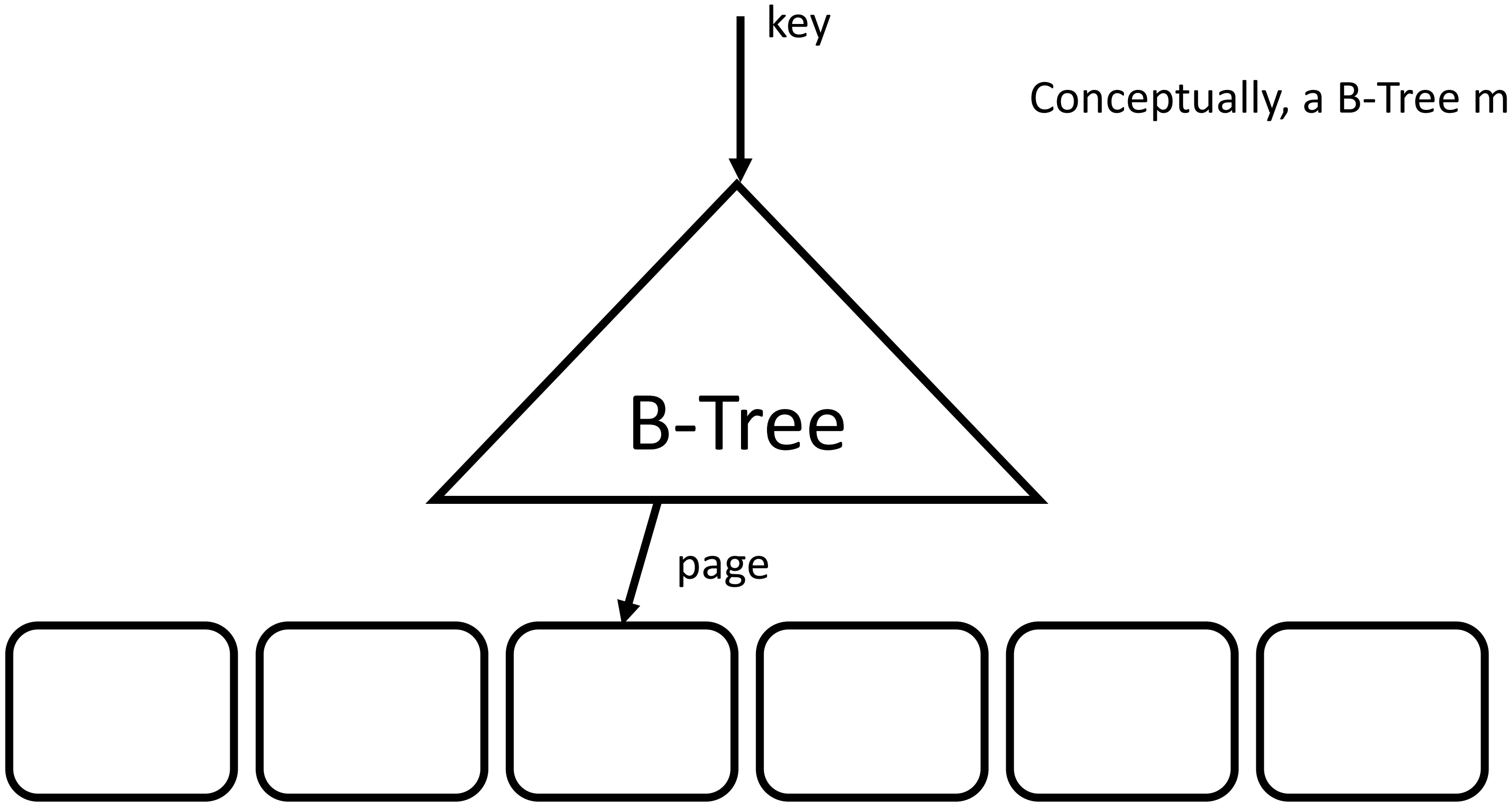

application/SQL
access patterns
complex queries
modules
Query Parser
Query Compiler
Optimizer
Evaluation Engine
Memory/Storage Management
Indexing
Transaction Management
Tuner


application/SQL
access patterns
complex queries
modules
Query Parser
Query Compiler
Optimizer
Memory/Storage Management
Tuner
Use ML models to replace the
navigational part of an ***Index***
Indexing
Transaction Management

B-Trees vs. Learned Indexes

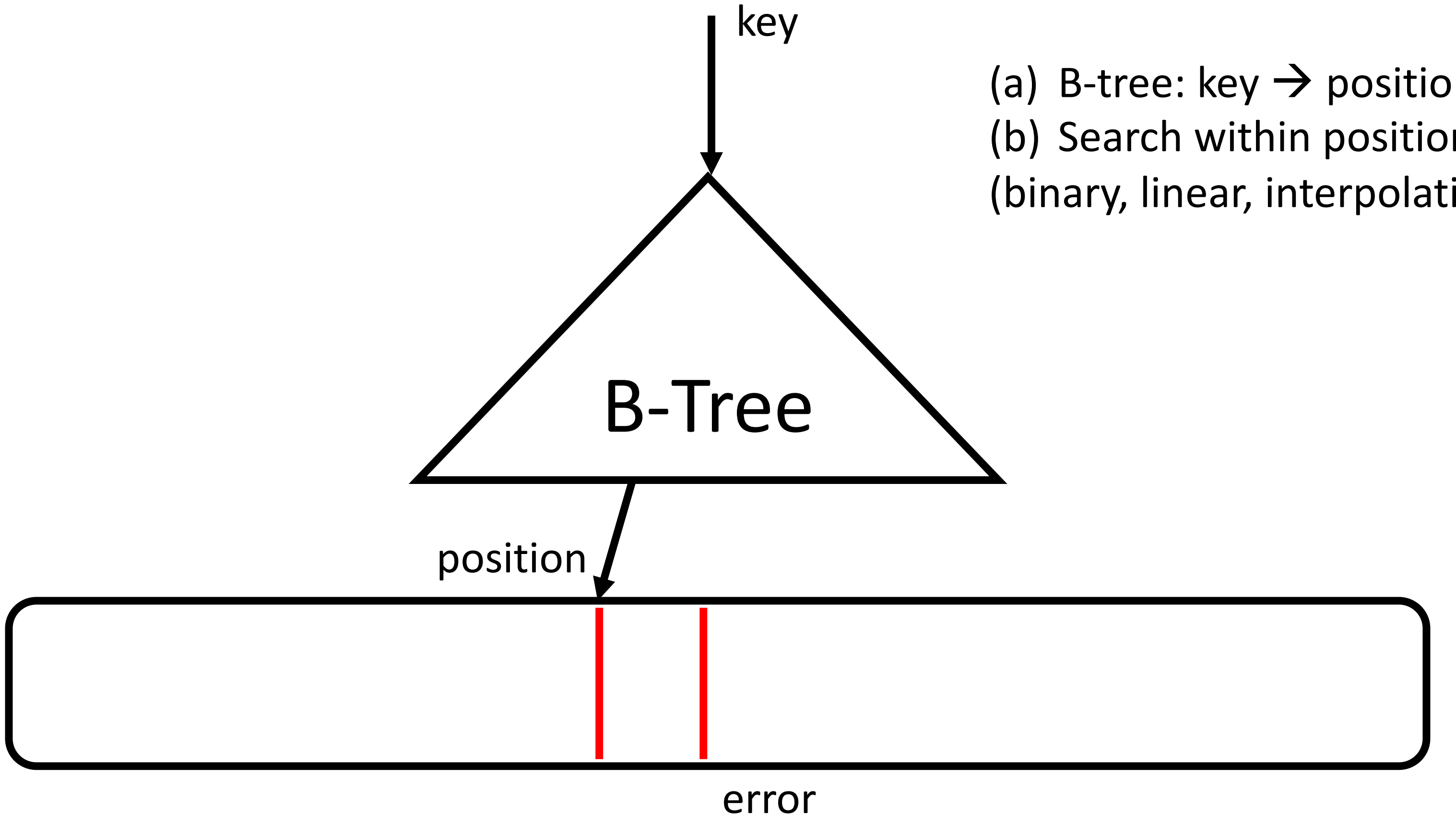


What is the difference?



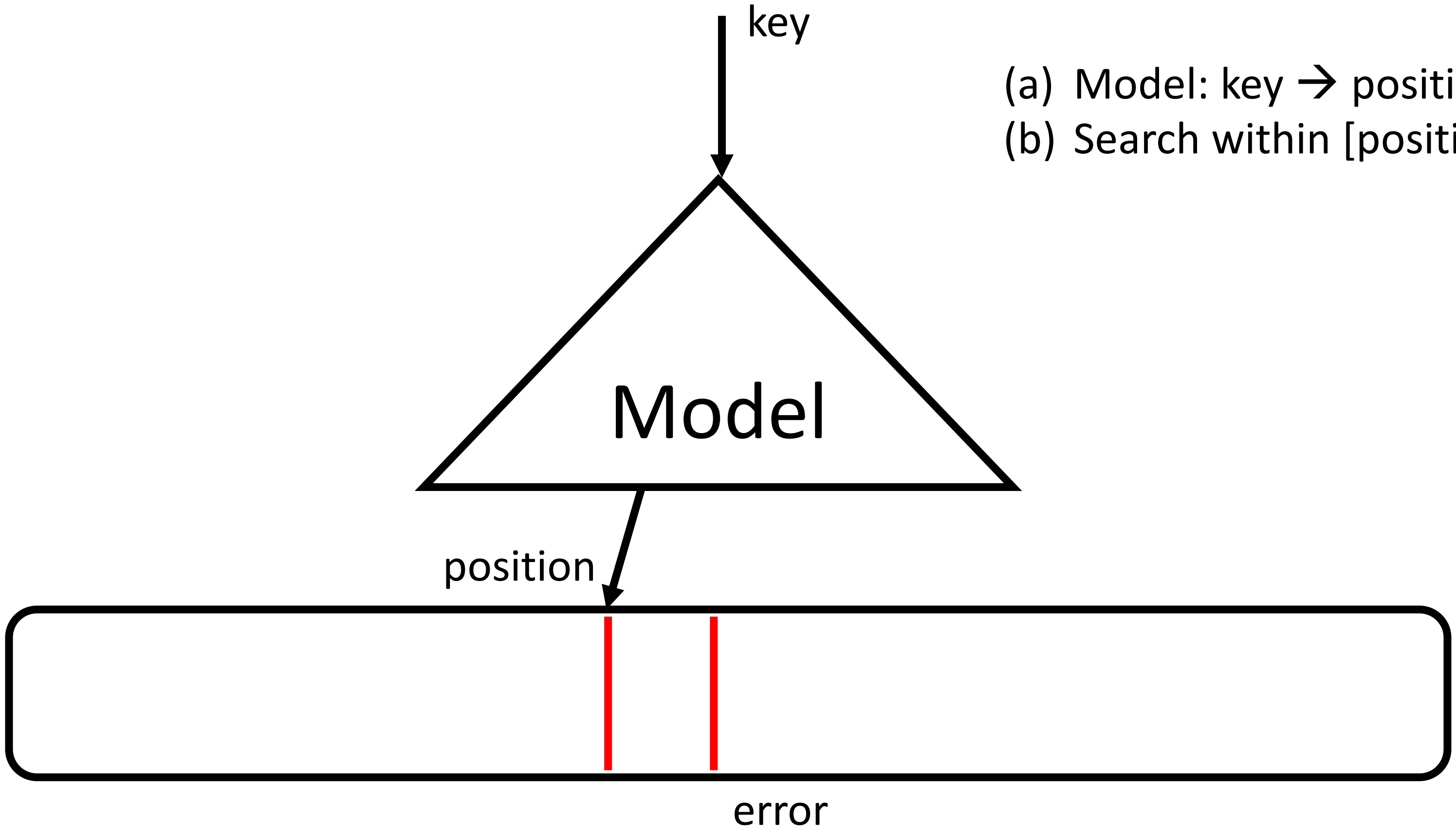
Conceptually, a B-Tree maps a key to a location (page)

Alternative view: data is sorted



(a) B-tree: key → position
(b) Search within position, position+error
(binary, linear, interpolation, exponential search)

A B-Tree is a Model



(a) Model: key → position estimate
(b) Search within [position-error, position+error]

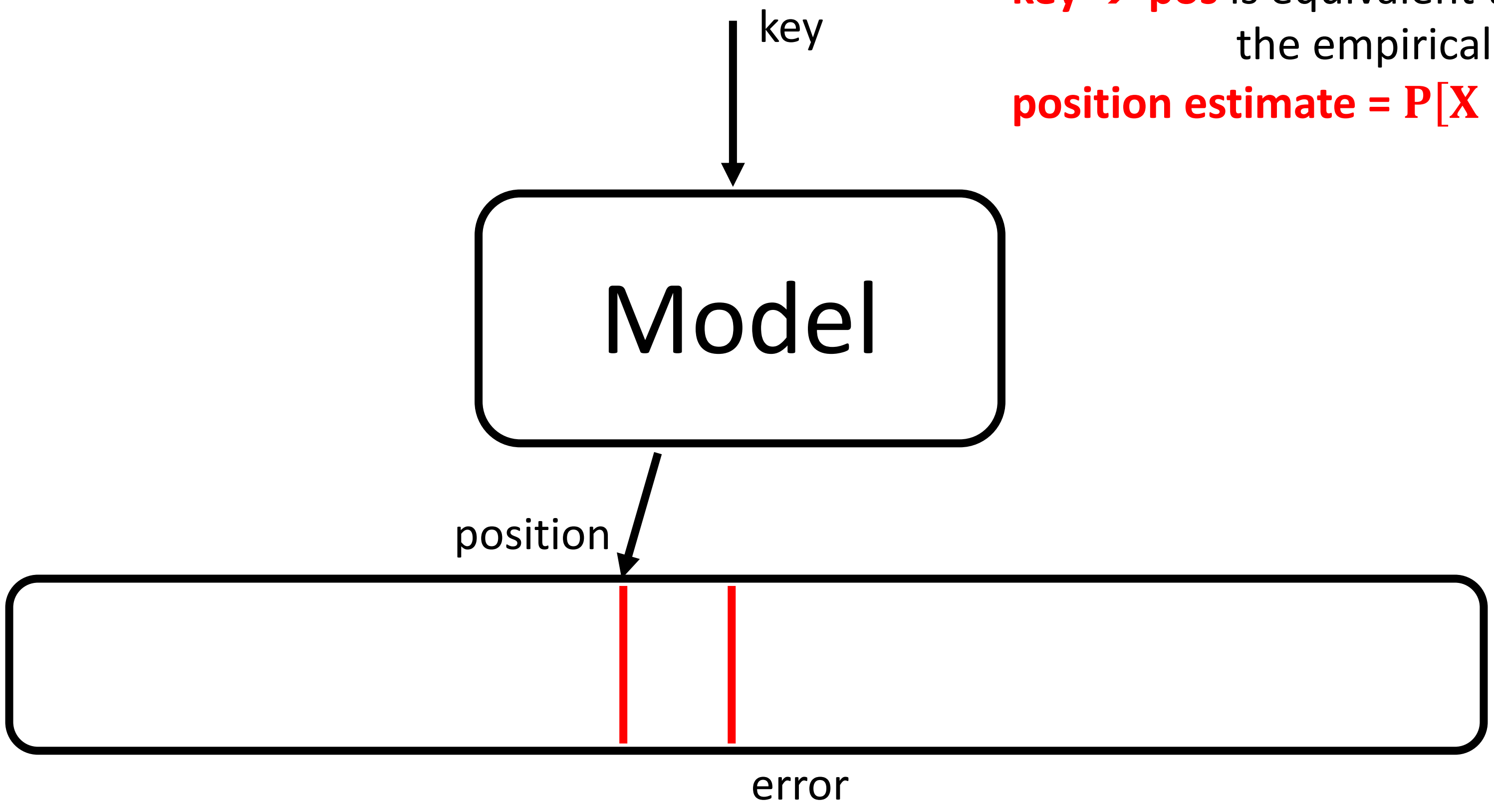
A B-Tree is already a model!

A B-Tree is a Model

A form of regression model

key → pos is equivalent to modeling the empirical CDF of the data

position estimate = $\mathbf{P[X \le \mathit{key}] * \#\mathit{keys}}$



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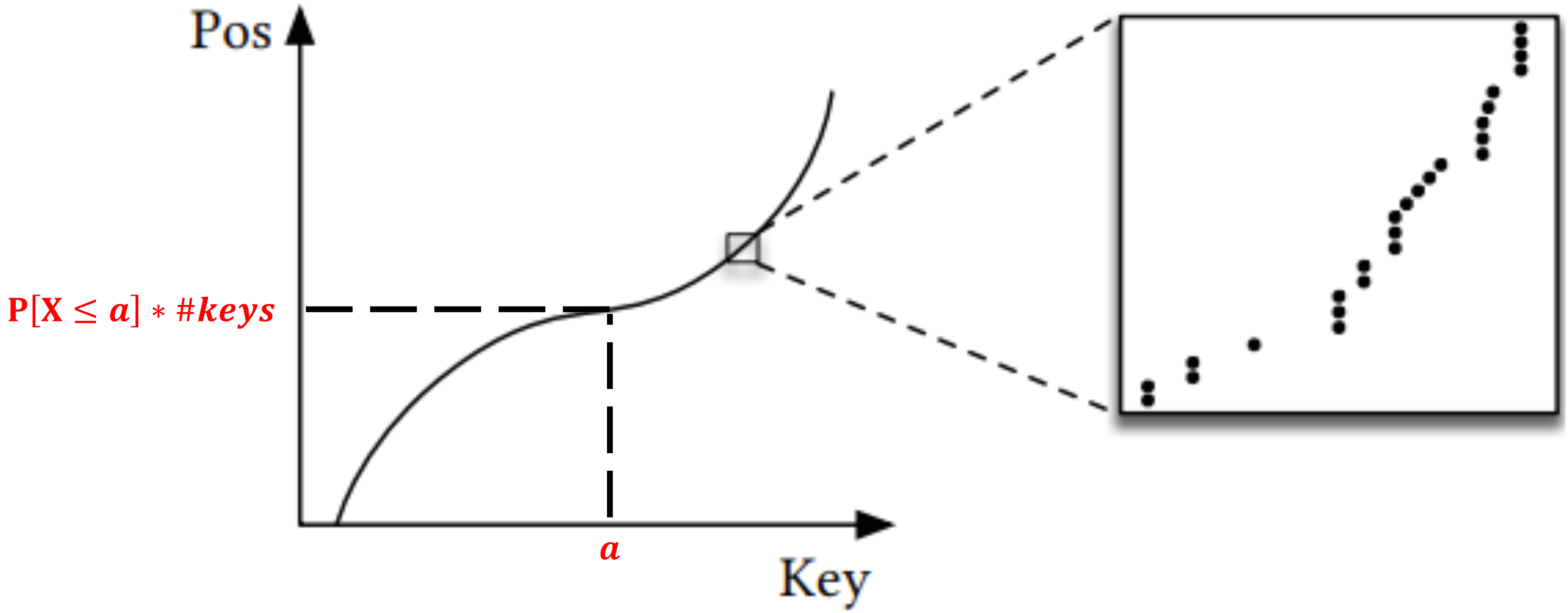
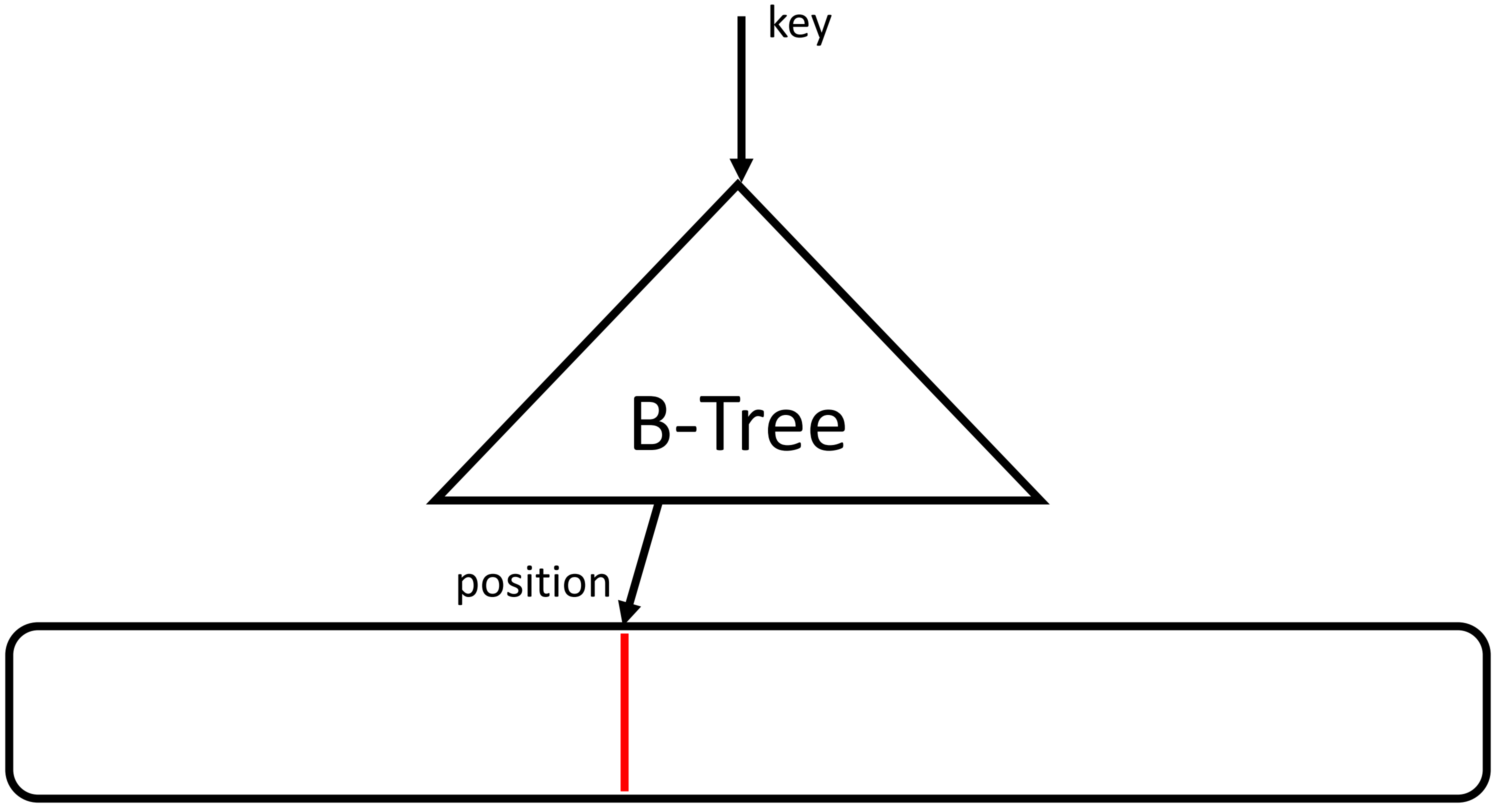

Pos
$P[X \leq a] * \#keys$
a
Key

B-Trees are regression trees



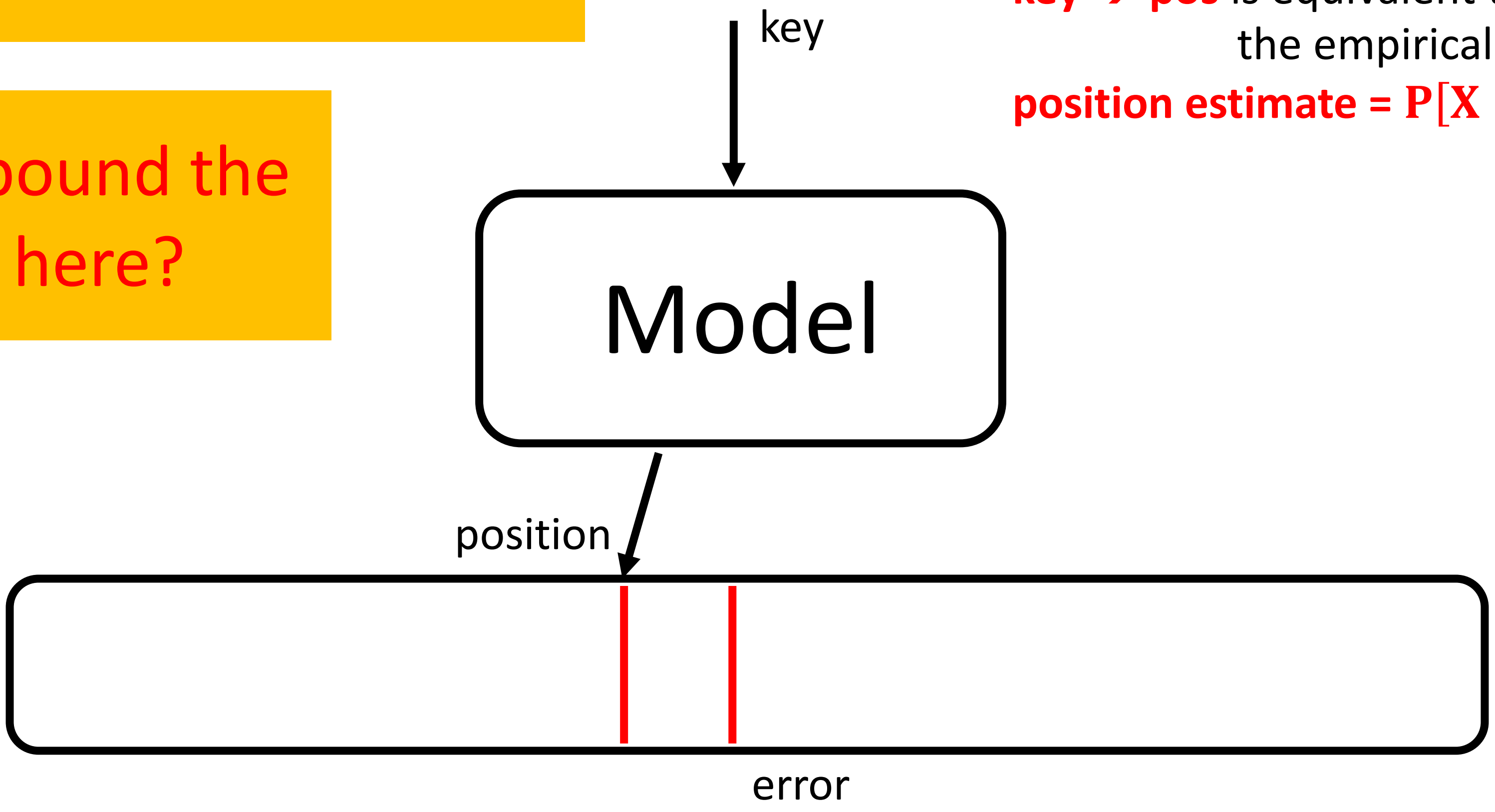
B-Trees is ***already*** a form of a learned index

What does this mean?

Learned Indexes

B-Trees have bounded error

Can we bound the error here?



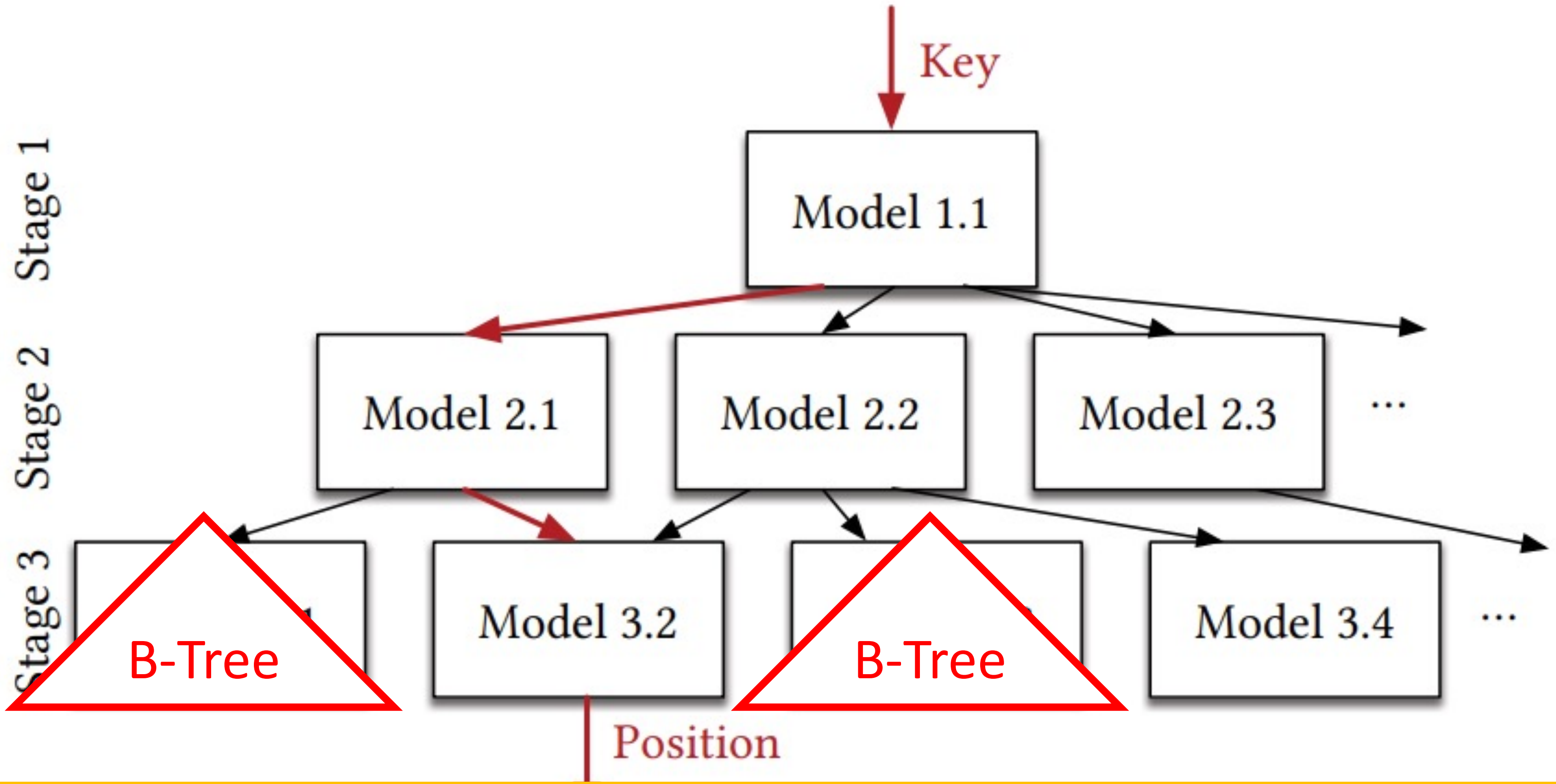
A form of regression model

key → pos is equivalent to modeling the empirical CDF of the data

position estimate = $\mathbf{P[X \leq \mathit{key}] * \#\mathit{keys}}$

What is the problem if we use an arbitrary model?

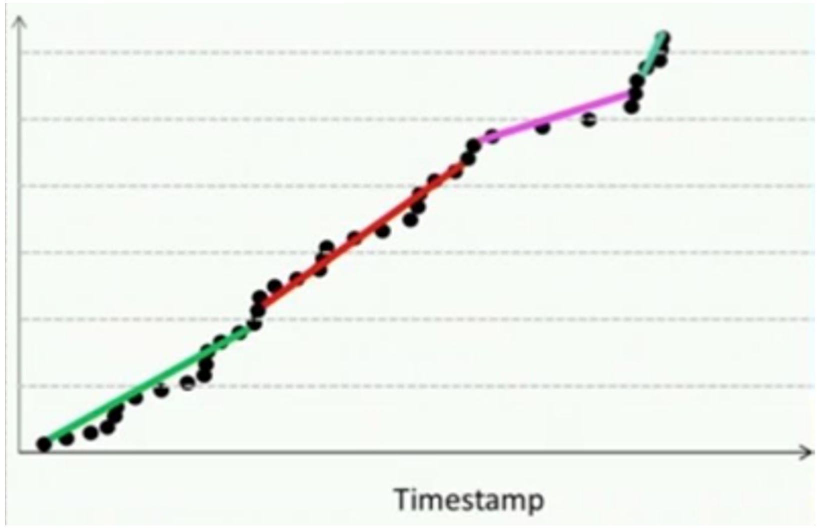
Last-mile indexing



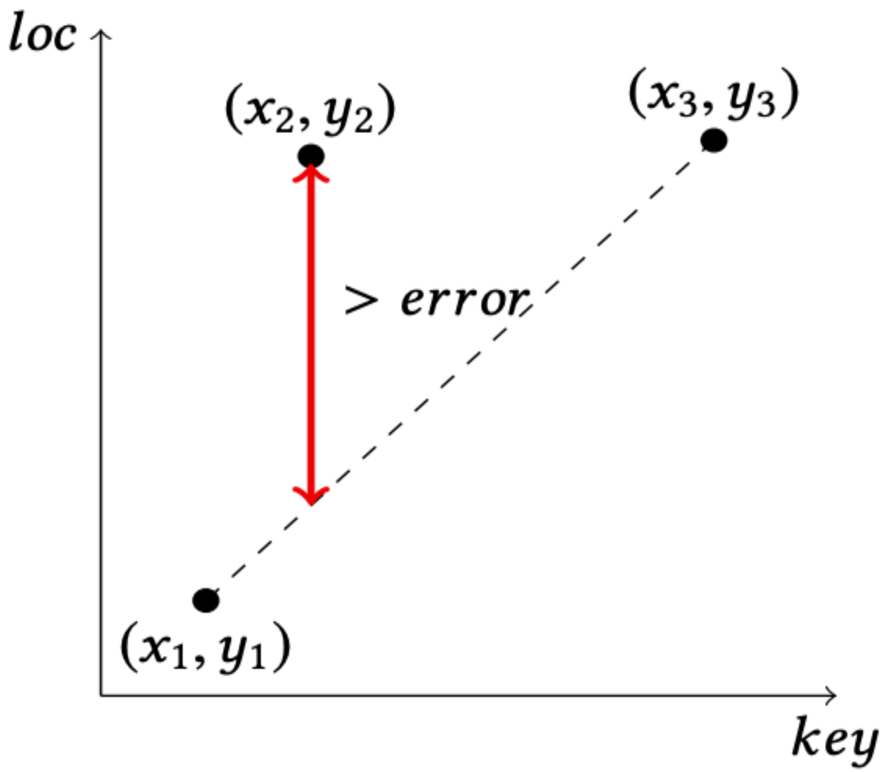
Some models can be replaced sub-B-Trees

Every level provides gain in accuracy

Use case: FITing-Tree



Piece-wise linear approximation



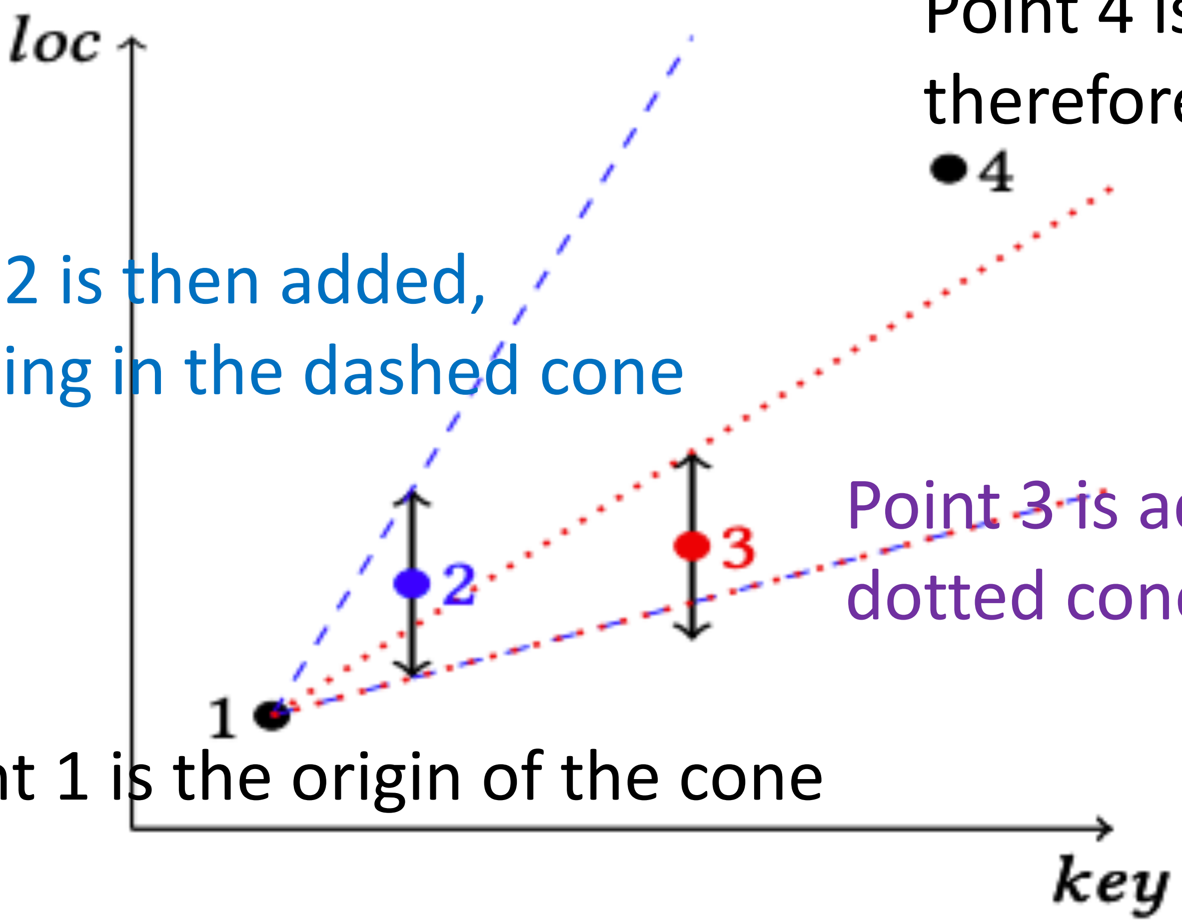
A segment from (x_1,y_1) to (x_3,y_3) is **not valid** if (x_2,y_2) is further than ***error*** from the interpolated line.

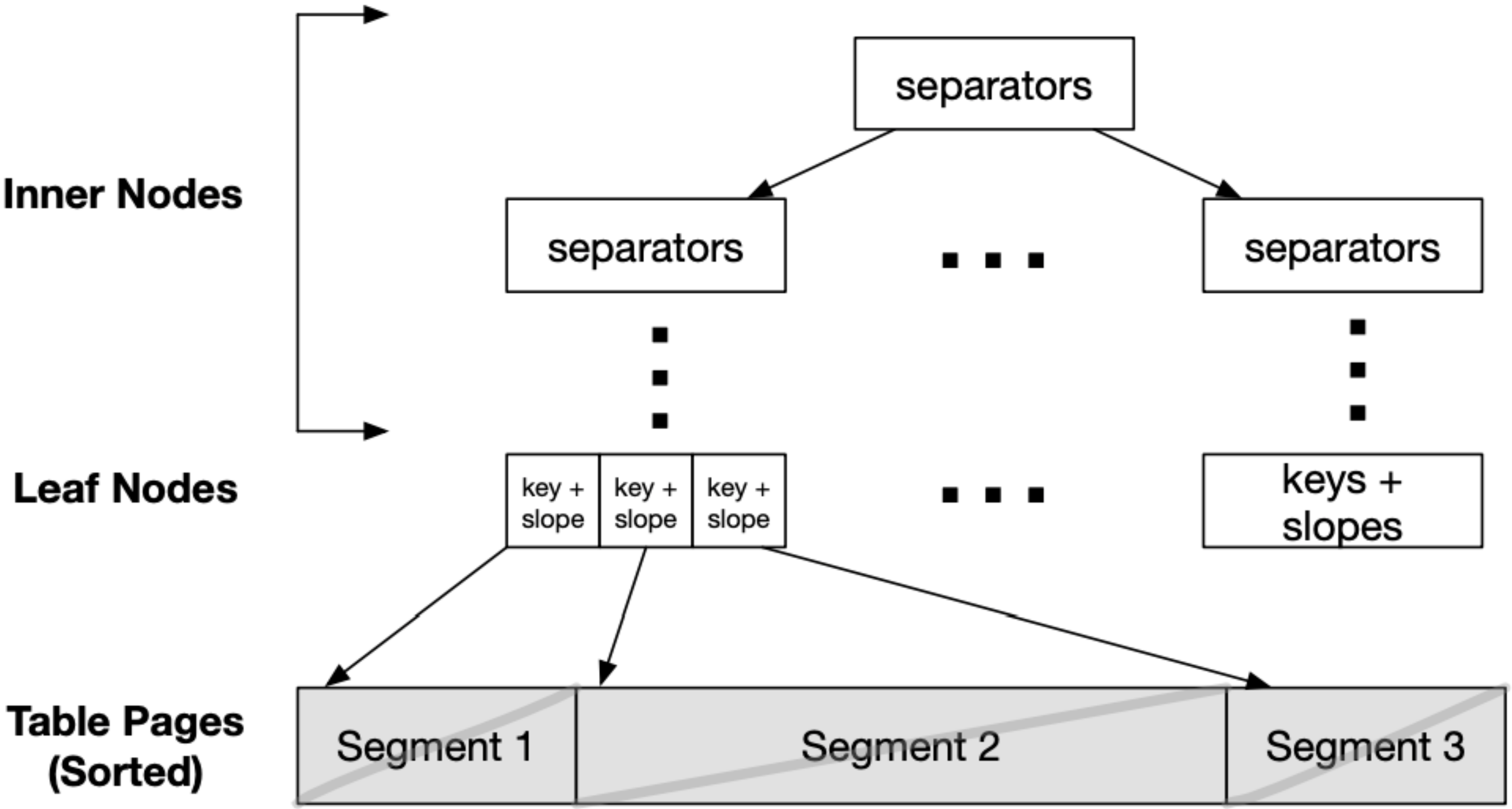
Point 4 is outside the dotted cone and therefore starts a new segment

Point 2 is then added, resulting in the dashed cone

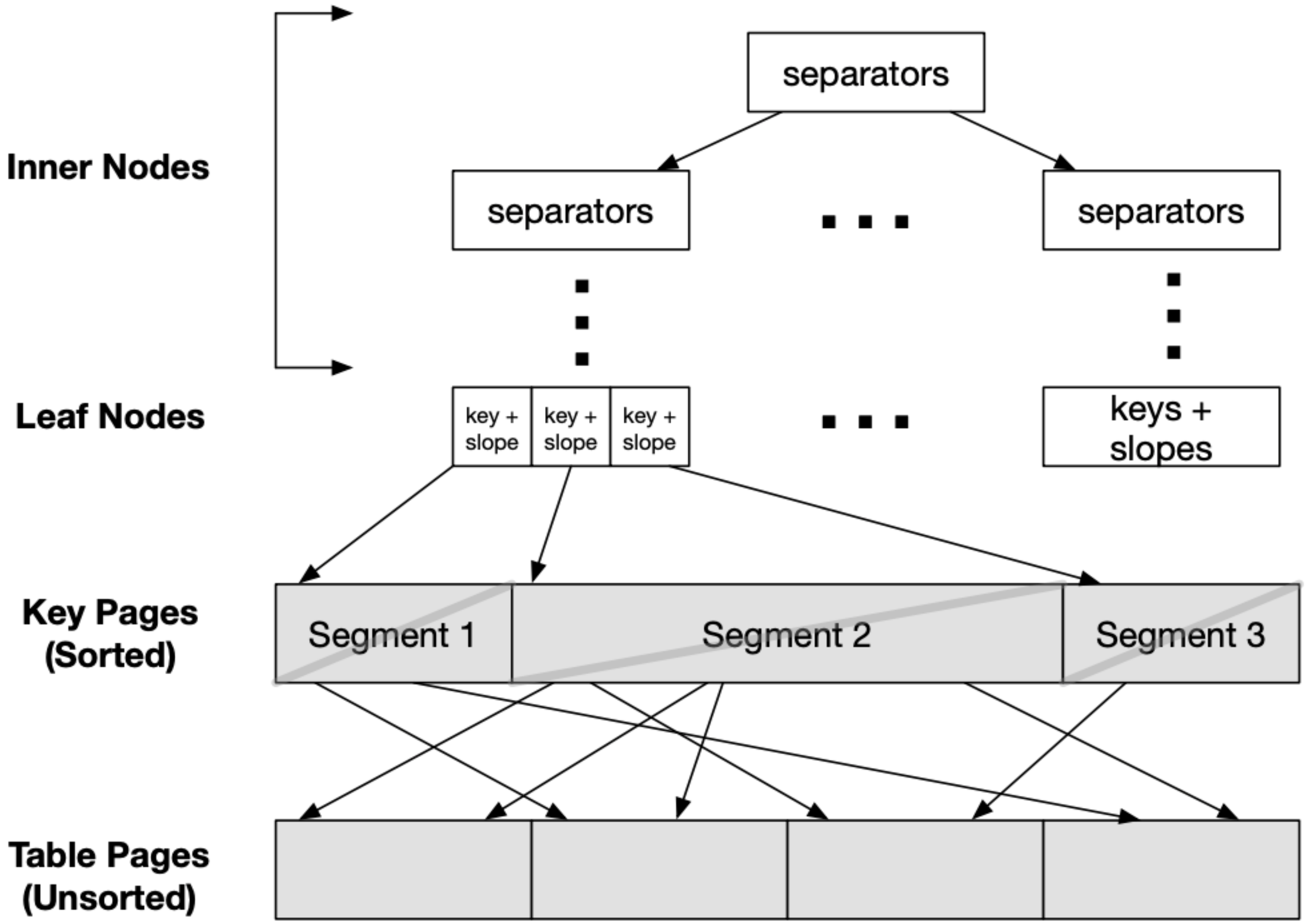
Point 3 is added next, yielding in the dotted cone

Point 1 is the origin of the cone





What if base data is not sorted?

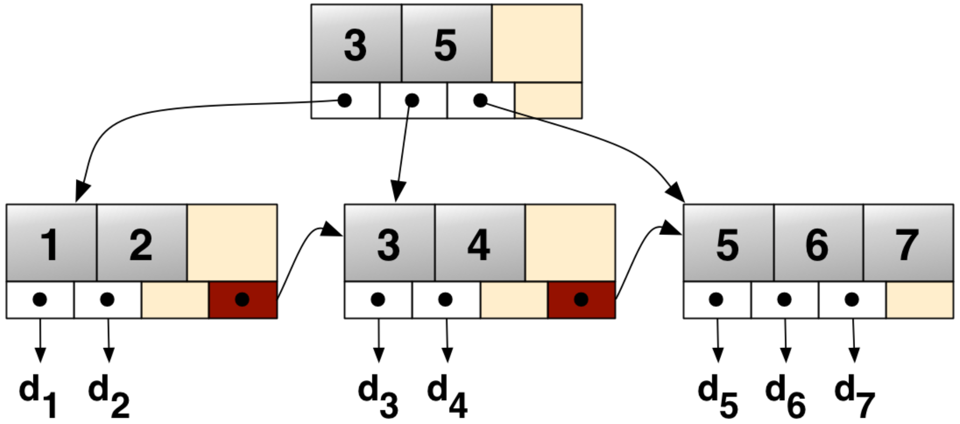


Need to materialize sorted data

What about updates and learned indexes?

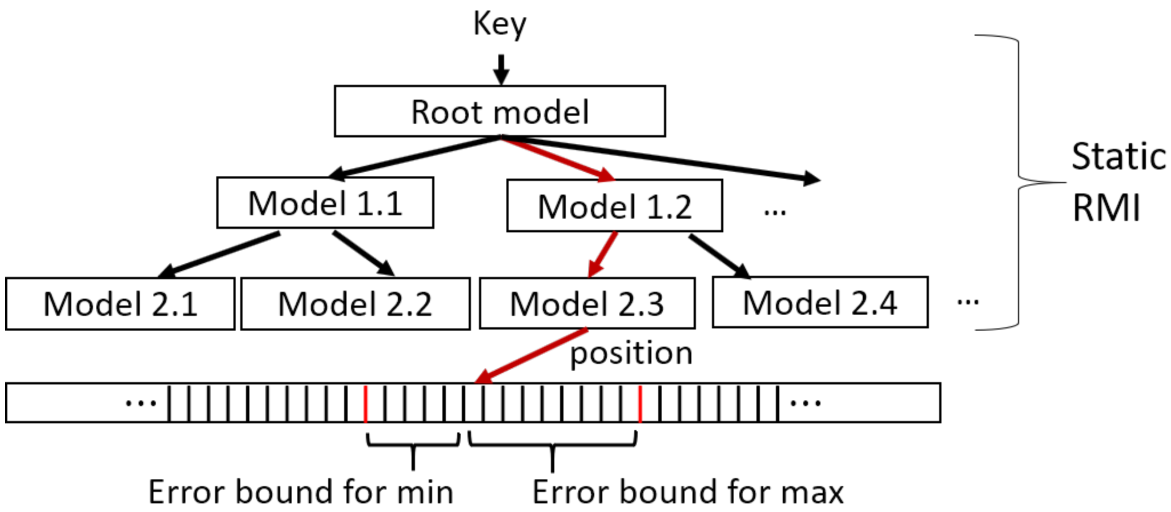
B+ Tree

- Traverses tree using comparisons
- Supports OLTP-style mixed workloads
  - Point lookups, range queries
  - Inserts, updates, deletes



Learned Index (Kraska et al., 2018)

- Traverses tree using computations (models)
- Supports point lookups and range queries
- Advantages: 3X faster reads, 10X smaller size
- Limitation: does not support writes



ALEX goals

	B+ Tree	Learned Index	ALEX
Lookup time	Slow	Fast	Faster
Insert time	Fast	Not Supported	Fast
Space usage	High	Low	Low

(every row should be read independently)

ALEX goals

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(every row should be read independently)

ALEX design overview

Structure
- **Dynamic tree** structure
- Each **node** contains a **linear model**
  - **internal** nodes → **models select the child** node
  - **data** nodes → **models predict the position** of a key

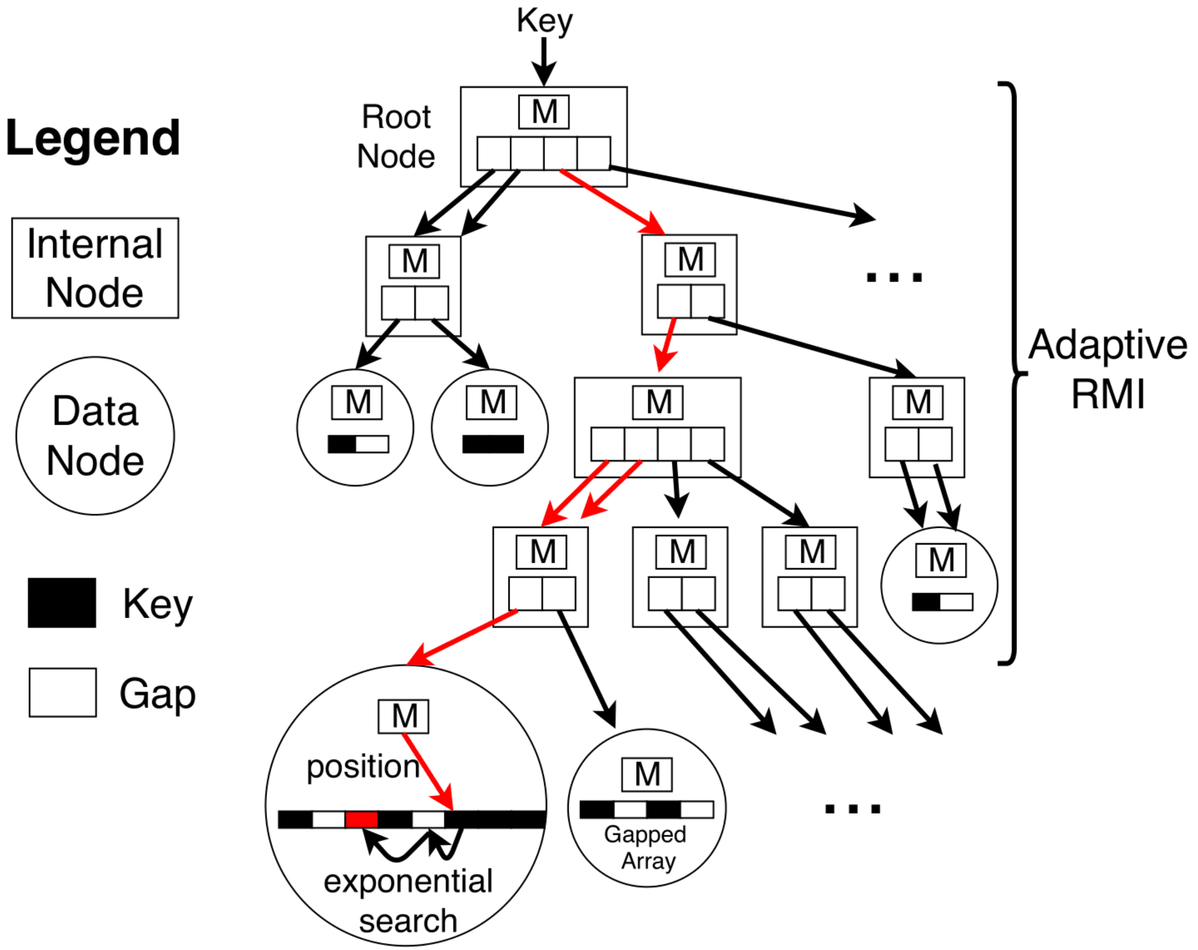
Core operations
- Lookup
  - Use **RMI to predict location of key** in a data node
  - Do **local search to correct** for prediction **error**
- Insert
  - Do a **lookup to find the insert** position
  - **Insert the new key/value** (might require shifting)

Current design constraints

a) In memory

b) Numeric data types

c) Single threaded



ALEX Core Ideas

	Faster Reads	Faster Writes	Adaptiveness
1. Gapped Array		✓	
2. Model-based Inserts	✓		
3. Exponential Search	✓		
4. Adaptive Tree Structure	✓	✓	✓

1. Gapped Array

How should data be stored in data nodes?

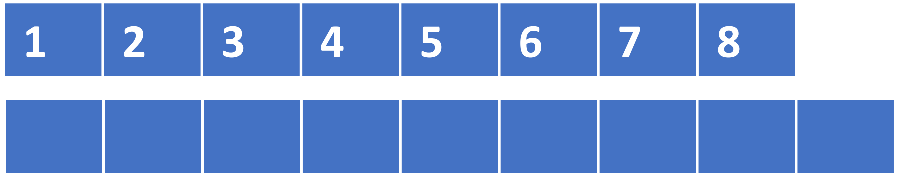
1. Gapped Array

Dense Array

1	2	3	4	5	6	7	8

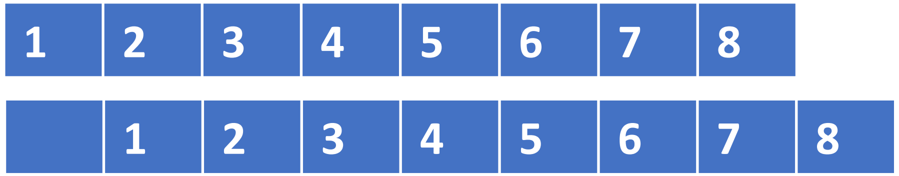
1. Gapped Array

Dense Array



1. Gapped Array

Dense Array



1. Gapped Array

Dense Array

1	2	3	4	5	6	7	8	
0	1	2	3	4	5	6	7	8

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Dense Array

0	1	2	3	4	5	6	7	8

1. Gapped Array

Insertion Time

Dense Array

0	1	2	3	4	5	6	7	8

$O(n)$

1. Gapped Array

Insertion Time

Dense Array

0	1	2	3	4	5	6	7	8

$O(n)$

B+ Tree Node

1	2	3	4	5	6	7	8				

1. Gapped Array

Insertion Time

Dense Array

0	1	2	3	4	5	6	7	8

$O(n)$

B+ Tree Node

	1	2	3	4	5	6	7	8			

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													Insertion Time
Dense Array	0	1	2	3	4	5	6	7	8				$O(n)$
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B+ Tree Node	0	1	2	3	4	5	6	7	8				$O(n)$

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1. Gapped Array

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Storing data in Gapped Arrays achieves inserts using fewer shifts, leading to faster writes

1. Gapped Array

													Insertion Time
Dense Array	0	1	2	3	4	5	6	7	8				$O(n)$
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Storing data in Gapped Arrays achieves inserts using fewer shifts, leading to faster writes

2. Model-based Inserts

Where do we put gaps in the Gapped Array?

2. Model-based Inserts

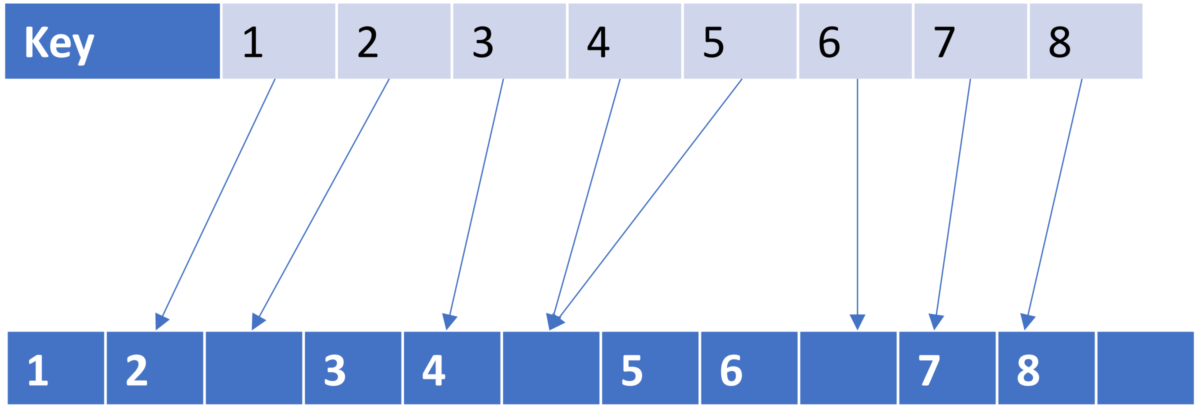
Gapped Array

1	2		3	4		5	6		7	8	

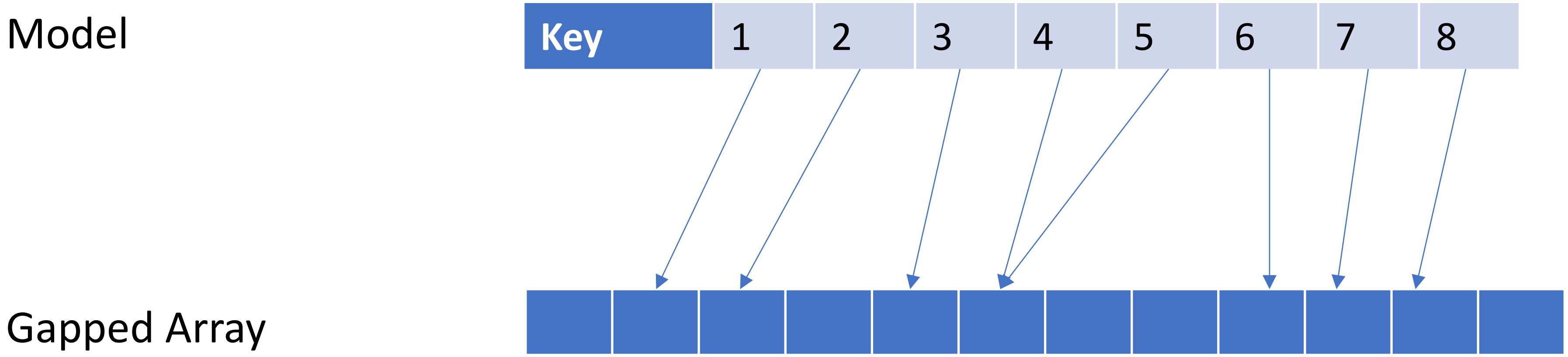
2. Model-based Inserts

Model

Gapped Array



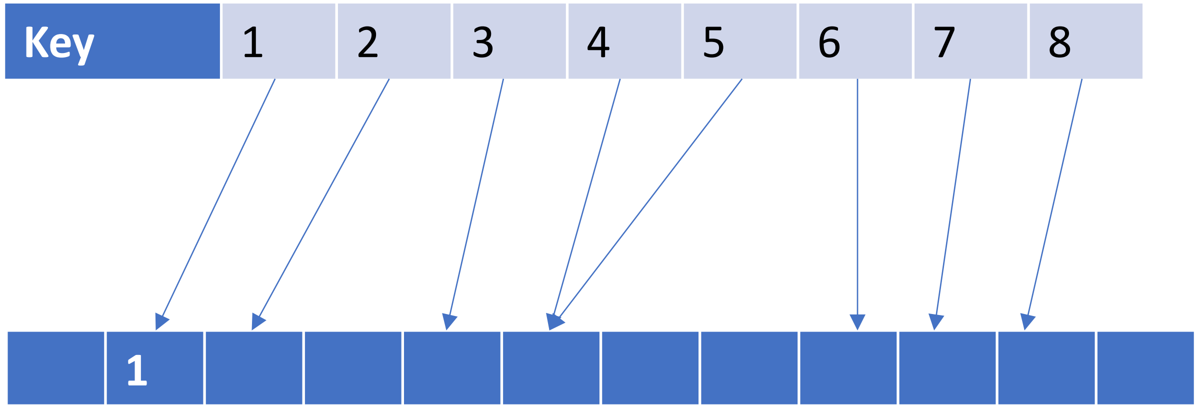
2. Model-based Inserts



2. Model-based Inserts

Model

Gapped Array



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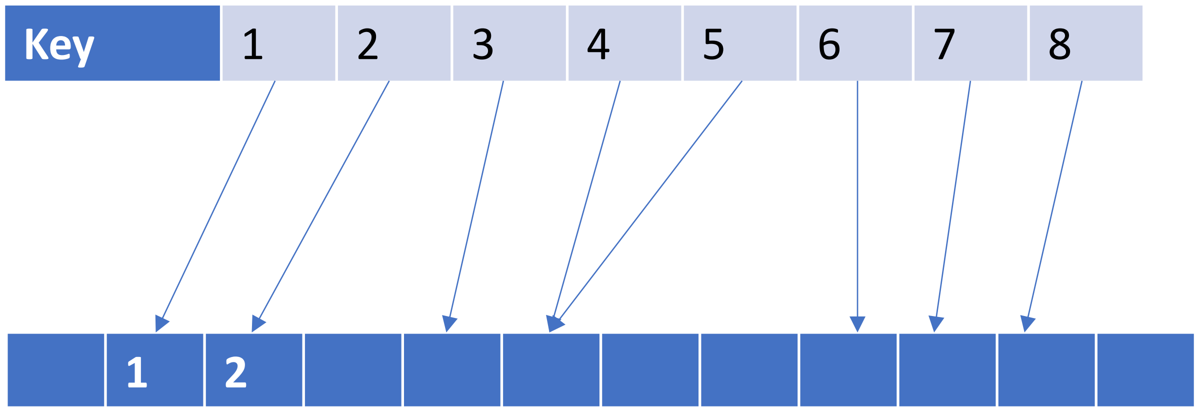
Model

Gapped Array



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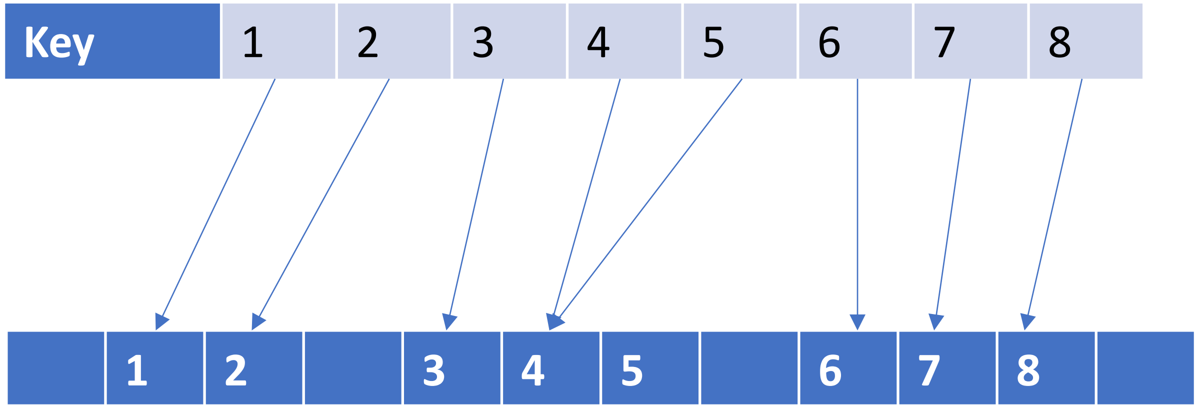

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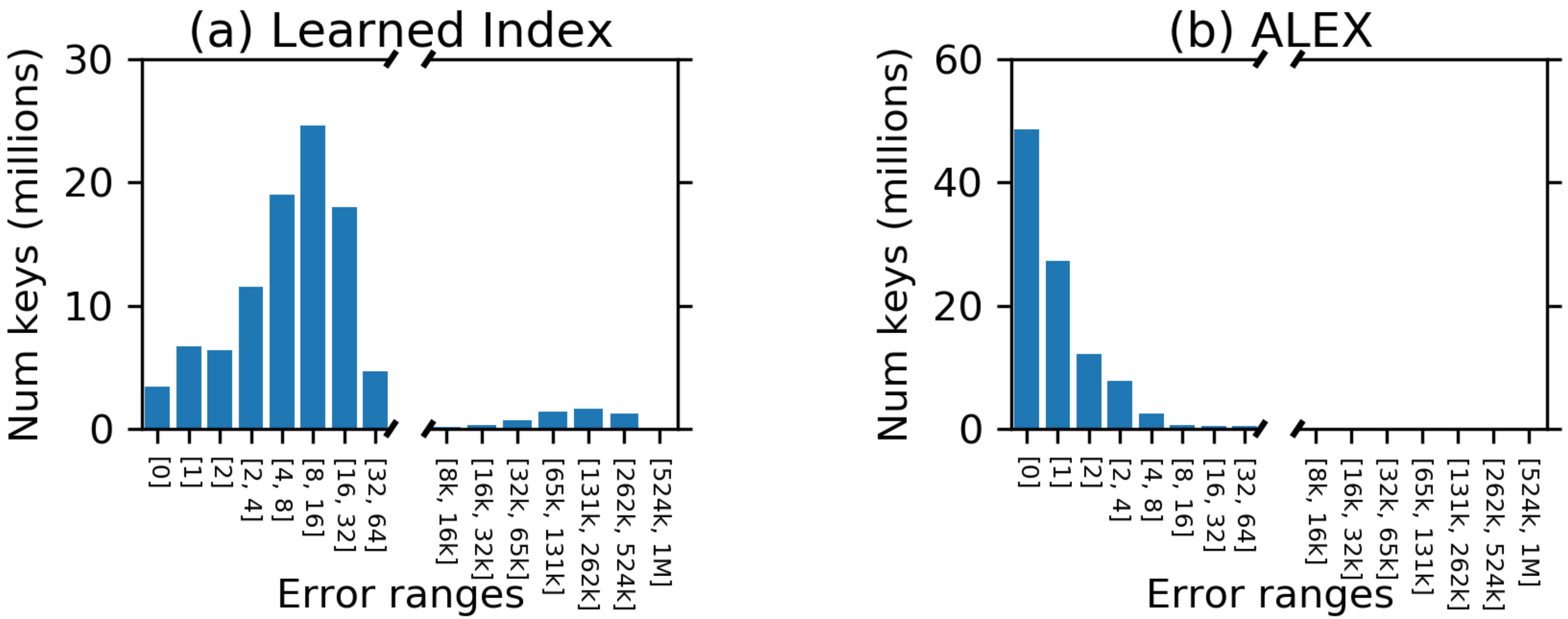
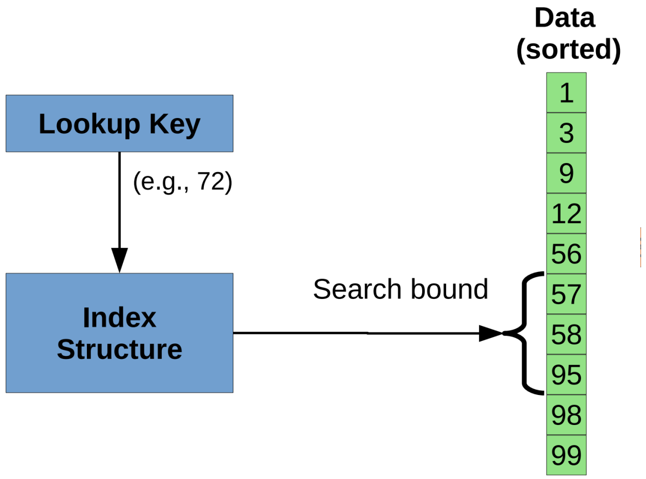


Model-based inserts achieve lower prediction error, leading to faster reads

3. Exponential Search



Can we do better than binary search?

Explanation: Exponential Search

1	2	3	5	6	7	8	13	15	17	18	21	22	23

```
int exponential_search(T arr[], int size, T key)
{
    if (size == 0) {
        return NOT_FOUND;
    }

    int bound = 1;
    while (bound < size && arr[bound] < key) {
        bound *= 2;
    }

    return binary_search(arr, key, bound/2, min(bound + 1, size));
}
```


Core algorithm: binary search!

Key difference: exp. increasing search bound

Explanation: Exponential Search

1	2	3	5	6	7	8	13	15	17	18	21	22	23

search for 3

```
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Explanation: Exponential Search

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search for 22

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Explanation: Exponential Search

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search for 22

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Explanation: Exponential Search

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Explanation: Exponential Search

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```
int exponential_search(T arr[], int size, T key)
{
    if (size == 0) {
        return NOT_FOUND;
    }
```

We begin our search from the "predicted" location, ***low error expected***!

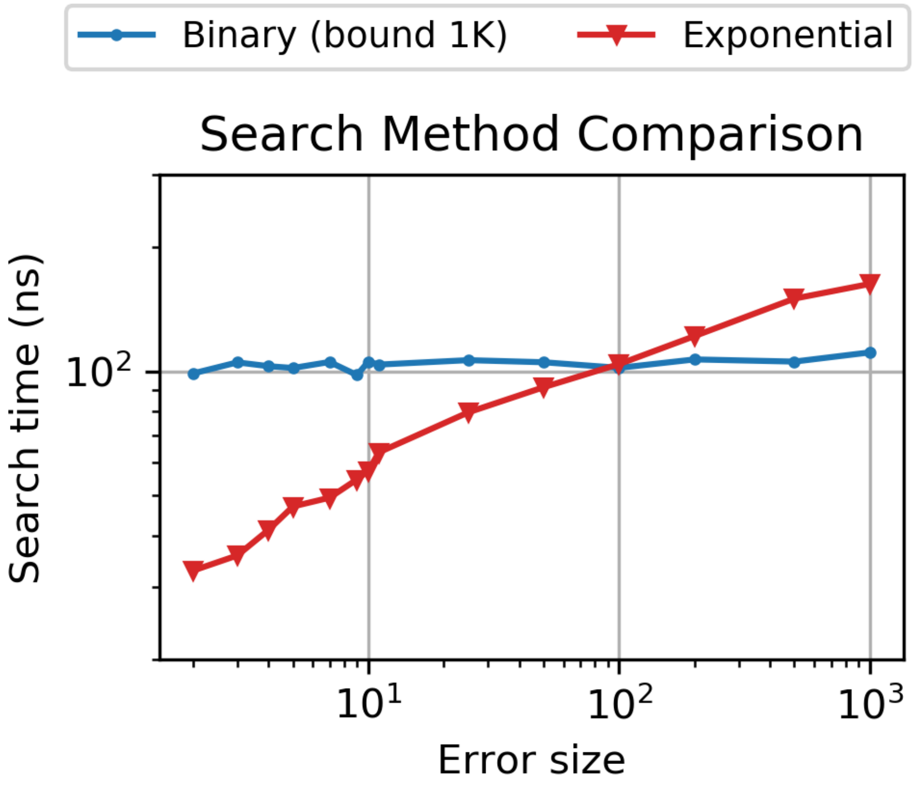
```
    }
```

Why is this helpful in our case?

```
}
```

Exp. Search is ***ideal*** for a **search key at the beginning** of the array!

3. Exponential Search



Model errors are low, so exponential search is faster than binary search

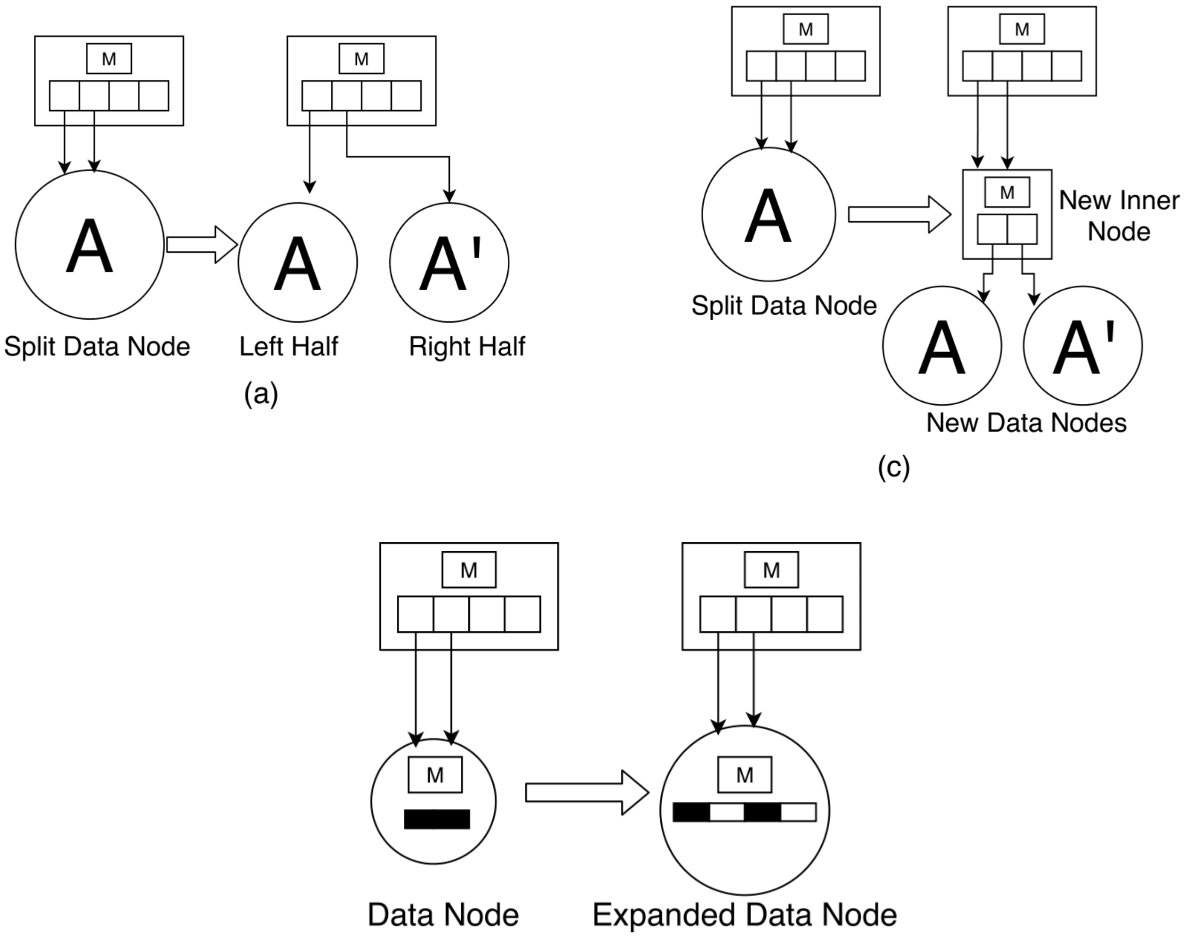
4. Adaptive Structure

What happens if data nodes become full?

What happens if models become inaccurate?

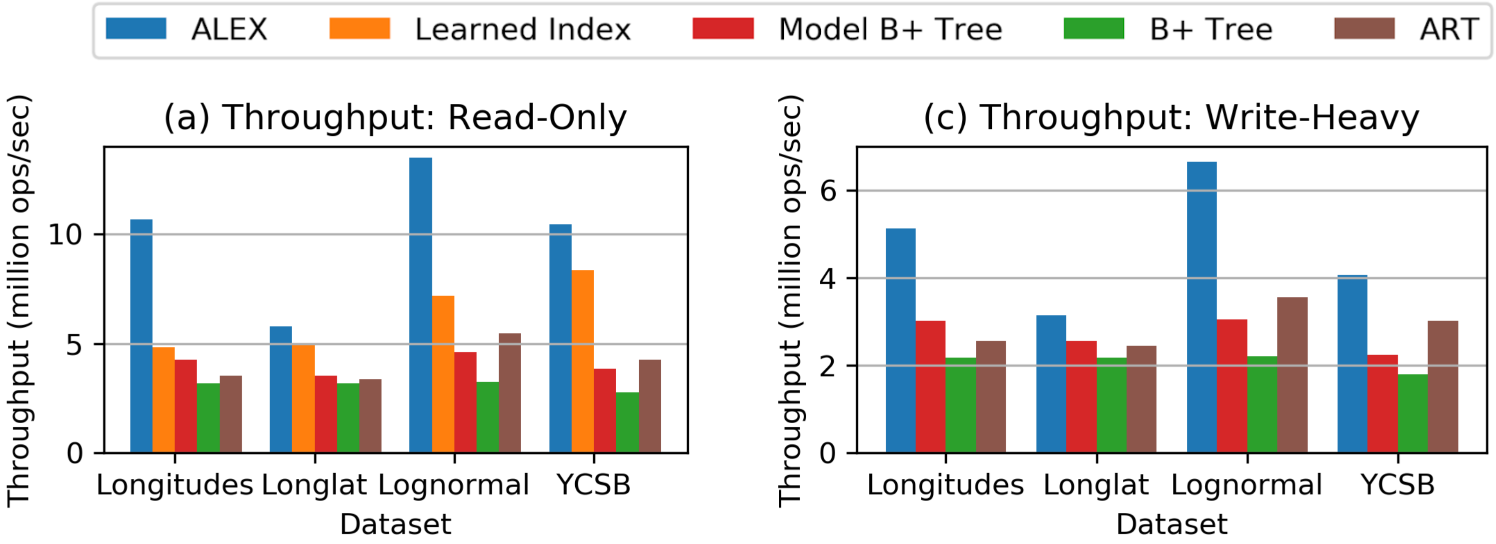
4. Adaptive Structure

- Flexible tree structure
  - Split nodes sideways
  - Split nodes downwards
  - Expand nodes
  - Merge nodes, contract nodes
- Key idea: all decisions are made to maximize performance
  - Use cost model of query runtime
  - No hand-tuning
  - Robust to data and workload shifts



Results

- High-level results
  - Fast reads
  - Fast writes

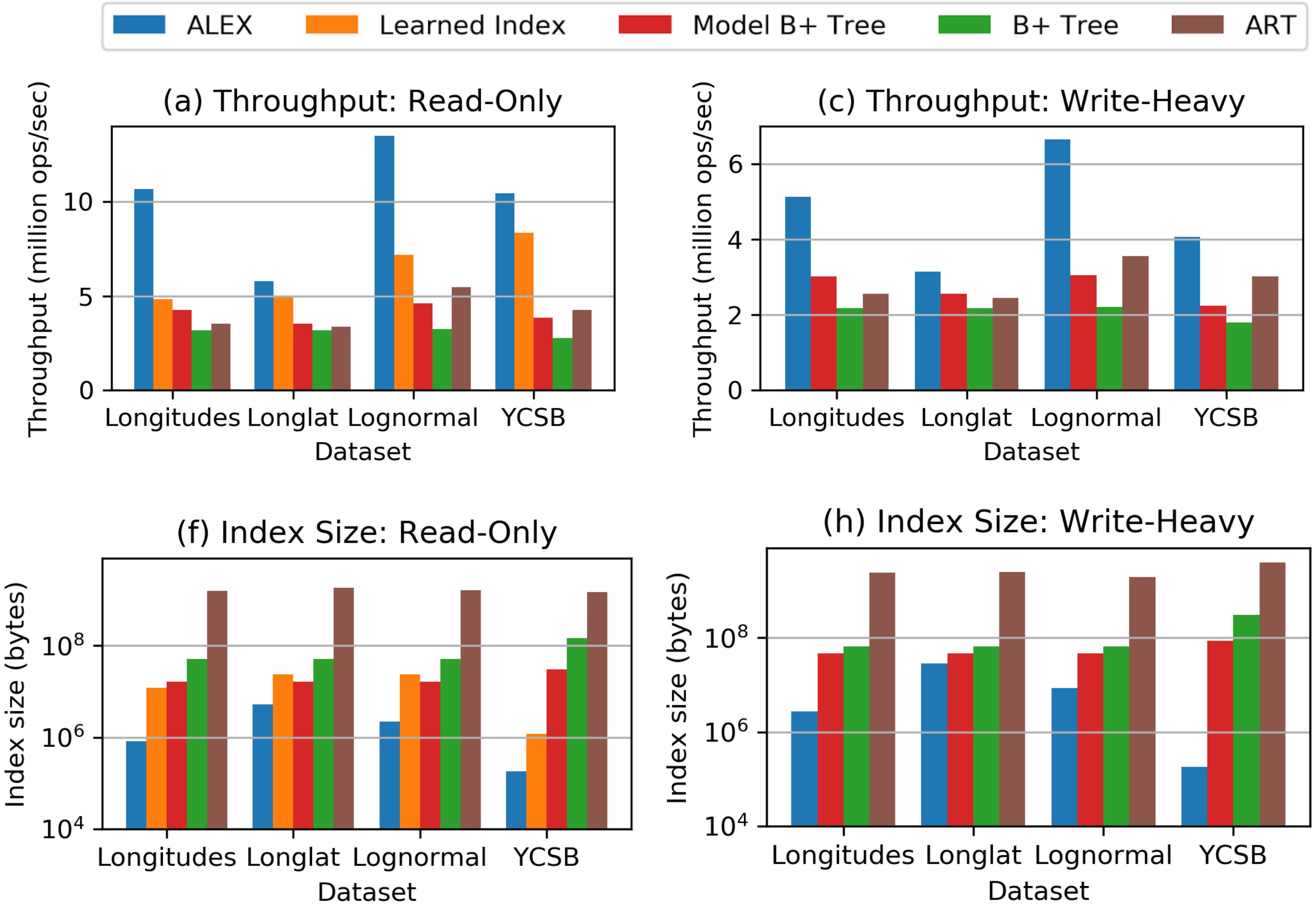


~4x faster than B+ Tree
~2x faster than Learned Index

~2-3x faster than B+ Tree

Results

- High-level results
  - Fast reads
  - Fast writes
  - Smaller index size
- Other results
  - Efficient bulk loading
  - Scales
  - Robust to data and workload shift



~3 orders of magnitude less space for index

ALEX Summary

- Combines the best of B+ Tree and Learned Indexes
  - Supports OLTP-style mixed workloads
    - Point lookups, range queries
    - Inserts, updates, deletes
  - Up to 4X faster, 2000X smaller than B+ Tree
- Current research
  - String keys
  - Concurrency
  - Persistence

	Faster Reads	Faster Writes	Adaptiveness
Gapped Array		✓	
Model-based Inserts	✓		
Exponential Search	✓		
Adaptive Tree Structure	✓	✓	✓

github.com/microsoft/ALEX

Learned Indexes

Replace data structure with **learned models**

- ✓ Simple approaches like linear approximation work well
- ✓ Empty space for updates
- ✓ Error bounds to split model nodes
- ✓ Exponential search for last-mile searching

- ➢ A very fertile area of research!
- ➢ A comprehensive list of papers: http://dsg.csail.mit.edu/mlforsystems/papers/#learned-range-indexes