L13: Application Case Studies

Administrative Issues

- Next assignment, triangular solve
  - Due 5PM, Monday, March 8
  - handin cs6963 lab 3 <probfile>"
- Project proposals
  - Due 5PM, Wednesday, March 17 (hard deadline)
  - handin cs6963 prop <pdffile>

Outline

- Discussion of strsm (more on Monday)
- How to approach your projects
- Application Case Studies
  - Advanced MRI Reconstruction
    Reading: Kirk and Hwu, Chapter 7, http://courses.ece.illinois.edu/ece498/al/textbook/Chapter7-MRI-Case-Study.pdf

Triangular Solve (STRSM)

```
for (j = 0; j < n; j++)
   for (k = 0; k < n; k++)
       if (B[j*n+k] != 0.0f) {
            for (i = k+1; i < n; i++)
               B[j*n+i] -= A[k * n + i] * B[j * n + k];
       }
```

Equivalent to:

```
cublasStrsm('l' /* left operator */, 'l' /* lower triangular */,
            'N' /* not transposed */, 'u' /* unit triangular */,
            N, N, alpha, d_A, N, d_B, N);
```

See: http://www.netlib.org/blas/strsm.f

Approaching Projects/STRSM/Case Studies

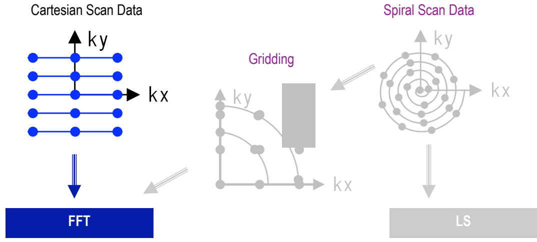
1. Parallelism?
   - How do dependences constrain partitioning strategies?
2. Analyze data accesses for different partitioning strategies
   - Start with global memory: coalesced?
   - Consider reuse: within a thread? Within a block? Across blocks?
3. Data Placement (adjust partitioning strategy?)
   - Registers, shared memory, constant memory, texture memory or just leave in global memory
4. Tuning
   - Unrolling, fine-tune partitioning, floating point, control flow, ...

Step 1. Simple Partition for STRSM

```
__global__ void strsm1( int n, float *A, float *B )
{
  int bx = blockIdx.x;
  int tx = threadIdx.x;
  int j = bx*THREADSPERBLOCK + tx; // // one thread per column, columns work
     independently
  int JN = j * n;
  int i, k;

  for (k = 0; k < n; ++k) {    // ROW
   int KN = k * n;
   for (i = k+1; i < n; ++i) {  // ALSO row
    // B[i][j] -= A[i][k] * B[k][j] element depends on elts in ROWS above it in same col
    B[ JN + i ] -= A[ KN + i ] * B[ JN + k ];
   }
  }
}
```

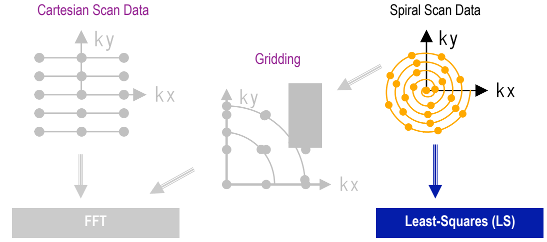
Slide source: Mark Hall

Reconstructing MR Images



Cartesian scan data + FFT:
Slow scan, fast reconstruction, images may be poor

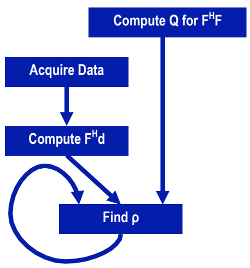
Reconstructing MR Images



Spiral scan data + LS
Superior images at expense of significantly more computation

Least-Squares Reconstruction

$$F^H F \rho = F^H d$$



- Q depends only on scanner configuration
- F^Hd depends on scan data
- ρ found using linear solver
- Accelerate Q and F^Hd on G80
  - Q: 1-2 days on CPU
  - F^Hd: 6-7 hours on CPU
  - ρ: 1.5 minutes on CPU


```
for (m = 0; m < M; m++) {

  phiMag[m] = rPhi[m]*rPhi[m] +
              iPhi[m]*iPhi[m];

  for (n = 0; n < N; n++) {
    expQ = 2*PI*(kx[m]*x[n] +
                 ky[m]*y[n] +
                 kz[m]*z[n]);

    rQ[n] +=phiMag[m]*cos(expQ);
    iQ[n] +=phiMag[m]*sin(expQ);
  }
}
```

(a) Q computation

```
for (m = 0; m < M; m++) {

  rMu[m] = rPhi[m]*rD[m] +
           iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] -
           iPhi[m]*rD[m];

  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg -
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}
```

(b) F^Hd computation

Q v.s. F^HD

Algorithms to Accelerate

```
for (m = 0; m < M; m++) {

  rMu[m] = rPhi[m]*rD[m] +
           iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] -
           iPhi[m]*rD[m];

  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg -
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}
```

- Scan data
  - M = # scan points
  - kx, ky, kz = 3D scan data
- Pixel data
  - N = # pixels
  - x, y, z = input 3D pixel data
  - rFhD, iFhD= output pixel data
- Complexity is O(MN)
- Inner loop
  - 13 FP MUL or ADD ops
  - 2 FP trig ops
  - 12 loads, 2 stores

Step 1. Consider Parallelism to Evaluate Partitioning Options

```
for (m = 0; m < M; m++) {

  rMu[m] = rPhi[m]*rD[m] +
           iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] -
           iPhi[m]*rD[m];

  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg -
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}
```

What about M total threads?

Note: M is O(millions)

(Step 2) What happens to data accesses with this strategy?

One Possibility

```
__global__ void cmpFHd(float* rPhi, iPhi, phiMag,
    kx, ky, kz, x, y, z, rMu, iMu, int N) {

  int m = blockIdx.x * FHD_THREADS_PER_BLOCK + threadIdx.x;

  rMu[m] = rPhi[m]*rD[m] + iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] - iPhi[m]*rD[m];

  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] + ky[m]*y[n] + kz[m]*z[n]);

    cArg = cos(expFhD);  sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg - iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg + rMu[m]*sArg;
  }
}
```

This code does not work correctly! The accumulation needs to use atomic operation.

Back to the Drawing Board – Maybe map the n loop to threads?

```
for (m = 0; m < M; m++) {

  rMu[m] = rPhi[m]*rD[m] + iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] - iPhi[m]*rD[m];

  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] + ky[m]*y[n] + kz[m]*z[n]);
    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg - iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg + rMu[m]*sArg;
  }
}
```



```
for (m = 0; m < M; m++) {

  rMu[m] = rPhi[m]*rD[m] +
           iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] -
           iPhi[m]*rD[m];

  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg -
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}
```

(a) $F^{H}d$ computation

```
for (m = 0; m < M; m++) {

  rMu[m] = rPhi[m]*rD[m] +
           iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] -
           iPhi[m]*rD[m];

for (m = 0; m < M; m++) {
  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg -
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}
```

(b) after loop fission

A Separate cmpMu Kernel

```
__global__ void cmpMu(float* rPhi, iPhi, rD, iD, rMu, iMu)
{
  int m = blockIdx.x * MU_THREAEDS_PER_BLOCK + threadIdx.x;

  rMu[m] = rPhi[m]*rD[m] + iPhi[m]*iD[m];
  iMu[m] = rPhi[m]*iD[m] - iPhi[m]*rD[m];
}
```

```
for (m = 0; m < M; m++) {
  for (n = 0; n < N; n++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg –
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}  (a) before loop interchange
```

```
for (n = 0; n < N; n++) {
  for (m = 0; m < M; m++) {
    expFhD = 2*PI*(kx[m]*x[n] +
                   ky[m]*y[n] +
                   kz[m]*z[n]);

    cArg = cos(expFhD);
    sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg –
                iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg +
                rMu[m]*sArg;
  }
}  (b) after loop interchange
```

Figure 7.9 Loop interchange of the FHD computation

Step 2. New FHd kernel

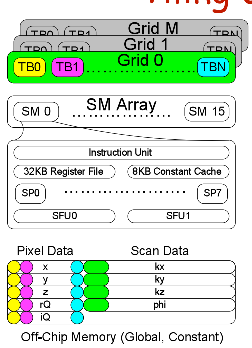
```
__global__ void cmpFHd(float*
    kx, ky, kz, x, y, z, rMu, iMu, int M) {

  int n = blockIdx.x * FHD_THREADS_PER_BLOCK + threadIdx.x;

  for (m = 0; m < M; n++) {
    float expFhD = 2*PI*(kx[m]*x[n]+ky[m]*y[n]+kz[m]*z[n]);

    float cArg = cos(expFhD);
    float sArg = sin(expFhD);

    rFhD[n] +=  rMu[m]*cArg – iMu[m]*sArg;
    iFhD[n] +=  iMu[m]*cArg + rMu[m]*sArg;
  }
}
```


Step 3. Using Registers to Reduce Global Memory Traffic

```
__global__ void cmpFHd(float* rPhi, iPhi, phiMag,
    kx, ky, kz, x, y, z, rMu, iMu, int M) {

  int n = blockIdx.x * FHD_THREADS_PER_BLOCK + threadIdx.x;

  float xn_r = x[n]; float yn_r = y[n]; float zn_r = z[n];
  float rFhDn_r = rFhD[n]; float iFhDn_r = iFhD[n];

  for (m = 0; m < M; m++) {
    float expFhD = 2*PI*(kx[m]*xn_r+ky[m]*yn_r+kz[m]*zn_r);

    float cArg = cos(expFhD);
    float sArg = sin(expFhD);

    rFhDn_r +=  rMu[m]*cArg – iMu[m]*sArg;
    iFhDn_r +=  iMu[m]*cArg + rMu[m]*sArg;
  }
  rFhD[n] = rFhD_r; iFhD[n] = iFhD_r;
}
```

Still too much stress on memory! Note that kx, ky and kz are read-only and based on m

Tiling of Scan Data



LS recon uses multiple grids
- Each grid operates on all pixels
- Each grid operates on a distinct subset of scan data
- Each thread in the same grid operates on a distinct pixel

Thread n operates on pixel n:

```
for (m = 0; m < M/32; m++) {
  exQ = 2*PI*(kx[m]*x[n] +
              ky[m]*y[n] +
              kz[m]*z[n])
  rQ[n] += phi[m]*cos(exQ)
  iQ[n] += phi[m]*sin(exQ)
}
```

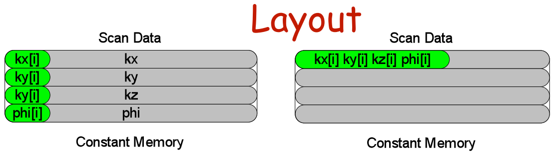
Tiling k-space data to fit into constant memory

```
__constant__ float      kx_c[CHUNK_SIZE],
        ky_c[CHUNK_SIZE], kz_c[CHUNK_SIZE];
…
__ void main() {

  int i;
  for (i = 0; i < M/CHUNK_SIZE; i++);
    cudaMemcpyToSymbol(kx_c,&kx[i*CHUNK_SIZE],4*CHUNK_SIZE);
    cudaMemcpyToSymbol(ky_c,&ky[i*CHUNK_SIZE],4*CHUNK_SIZE);
    cudaMemcpyToSymbol(kz_c,&ky[i*CHUNK_SIZE],4*CHUNK_SIZE);
    …
    cmpFHD<<<N/FHD_THREADS_PER_BLOCK, FHD_THREADS_PER_BLOCK>>>
      (rPhi, iPhi, phiMag, x, y, z, rMu, iMu, int M);
  }
  /* Need to call kernel one more time if M is not */
  /* perfect multiple of CHUNK SIZE */
}
```


Revised Kernel for Constant Memory

```
__global__ void cmpFHd(float*
   x, y, z, rMu, iMu, int M) {

  int n = blockIdx.x * FHD_THREADS_PER_BLOCK + threadIdx.x;

  float xn_r = x[n]; float yn_r = y[n]; float zn_r = z[n];
  float rFhDn_r = rFhD[n]; float iFhDn_r = iFhD[n];

  for (m = 0; m < M; m++) {
    float expFhD = 2*PI*(kx_c[m]*xn_r+ky_c[m]*yn_r
+kz_c[m]*zn_r);

    float cArg = cos(expFhD);
    float sArg = sin(expFhD);

    rFhDn_r +=  rMu[m]*cArg – iMu[m]*sArg;
    iFhDn_r +=  iMu[m]*cArg + rMu[m]*sArg;
  }
  rFhD[n] = rFhD_r; iFhD[n] = iFhD_r;
}
```


Sidebar: Cache-Conscious Data Layout



- kx, ky, kz, and phi components of same scan point have spatial and temporal locality
  - Prefetching
  - Caching
- Old layout does not fully leverage that locality
- New layout does fully leverage that locality

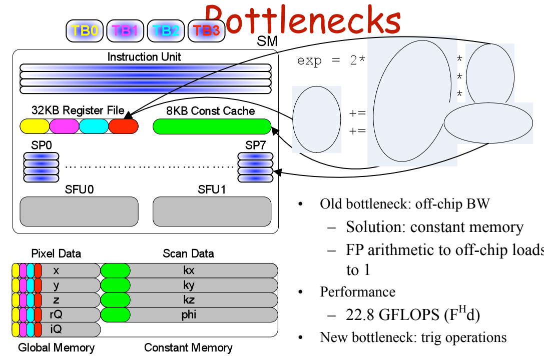
Adjusting K-space Data Layout

```
struct kdata {
   float x, float y, float z;
} k;

__constant__ struct kdata k_c[CHUNK_SIZE];
…

__ void main() {

 int i;

 for (i = 0; i < M/CHUNK_SIZE; i++);
   cudaMemcpyToSymbol(k_c, k, 12*CHUNK_SIZE);

   cmpFHD<<<FHD_THREADS_PER_BLOCK,N/FHD_THREADS_PER_BLOCK>>>
      ();

  }
```

```
__global__ void cmpFHd(float* rPhi, iPhi, phiMag,
    x, y, z, rMu, iMu, int M) {

  int n = blockIdx.x * FHD_THREADS_PER_BLOCK + threadIdx.x;

  float xn_r = x[n]; float yn_r = y[n]; float zn_r = z[n];
  float rFhDn_r = rFhD[n]; float iFhDn_r = iFhD[n];

  for (m = 0; m < M; m++) {
    float expFhD = 2*PI*(k[m].x*xn_r+k[m].y*yn_r+k[m].z*zn_r);

    float cArg = cos(expFhD);
    float sArg = sin(expFhD);

    rFhDn_r +=  rMu[m]*cArg – iMu[m]*sArg;
    iFhDn_r +=  iMu[m]*cArg + rMu[m]*sArg;
  }
  rFhD[n] = rFhD_r; iFhD[n] = iFhD_r;
}
```

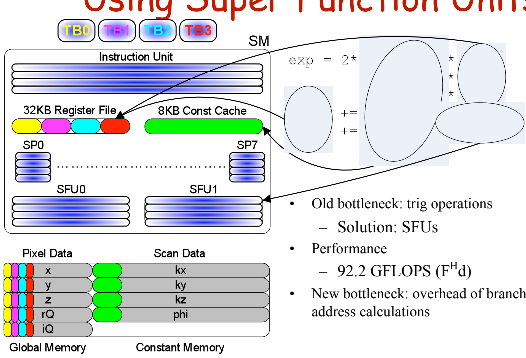
Figure 7.16 Adjusting the k-space data memory layout in the F^Hd kernel

Overcoming Mem BW Bottlenecks



- Old bottleneck: off-chip BW
  - Solution: constant memory
  - FP arithmetic to off-chip loads: 421 to 1
- Performance
  - 22.8 GFLOPS (F^Hd)
- New bottleneck: trig operations

Using Super Function Units



- Old bottleneck: trig operations
  - Solution: SFUs
- Performance
  - 92.2 GFLOPS (F^Hd)
- New bottleneck: overhead of branches and address calculations

Sidebar: Effects of Approximations

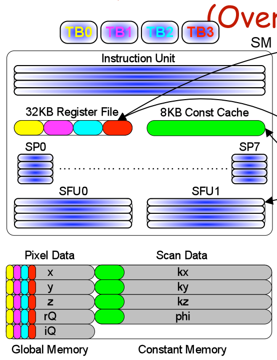
- Avoid temptation to measure only absolute error ($I_0 – I$)
  - Can be deceptively large or small
- Metrics
  - PSNR: Peak signal-to-noise ratio
  - SNR: Signal-to-noise ratio
- Avoid temptation to consider only the error in the computed value
  - Some apps are resistant to approximations; others are very sensitive

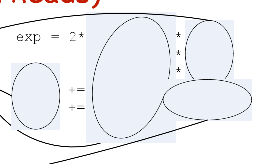
$$MSE = \frac{1}{mn}\sum_i\sum_j (I(i,j) - I_0(i,j))^2 \qquad A_s = \frac{1}{mn}\sum_i\sum_j I_0(i,j)^2$$

$$PSNR = 20\log_{10}\left(\frac{\max(I_0(i,j))}{\sqrt{MSE}}\right) \qquad SNR = 20\log_{10}\left(\frac{\sqrt{A_s}}{\sqrt{MSE}}\right)$$

A.N. Netravali and B.G. Haskell, Digital Pictures: Representation, Compression, and Standards (2nd Ed), Plenum Press, New York, NY (1995).

Step 4: Overcoming Bottlenecks (Overheads)





- Old bottleneck: Overhead of branches and address calculations
  - Solution: Loop unrolling and experimental tuning
- Performance
  - 145 GFLOPS (F^Hd)

What's Coming

- Before Spring Break
  - Complete MRI application study + more STRSM
  - MPM/GIMP application study from last year (read

    GPU Acceleration of the Generalized Interpolation Material Point Method Wei-Fan Chiang, Michael DeLisi, Todd Hummel, Tyler Prete, Kevin Tew, Mary Hall, Phil Wallstedt, and James Guilkey (http://saahpc.ncsa.illinois.edu/09/papers/Chiang_paper.pdf)
  - Review for Midterm (week after spring break)