L11: Dense Linear Algebra on GPUs, cont.

Administrative Issues

- Next assignment, triangular solve
  - Due 5PM, Friday, March 5
  - handin cs6963 lab 3 <probfile>"
- Project proposals (discussed today)
  - Due 5PM, Wednesday, March 17 (hard deadline)
  - handin cs6963 prop <pdffile>
  - A few minutes at end of class to help form groups

Outline

- Project
  - Suggested topics
- Triangular solve assignment
- More on dense linear algebra
  - LU

Project Ideas

- Suggestions from Nvidia (Nathan Bell via Pete Shirley, Steve Parker):
  - Sparse solvers
    - Tailor computation to matrix structure
    - Optimized SpMV for multiple vectors
    - Block Krylov methods
    - Block eigensolvers
  - Parallel preconditions
  - Parallel Graph Algorithms
    - (Maximal) Independent sets
    - Graph colorings
    - Partitioning of graphs that minimize a given metric (e.g. edge cut)
    - Optimal network flows
    - Graph clustering

More Project Ideas

- Suggestions from Steve Parker:
  - Non-uniform, unstructured
  - Double precision, cluster-based

Triangular Solve (STRSM)

```
for (j = 0; j < n; j++)
   for (k = 0; k < n; k++)
       if (B[j*n+k] != 0.0f) {
            for (i = k+1; i < n; i++)
               B[j*n+i] -= A[k * n + i] * B[j * n + k];
       }
```

Equivalent to:

```
cublasStrsm('l' /* left operator */, 'l' /* lower triangular */,
            'N' /* not transposed */, 'u' /* unit triangular */,
             N, N, alpha, d_A, N, d_B, N);
```

See: http://www.netlib.org/blas/strsm.f

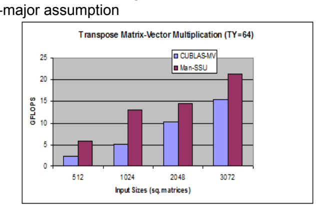
A Few Details

- C stores multi-dimensional arrays in row major order
- Fortran (and MATLAB) stores multi-dimensional arrays in column major order
  - **Confusion alert:** BLAS libraries were designed for FORTRAN codes, so column major order is implicit in CUBLAS!

SGEMV (single-precision

sgemv (matrix vector multiplication) and transposed sgemv
Recall transpose vs. column-major assumption

```
// This is transposed due to
// Fortran assumption
// Follows coalesced accesses
// Place b in shared memory
for (i = 0; i < n; i++)
   for (j = 0; j < n; j++)
      c[j] += a[j*n+i]*b[i];
```



Results from Malik Khan

```
// Non-transposed
// Store a in shared memory
// and access in a different order
for (i = 0; i < n; i++)
   for (j = 0; j < n; j++)
      c[i] += a[i*n+j]*b[j];
```

Non-transposed more than 2X slower for 8K matrix!

Reference: An Optimizing Compiler for GPGPU Programs with Input-Data Sharing, Yang et al., POPL 2010 Poster, Jan. 2010.

Dependences in STRSM

```
for (j = 0; j < n; j++)
  for (k = 0; k < n; k++)
      if (B[j*n+k] != 0.0f) {
           for (i = k+1; i < n; i++)
              B[j*n+i] -= A[k * n + i] * B[j * n + k];
      }
```

Which loop(s) "carry" dependences?
Which loop(s) is(are) safe to execute in parallel?

Assignment

- Details:
  - Integrated with simpleCUBLAS test in SDK
  - Reference sequential version provided

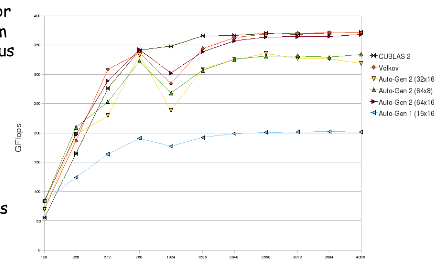
1. Rewrite in CUDA
2. Compare performance with CUBLAS 2.0 library

Performance Issues?

- + Abundant data reuse
- - Difficult edge cases
- - Different amounts of work for different <j,k> values
- - Complex mapping or load imbalance

Latest Research: CUDA-CHiLL

- Automatically generate CUDA for NVIDIA GPU from sequential code plus script
- Abstraction permits parallel threads & staging of data
- Heterogeneous code generation: Alternative scripts generate CUDA, OpenMP or sequential code tuned for memory hierarchy



Results provided by Malik Khan, Gabe Rudy, Chun Chen

Recent Results

Suppose your matrix is not evenly divided by the tile sizes (64 and 16)?

512x512 matrix sgemm:
 CUBLAS 258 Gflops
 Our compiler 284 Gflops

500x500 matrix sgemm:
 CUBLAS 167 Gflops
 Our compiler 223 Gflops

CUDA-CHiLL: Higher-Level Interface (in progress)

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      c[j][i] += a[k][i]*b[j][k];
```

```
init("mm.sp2", "MarkedLoop")

tile_control({"i","j"}, {TI,TJ},
             {l1_control="ii", l2_control="jj"},
             {"ii", "jj", "i", "j"})
tile_control({"k"}, {TK}, {l1_control="kk"},
             {"ii", "jj", "kk","i", "j","k"}, strided)
tile_control({"i"}, {TJ},
             {l1_control="ty",l1_tile="tx"},
             {"ii", "jj", "kk","tx","ty","j","k"})

--Assign loop levels to thread space and name the kernel
cudaize("mm_GPU",
        {a=N*N, b=N*N, c=N*N},--array sizes for data copying
        {block={"ii","jj"}, thread={"tx","ty"}})
--Copy the "c" array usage to registers
copy_to_registers("kk", "c", {"tx","ty"})
copy_to_shared("ty", "b")
--Unroll two innermost loop levels fully
unroll_to_depth(2)
```

Gabe Rudy Master's thesis

Breaking it Down: Tiling

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      c[j][i] += a[k][i]*b[j][k];
```

1. Tile I and J loops

```
for (ii=0; i<n/TI; i++)
  for (jj=0; j<n/TJ; j++)
    for (i = ii*TI; i < min(n,(ii+1)*TJ); i++)
      for (j = jj*TJ; j < min(n,(jj+1)*TJ); j++)
        for (k = 0; k < n; k++)
          c[j][i] += a[k][i]*b[j][k];
```

```
1. tile_control({"i","j"}, {TI,TJ},
             {l1_control="ii", l2_control="jj"},
             {"ii", "jj", "i", "j"})
tile_control({"k"}, {TK}, {l1_control="kk"},
             {"ii", "jj", "kk","i", "j","k"}, strided)
tile_control({"i"}, {TJ},
             {l1_control="ty",l1_tile="tx"},
             {"ii", "jj", "kk","tx","ty","j","k"})

--Assign loop levels to thread space and name the kernel
cudaize("mm_GPU",
        {a=N*N, b=N*N, c=N*N},--array sizes for data copying
        {block={"ii","jj"}, thread={"tx","ty"}})
-
```


Breaking it Down: Tiling

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      c[j][i] += a[k][i]*b[j][k];
```

2. Tile k loop (strided, move kk above i&j)

```
for (ii=0; i<n/TI; i++)
  for (jj=0; j<n/TJ; j++)
    for (kk = 0; kk < n; kk+=TK)
      for (i = ii*TI; i < min(n,(ii+1)*TI); i++)
        for (j = jj*TJ; j < min(n,(jj+1)*TJ); j++)
          for (k = kk; k < min(n,kk+TK); k++)
            c[j][i] += a[k][i]*b[j][k];
```

```
1. tile_control({"i","j"}, {TI,TJ},
             {l1_control="ii", l2_control="jj"},
             {"ii", "jj", "i", "j"})
2. tile_control({"k"}, {TK}, {l1_control="kk"},
             {"ii", "jj", "kk","i", "j","k"}, strided)
tile_control({"i"}, {TJ},
             {l1_control="ty",l1_tile="tx"},
             {"ii", "jj", "kk","tx","ty","j","k"})

--Assign loop levels to thread space and name the kernel
cudaize("mm_GPU",
        {a=N*N, b=N*N, c=N*N},--array sizes for data copying
        {block={"ii","jj"}, thread={"tx","ty"}})
-
```

Breaking it Down: Tiling

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      c[j][i] += a[k][i]*b[j][k];
```

```
3. Create tx and ty by further tiling of i
for (ii=0; i<n/TI; i++)
  for (jj=0; j<n/TJ; j++)
    for (kk = 0; kk < n; kk+=TK)
      for (tx = fn(i,TJ))
        for (ty = fn(i,TJ))
          for (j = jj*TJ; j < min(n,(jj+1)*TJ); j++)
            for (k = kk; k < min(n,(kk+1)*TK); k++)
              c[j][…] += a[k][…]*b[j][k];
```

```
1. tile_control({"i","j"}, {TI,TJ},
             {l1_control="ii", l2_control="jj"},
             {"ii", "jj", "i", "j"})
2. tile_control({"k"}, {TK}, {l1_control="kk"},
             {"ii", "jj", "kk","i", "j","k"}, strided)
3. tile_control({"i"}, {TJ},
             {l1_control="ty",l1_tile="tx"},
             {"ii", "jj", "kk","tx","ty","j","k"})

--Assign loop levels to thread space and name the kernel
cudaize("mm_GPU",
        {a=N*N, b=N*N, c=N*N},--array sizes for data copying
        {block={"ii","jj"}, thread={"tx","ty"}})
```


Breaking it Down: "cudaize"

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      c[j][i] += a[k][i]*b[j][k];
```

```
4. Remove loops representing bx, by, tx, ty
  for (kk = 0; kk < n; kk+=TK)
    for (j = jj*TJ; j < min(n,(jj+1)*TJ); j++)
      for (k = kk; k < min(n,(kk+1)*TK); k++)
        c[j][?] += a[k][?]*b[j][k];
```

```
1. tile_control({"i","j"}, {TI,TJ},
             {l1_control="ii", l2_control="jj"},
             {"ii", "jj", "i", "j"})
2. tile_control({"k"}, {TK}, {l1_control="kk"},
             {"ii", "jj", "kk","i", "j","k"}, strided)
3. tile_control({"i"}, {TJ},
             {l1_control="ty",l1_tile="tx"},
             {"ii", "jj", "kk","tx","ty","j","k"})

--Assign loop levels to thread space and name the kernel
4. cudaize("mm_GPU",
        {a=N*N, b=N*N, c=N*N},--array sizes for data copying
        {block={"ii","jj"}, thread={"tx","ty"}})
```


Final Result: Copy C to registers, B to shared memory and unroll

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      c[j][i] += a[k][i]*b[j][k];
```

```
--Copy the "c" array usage to registers
5. copy_to_registers("kk", "c", {"tx","ty"})
6. copy_to_shared("ty", "b")
--Unroll two innermost loop levels fully
7. unroll_to_depth(2)
```

B goes into shared memory

C goes into registers and is copied back at end

A is used from registers, but not reused

```
float P1[16];
__shared__ float P2[16][17];
bx = blockIdx.x, by = blockIdx.y;
tx = threadIdx.x, ty = threadIdx.y;
P1[0:15] = c[16*by:16*by+15][tx+64*bx+16*ty];
for (t6 = 0; t10 <= 1008; t6+=16) {
  P2[tx][4*ty:4*ty+3] = b[16*by+4*ty:16*by+4*ty+3]
[tx+t6];
  __syncthreads();
  P1[0:15] += a[t6][64*bx+16*ty+tx]*P2[0][0:15]
  P1[0:15] += a[t6+1][64*bx+16*ty+tx]*P2[1][0:15]
  ...
  P1[0:15] += a[t6+15][64*bx+16*ty+tx]*P2[15][0:15]
  __syncthreads();
}
c[16*by:16*by+15][tx+64*bx+16*ty] = P1[0:15];
```


LU Decomposition and Other Matrix Factorizations

- Volkov talks about matrix factorization algorithms
- What are these:
  - LU Decomposition example
  - Also, Cholesky and QR in paper

LU Decomposition (no pivoting)

```
DO K=1,N-1
 DO I=K+1,N
  A(I,K)=A(I,K)/A(K,K)
 DO I=K+1,N
  DO J=K+1,N
   A(I,J)=A(I,J)-A(I,K)*A(K,J)
```

- LAPACK version decomposes this into three parts: mini-LU, (S/D)TRSM, & (S/D) GEMM
- Row pivoting (used for numeric stability) reorders matrix and computation

What's Coming

- See Khan/Rudy poster on Friday!
- Before Spring Break
  - Two application case studies from newly published Kirk and Hwu
  - Sparse linear algebra
  - Review for Midterm