L5: Writing Correct Programs, cont.

Outline

- How to tell if your parallelization is correct?
- Race conditions and data dependences
- Tools that detect race conditions
- Abstractions for writing correct parallel code
  – Mapping these abstractions to CUDA
- Reading (if you can find it):
  – "Optimizing Compilers for Modern Architectures: A Dependence-Based Approach", Allen and Kennedy, 2002, Ch. 2.

Administrative

- Next assignment (a homework) given out on Monday

Is this CUDA code correct?

```
__host callkernel() {
  dim3 blocks(10);
  dim3 threads(100);
  float *d_array;
  …
  cudaMalloc(&d_array1,…);
  cudaMalloc(&d_array2,…);
  kernelcode<<<blocks,threads,
    0>>>(d_array1, d_array2,
    1000);
}
```

```
__global kernelcode(float * d_array1,
    d_array2, int N) {
  float result;

  for (int i=0; i<N; i++) {
      d_array1[threadIdx] +=
    d_array2[blockIdx][i];
  }

  for (int i=1; i<N; i++) {
      result += d_array1[threadIdx-1];
  }
}
```

Threads Access Shared Memory!

- Global memory and shared memory within an SM can be freely accessed by multiple threads
- Requires appropriate sequencing of memory accesses across threads to same location ***if at least one access is a write***
  - Recall using __syncthreads() within a thread block for synchronization
  - Not to be used for different blocks within a grid

Is this CUDA code correct?

```
__host callkernel() {
  dim3 blocks(10);
  dim3 threads(100);
  float *d_array;
  …
  cudaMalloc(&d_array1,…);
  cudaMalloc(&d_array2,…);
  kernelcode<<<blocks,threads,
    0>>>(d_array1, d_array2,
    1000);
}
```

```
__global kernelcode(float * d_array1,
    d_array2, int N) {

  for (int i=0; i<N; i++) {
        d_array1[threadIdx] +=
     d_array2[blockIdx][i];
  }
  __syncthreads();
 for (int i=1; i<N; i++) {
        result += d_array1[threadIdx-1];
  }
}
```


More Formally: Race Condition or Data Dependence

- A ***race condition*** exists when the result of an execution depends on the ***timing*** of two or more events.
- A ***data dependence*** is an ordering on a pair of memory operations that must be preserved to maintain correctness.

How about other Shared Memory Architectures?

- Race detection software (*e.g.,* Intel ThreadChecker)
  - Trace memory accesses for each thread
  - Compare addresses accessed by each thread
  - Race condition exists if, between synchronization points,
    - multiple threads access the same memory location
    - and, at least one access is a write

What can we do to debug parallelization in CUDA?

- -deviceemu code (to be emulated on host)
  - Support for pthread debugging?
- Can compare GPU output to CPU output, or compare GPU output to device emulation output
  - Race condition may still be present
- Or can (try to) prevent introduction of race conditions (remainder of lecture)

Data Dependence

- **Definition:**
  Two memory accesses are involved in a data dependence if they may refer to the same memory location and one of the references is a write.

  A data dependence can either be between two distinct program statements or two different dynamic executions of the same program statement.

- Two important uses of data dependence information (among others):
  Parallelization: no data dependence between two computations → parallel execution safe
  Locality optimization: absence of data dependences & presence of reuse → reorder memory accesses for better data locality (next week)

Data Dependence of Scalar Variables

True (flow) dependence

```
a    =
     = a
```

Anti-dependence

```
     = a
a    =
```

Output dependence

```
a    =
a    =
```

Input dependence (for locality)

```
     = a
     = a
```

Definition: Data dependence exists from a reference instance i to i' iff
either i or i' is a write operation
i and i' refer to the same variable
i executes before i'

Some Definitions (from Allen & Kennedy)

- **Definition 2.5:**
  - Two computations are equivalent if, on the same inputs,
    - they produce identical outputs
    - the outputs are executed in the same order
- **Definition 2.6:**
  - A reordering transformation
    - changes the order of statement execution
    - without adding or deleting any statement executions.
- **Definition 2.7:**
  - A reordering transformation preserves a dependence if
    - it preserves the relative execution order of the dependences' source and sink.

Fundamental Theorem of Dependence

- **Theorem 2.2:**
  - Any reordering transformation that preserves every dependence in a program preserves the meaning of that program.
- Now we will discuss abstractions and algorithms to determine whether reordering transformations preserve dependences...

Parallelization as a Reordering Transformation in CUDA

```
__host callkernel() {
  dim3 blocks(bx,by);
  dim3 threads(tx,ty,tz);
  ...
    kernelcode<<<blocks,threads0>>>(<args>);
}
__global kernelcode(<args>) {
    /* code refers to threadIdx.x, threadIdx.y, threadIdx.z, blockIdx.x, blockIdx.y */
}
```

```
__host callkernel() {

for (int bIdx_x=0; bIdx_x<bx; bIdx_x++) {
for (int bIdx_y=0; bIdx_y<by; bIdx_y++) {
for (int tIdx_x=0; tIdx_x<tx; tIdx_x++) {
for (int tIdx_y=0; tIdx_y<ty; tIdx_y++) {
for (int tIdx_z=0; tIdx_z<tz; tIdx_z++) {

/* code refers to tIdx_x, tIdx_y, tIdx_z,
    bIdx_x, bIdx_y */
}}}}}
```

EQUIVALENT?

In Today's Lecture: Parallelizable Loops

Forall (or CUDA kernels or Doall) loops:
Loops whose iterations can execute in parallel (a particular reordering transformation)

Example

```
forall (i=1; i<=n; i++)
  A[i] = B[i] + C[i];
```

Meaning?

Each iteration can execute independently of others
Free to schedule iterations in any order

Why are parallelizable loops an important concept for data-parallel programming models?

CUDA Equivalent to "Forall"

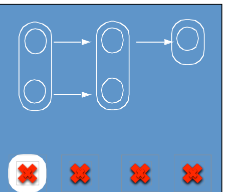
```
__host callkernel() {

forall (int bIdx_x=0; bIdx_x<bx; bIdx_x++) {
forall (int bIdx_y=0; bIdx_y<by; bIdx_y++) {
forall (int tIdx_x=0; tIdx_x<tx; tIdx_x++) {
forall (int tIdx_y=0; tIdx_y<ty; tIdx_y++) {
forall (int tIdx_z=0; tIdx_z<tz; tIdx_z++) {

/* code refers to tIdx_x, tIdx_y, tIdx_z,
    bIdx_x, bIdx_y */
}}}}}
```

Data Dependence for Arrays

```
for (i=2; i<5; i++)
   A[i] = A[i-2]+1;
```

Loop-Carried dependence

```
for (i=1; i<=3; i++)
   A[i] = A[i]+1;
```



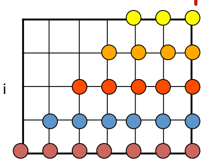
Loop-Independent dependence

- Recognizing parallel loops (intuitively)
  - Find data dependences in loop
  - No dependences crossing iteration boundary ➔ parallelization of loop's iterations is safe

1. Characterize Iteration Space

```
for (i=1;i<=5; i++)
  for (j=i;j<=7; j++)
   ...
```

i



j

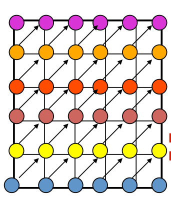
$1 \le i \le 5$
$i \le j \le 7$

- **Iteration instance:** represented as coordinates in iteration space
  - *n*-dimensional discrete cartesian space for *n* deep loop nests
- **Lexicographic order:** Sequential execution order of iterations
  [1,1], [1,2], ..., [1,6],[1,7],
  [2,2], [2,3], ..., [2,6], ...
- Iteration I (a vector) is lexicographically less than I', I<I' , iff there exists c (i_1, …, i_{c-1}) = (i'_1, …, i'_{c-1}) and $i_c < i'_c$.

2. Compare Memory Accesses across Dynamic Instances in Iteration Space

```
N = 6;
for (i=1; i<N; i++)
  for (j=1; j<N; j++)
     A[i+1,j+1] = A[i,j] * 2.0;
```

i



I'=[2,2],
Read A[2,2]

I=[1,1],
Write A[2,2]

j

How to describe relationship between two dynamic instances?
e.g., **I=[1,1]** and **I'=[2,2]**

Distance Vectors

```
N = 6;
for (i=1; i<N; i++)
  for (j=1; j<N; j++)
     A[i+1,j+1] = A[i,j] * 2.0;
```

- Distance vector = **[1,1]**
- A loop has a distance vector D if there exists data dependence from iteration vector **I** to a later vector **I'**, and **D = I' - I**.
- Since I' > I, **D >= 0**.
  (D is lexicographically greater than or equal to 0).

Distance and Direction Vectors

- Distance vectors: (infinitely large set)

$$\left(\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\2\end{bmatrix}\cdots\begin{bmatrix}0\\n\end{bmatrix}\right)\left(\begin{bmatrix}1\\-n\end{bmatrix}\cdots\begin{bmatrix}1\\0\end{bmatrix}\cdots\begin{bmatrix}1\\n\end{bmatrix}\right)\cdots\left(\begin{bmatrix}n\\-n\end{bmatrix}\cdots\begin{bmatrix}n\\0\end{bmatrix}\cdots\begin{bmatrix}n\\n\end{bmatrix}\right)$$

- Direction vectors: (realizable if 0 or lexicographically positive)
  ([=,=],[=,<],[<,>], [<,=], [<.<])
- Common notation:
  0 =
  \+ <
  \- >
  +/- *

Parallelization Test: 1-Dimensional Loop

- **Examples:**

```
for (j=1; j<N; j++)
    A[j] = A[j] + 1;
```

```
for (j=1; j<N; j++)
    B[j] = B[j-1] + 1;
```

- **Dependence (Distance and Direction) Vectors?**
- **Test for parallelization:**
  - A loop is parallelizable if for all data dependences D e **D**, D = 0

n-Dimensional Loop Nests

```
for (i=1; i<=N; i++)
   for (j=1; j<=N; j++)
    A[i][j] = A[i][j-1]+1;

for (i=1; i<=N; i++)
   for (j=1; j<=N; j++)
    A[i][j] = A[i-1][j+1]+1;
```

- **Distance and direction vectors?**
- **Definition:**
  $D = (d_1, \ldots d_n)$ is loop-carried at level i if d_i is the first nonzero element.

A Few Words about n-Dimensional Arrays in C

- Largely conceptual, due to difficulty in expressing this in C for dynamically allocated data structures
- Imagine the following macros,

```
#define 2dAccess(i,j,dim_i) \
      i+j*dim_i
#define 3dAccess(i,j,k,dim_i,dim_j) \
      i+j*dim_i + k*dim_i*dim_j
```

Test for Parallelization

The i th loop of an n-dimensional loop is parallelizable if there does not exist any level i data dependences.

The i th loop is parallelizable if for all dependences

$D = (d_1, \dots d_n)$,

either

$(d_1, \dots d_{i-1}) > 0$

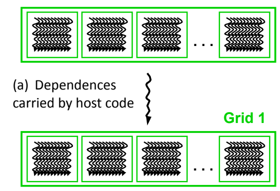
or

$(d_1, \dots d_i) = 0$

Safe Parallelization of CUDA Code

- Dependences must be carried by
  - (a) Loops that execute on the host
  - OR, loops that execute within a kernel function

*May be able to use synchronization for dependences across threads, but not across blocks (subtle distinction)




```
__global compute() {
  for (j=1; j<n; j++)
     A[threadIdx][j] =
         A[threadIdx][j-1];
}
```

(b) Dependence carried within thread code

Parallelization Algorithm

- **For each pair of dynamic accesses to the same array within a loop nest:**
  - determine if there exists a dependence between that pair
- **Key points:**
  - n^2 tests for n accesses in loop!
  - a single access is compared with itself
  - includes accesses in all loops within a nest

Dependence Testing

- **Question so far:**
  - What is the distance/direction (in the iteration space) between two dynamic accesses to the same memory location?
- **Simpler question:**
  - Can two data accesses ever refer to the same memory location?

```
for (i=11;i<=20;i++)
  A[i] = A[i-1]+ 3;
```

```
for (i=11;i<=20;i++)
  A[i] = A[i-10]+ 1;
```

Restrict to an Affine Domain

```
for (i=1; i<N; i++)
   for (j=1; j<N j++) {
     A[i+2*j+3, 4*i+2*j, 3*i] = ...;
     ... = A[1, 2*i+1, j];
}
```

- Only use loop bounds and array indices which are integer linear functions of loop variables.
- **Non-affine example:**

```
for (i=1; i<N; i++)
    for (j=1; j<N j++) {
      A[i*j] = A[i*(j-1)];
         A[B[i]] = A[B[j]];
  }
```


Equivalence to Integer Programming

- Need to determine if F(i) = G(i'), where i and i' are iteration vectors, with constraints i,i' >= L, U>= i, i'
- **Example:**
  for (i=1; i<=100; i++)
  A[i] = A[i-1];
- **Inequalities:**
  1 <= iw <= 100, ir = iw - 1, ir <= 100
  integer vector I, AI <= b
- **Integer Programing is NP-complete**
  - O(size of the coefficients)
  - $O(n^n)$

Example

0 ≤ iw ≤ 100, ir = iw-1, ir ≤ 100

Solve

$$\begin{bmatrix} -1 & 0 \\ 1 & 0 \\ -1 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} iw \\ ir \end{bmatrix} \leq \begin{bmatrix} 1 \\ 100 \\ -1 \\ 1 \\ 100 \end{bmatrix}$$

Solution exist?

Replace instances of iw with ir
Eliminate ir
Simplifies to 0 < 99 (yes, solution exists!)

Introducing Omega Calculator

- A software tool used in compilers
- Description:
  - Solves "Presburger formulas", logical formulas built from affine constraints over integer variables, logical connectives and quantifiers.
  - Can formulate these dependence problems, and derive existence of dependences and distances.
- Relevant to locality optimizations as well, next week's focus
- Can download from:
  - http://www.cs.utah.edu/~chunchen/omega/
- Also available from CADE Linux machines in:
  - ~cs6963/bin/oc

Using Omega Calculator

- **Example:**
  for (i=2; i<=100; i++)
  A[i] = A[i-1];
- **Define relation iw = i, and iw = ir-1 in the iteration space** 2 <= i <= 100.

```
R := {[iw] -> [ir] :
 2 <= iw, ir <= 100        /* iteration space */
 && iw < ir                /* looking for loop-carried dep */
 && iw = ir-1};            /* can they be the same? */
```

R := {[iw] -> [ir] : 2 <= iw, ir <= 100 && iw < ir && iw = ir - 1};

Result: {[iw] -> [iw+1] : 2 <= iw <= 99}

Using Omega Calculator, cont.

- **Example:**
  for (i=20; i<=29; i++)
  A[i] = A[i-10];
- **Define relation iw = i, and iw = ir-10 in the iteration space** 20 <= i <= 29.

```
R := {[iw] -> [ir] :
 20 <= iw, ir <= 29        /* iteration space */
 && iw < ir                /* looking for loop-carried dep */
 && iw = ir-10};           /* can they be the same? */
```

R := {[iw] -> [ir] : 20 <= iw, ir <= 29 && iw < ir && iw = ir - 10};

Result: {[iw] -> [ir] : FALSE }

2-D Omega Example

- **Example:**

```
for (i=0; i<n; i++)
  for (j=i; j<n; j++)
    a[i][j+1] = a[n][j];
```

- **Formula (more complex):**

```
R := {[iw,jw] -> [ir,jr] : exists(n :    /* unbound variable */
  1 <= iw <= n && iw <= jw <= n          /* iteration space */
  && 1 <= ir <= jr <= n                  /* loop-carried dependence? */
  && jw+1 = jr && ir = n)};              /* access expression */
```

Result: {[iw,jw] -> [ir,jr] : FALSE }

Calculating Distance Vectors

- Example from before:

```
for (i=1; i<=100; i++)
  for (j=1; j<=100; j++)
    A[i][j] = A[i-1][j+1]+1;
```

- Omega formula:

```
R := {[iw, jw] -> [di, dj] : exists (ir, jr :   /* read iters unbound */
  1 <= iw, ir <= 100 && 1 <= jw, jr <= 100      /* iteration space */
  && iw = ir -1 && jw = jr + 1 &&               /* access exprs */
  && di = ir - iw && dj = jr - jw)};            /* distances */
```

Result: {[iw,jw] -> [1,-1] : 1 <= iw <= 99 && 2 <= jw <= 100}

Aside: What about dependences for other data structures?

- Pointer-based
  - Pointer alias analysis
  - Shape analysis
- Objects
  - Escape analysis
- In practice
  - Lots of #pragma and special flags for programmer to assert no dependence

Homework Assigned Monday

- Example questions
  - Given some sequential code, do the following:
    - Show distance vectors and decide which loop can be parallelized
    - Show Omega dependence relations
    - Show correct CUDA code
  - Memory hierarchy optimization
    - Simple tiling example
    - Identify safety of code transformations
    - Given description of goal, show CUDA code to manage memory hierarchy

Summary of Lecture

- Data dependence can be used to determine the safety of reordering transformations such as parallelization
  - preserving dependences = preserving "meaning"
- Iteration, distance and direction vectors are abstractions for understanding whether reordering transformations preserve dependences.
  - Parallelization of CUDA kernel programs can be viewed as a reordering transformation of a sequential implementation
- Dependence testing on array accesses in loops has been shown to be equivalent to integer programming.
  - Omega calculator demonstrated

What's Ahead

- Next week
  - Homework assignment on Monday
  - Managing the memory hierarchy
  - Initial discussion of projects
- February 16: President's Day holiday
- February 18:
  - Jim Guilkey to present MPM for projects
- February 20 (Friday):
  - Make up class (we'll discuss control flow)