

Definition-Contexts Model

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$ast ::= var \mid \mathbf{APP}(ast, ast, \dots) \mid val$
 $var ::= \mathbf{VAR}(name)$
 $val ::= \mathbf{FUN}(var, ast) \mid atom \mid \mathbf{LIST}(val, \dots) \mid stx$
 $stx ::= \mathbf{STX}(atom, ctx) \mid \mathbf{STX}(\mathbf{LIST}(stx, \dots), ctx)$
 $id ::= \mathbf{STX}(sym, ctx)$
 $ctx ::=$ a mapping from ph to \overline{scp}
 $\overline{scp} ::= \{scp, \dots\}$
 $atom ::= sym \mid prim \mid \dots$
 $sym ::= 'name$
 $prim ::= \mathbf{stx-e} \mid \mathbf{mk-stx} \mid \dots$
 $\xi ::=$ a mapping from $name$ to $transform$
 $transform ::= \text{lambda} \mid \text{let-syntax} \mid \text{quote} \mid \text{syntax} \mid \mathbf{VAR}(id) \mid val$
 $\Sigma ::=$ binding store, $name \rightarrow (\overline{scp} \rightarrow name)$
 $name ::=$ a token such as `x`, `egg`, or `lambda`
 $scp ::=$ a token that represents a scope
 $ph ::= integer$
 $\widehat{scp} ::= scp \mid \bullet$
 $\widehat{\Sigma} ::= \langle \Sigma, \overline{scp}, \overline{scp} \rangle$

$$\text{eval} : ph \text{ ast } \widehat{scp} \xi \widehat{\Sigma} \rightarrow \langle val, \widehat{\Sigma} \rangle$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{new-defs}), scp_i, \xi, \langle \Sigma, \overline{scp}_p, \overline{scp}_u \rangle]\!] &= \langle \mathbf{DEFS}(scp_{defs}, addr_{env}), \widehat{\Sigma}_3 \rangle \\ \text{subject to } \text{alloc-scope}[\![\Sigma]\!] &= \langle scp_{defs}, \Sigma_2 \rangle, \text{alloc-def-env}[\![\Sigma_2]\!] = \langle addr_{env}, \Sigma_3 \rangle, \\ &\langle \Sigma_2 + \{addr_{env} \rightarrow \xi\}, \{scp_{defs}\} \cup \overline{scp}_p, \overline{scp}_u \rangle = \widehat{\Sigma}_3 \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{def-bind}, ast_{defs}, ast_{id}), scp_i, \xi, \widehat{\Sigma}]\!] &= \langle 0, \langle \Sigma_6, \overline{scp}_{p3}, \overline{scp}_{u3} \rangle \rangle \\ \text{subject to } \text{eval}_{ph}[\![ast_{defs}, scp_i, \xi, \widehat{\Sigma}]\!] &= \langle \mathbf{DEFS}(scp_{defs}, addr_{env}), \widehat{\Sigma}_2 \rangle, \\ \text{eval}_{ph}[\![ast_{id}, scp_i, \xi, \widehat{\Sigma}_2]\!] &= \langle id_{arg}, \widehat{\Sigma}_3 \rangle, \widehat{\Sigma}_3 = \langle \Sigma_3, \overline{scp}_{p3}, \overline{scp}_{u3} \rangle, \\ \text{add}_{ph}[\![\text{prune}_{ph}[\![\text{flip}_{ph}[\![id_{arg}, scp_i], \overline{scp}_{u3}]\!], scp_{defs}]\!] &= id_{defs}, \\ \text{alloc-name}[\![\Sigma_3]\!] &= \langle name_{new}, \Sigma_4 \rangle, \Sigma_4 + \{id_{defs} \rightarrow name_{new}\} = \Sigma_5, \\ \Sigma_5(addr_{env}) &= \xi_{defs}, \Sigma_5 + \{addr_{env} \rightarrow \xi_{defs} + \{name_{new} \rightarrow \text{VAR}(id_{defs})\}\} = \Sigma_6 \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{def-bind}, ast_{defs}, ast_{id}, ast_{stx}), scp_i, \xi, \widehat{\Sigma}]\!] &= \langle 0, \langle \Sigma_9, \overline{scp}_{p4}, \overline{scp}_{u4} \rangle \rangle \\ \text{subject to } \text{eval}_{ph}[\![ast_{defs}, scp_i, \xi, \widehat{\Sigma}]\!] &= \langle \mathbf{DEFS}(scp_{defs}, addr_{env}), \widehat{\Sigma}_2 \rangle, \\ \text{eval}_{ph}[\![ast_{id}, scp_i, \xi, \widehat{\Sigma}_2]\!] &= \langle id_{arg}, \widehat{\Sigma}_3 \rangle, \text{eval}_{ph}[\![ast_{stx}, scp_i, \xi, \widehat{\Sigma}_3]\!] = \langle stx_{arg}, \widehat{\Sigma}_4 \rangle, \\ \widehat{\Sigma}_4 &= \langle \Sigma_4, \overline{scp}_{p4}, \overline{scp}_{u4} \rangle, \\ \text{expand}_{ph+1}[\![\text{add}_{ph}[\![\text{flip}_{ph}[\![stx_{arg}, scp_i], \overline{scp}_{u4}]\!], scp_{defs}]\!] &= \langle \xi_{primitives}, \langle \Sigma_4, \emptyset, \emptyset \rangle \rangle = \langle stx_{exp}, \langle \Sigma_5, _, _ \rangle \rangle, \\ \text{eval}_{ph}[\![\text{parse}_{ph+1}[\![stx_{exp}, \Sigma_5], \bullet, \xi, \langle \Sigma_5, \overline{scp}_{p4}, \emptyset \rangle]\!] &= \langle val_{exp}, \langle \Sigma_6, _, _ \rangle \rangle, \\ \Sigma_6(addr_{env}) &= \xi_{defs}, \text{add}_{ph}[\![\text{prune}_{ph}[\![\text{flip}_{ph}[\![id_{arg}, scp_i], \overline{scp}_{u4}]\!], scp_{defs}]\!] = id_{defs}, \\ \text{alloc-name}[\![\Sigma_6]\!] &= \langle name_{new}, \Sigma_7 \rangle, \Sigma_7 + \{id_{defs} \rightarrow name_{new}\} = \Sigma_8, \\ \Sigma_8 + \{addr_{env} \rightarrow \xi_{defs} + \{name_{new} \rightarrow val_{exp}\}\} &= \Sigma_9 \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{lexpand}, ast_{expr}, ast_{stops}, ast_{defs}), scp_i, \xi, \widehat{\Sigma}]\!] &= \langle \text{flip}_{ph}[\![stx_{exp}, scp_i]\!], \widehat{\Sigma}_5 \rangle \\ \text{subject to } \text{eval}_{ph}[\![ast_{expr}, scp_i, \xi, \widehat{\Sigma}]\!] &= \langle stx, \widehat{\Sigma}_2 \rangle, \\ \text{eval}_{ph}[\![ast_{stops}, scp_i, \xi, \widehat{\Sigma}_2]\!] &= \langle \mathbf{LIST}(id_{stop}, \dots), \widehat{\Sigma}_3 \rangle, \\ \text{eval}_{ph}[\![ast_{defs}, scp_i, \xi, \widehat{\Sigma}_3]\!] &= \langle \mathbf{DEFS}(scp_{defs}, addr_{env}), \widehat{\Sigma}_4 \rangle, \widehat{\Sigma}_4 = \langle \Sigma_4, _, _ \rangle, \\ \Sigma_4(addr_{env}) &= \xi_{defs}, \{var \rightarrow \text{unstop}[\![\xi_{defs}(var)]\!] \mid var \in \text{dom}(\xi_{defs})\} = \xi_{unstops}, \\ \xi_{unstops} + \{\text{resolve}_{ph}[\![id_{stop}, \Sigma_4]\!] \rightarrow \text{STOP}(\xi_{defs}(\text{resolve}_{ph}[\![id_{stop}, \Sigma_4]\!]))\} &\dots = \xi_{stops}, \\ \text{expand}_{ph}[\![\text{add}_{ph}[\![\text{flip}_{ph}[\![stx, scp_i]\!], scp_{defs}]\!] &= \langle stx_{exp}, \widehat{\Sigma}_5 \rangle \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{lvalue}, ast_{id}), scp_i, \xi, \widehat{\Sigma}]\!] &= \langle \xi(\text{resolve}_{ph}[\![id_{result}, \Sigma_2]\!]), \widehat{\Sigma}_2 \rangle \\ \text{subject to } \text{eval}_{ph}[\![ast_{id}, scp_i, \xi, \widehat{\Sigma}]\!] &= \langle id_{result}, \widehat{\Sigma}_2 \rangle, \widehat{\Sigma}_2 = \langle \Sigma_2, _, _ \rangle \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{lexpand}, ast_{expr}, ast_{stops}), scp_i, \xi, \widehat{\Sigma}]\!] &= \langle \text{flip}_{ph}[\![stx_{exp}, scp_i]\!], \widehat{\Sigma}_4 \rangle \\ \text{subject to } \text{eval}_{ph}[\![ast_{expr}, scp_i, \xi, \widehat{\Sigma}]\!] &= \langle stx, \widehat{\Sigma}_2 \rangle, \\ \text{eval}_{ph}[\![ast_{stops}, scp_i, \xi, \widehat{\Sigma}_2]\!] &= \langle \mathbf{LIST}(id_{stop}, \dots), \widehat{\Sigma}_3 \rangle, \\ \{var \rightarrow \text{unstop}[\![\xi(var)]\!] \mid var \in \text{dom}(\xi)\} &= \xi_{unstops}, \widehat{\Sigma}_3 = \langle \Sigma_3, _, _ \rangle, \\ \xi_{unstops} + \{\text{resolve}_{ph}[\![id_{stop}, \Sigma_3]\!] \rightarrow \text{STOP}(\xi(\text{resolve}_{ph}[\![id_{stop}, \Sigma_3]\!]))\} &\dots = \xi_{stops}, \\ \text{expand}_{ph}[\![\text{flip}_{ph}[\![stx, scp_i]\!], \xi_{stops}, \widehat{\Sigma}_3]\!] &= \langle stx_{exp}, \widehat{\Sigma}_4 \rangle \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{lbinder}, ast_{id}), scp_i, \xi, \widehat{\Sigma}]\!] &= \langle \text{prune}_{ph}[\![id_{result}, \overline{scp}_{u2}]\!], \widehat{\Sigma}_2 \rangle \\ \text{subject to } \text{eval}_{ph}[\![ast_{id}, scp_i, \xi, \widehat{\Sigma}]\!] &= \langle id_{result}, \widehat{\Sigma}_2 \rangle, \widehat{\Sigma}_2 = \langle _, _, \overline{scp}_{u2} \rangle \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(\mathbf{FUN}(var, ast_{body}), ast_{arg}), \widehat{scp}, \xi, \widehat{\Sigma}]\!] &= \text{eval}_{ph}[\![ast_{body}[var \leftarrow val_{arg}], \widehat{scp}, \xi, \widehat{\Sigma}_2]\!] \\ \text{subject to } \text{eval}_{ph}[\![ast_{arg}, \widehat{scp}, \xi, \widehat{\Sigma}]\!] &= \langle val_{arg}, \widehat{\Sigma}_2 \rangle \end{aligned}$$

$$\begin{aligned} \text{eval}_{ph}[\![\mathbf{APP}(prim, ast_{arg}, \dots), \widehat{scp}, \xi, \widehat{\Sigma}]\!] &= \langle \delta(prim, val_{arg}, \dots), \widehat{\Sigma}_2 \rangle \\ \text{subject to } \text{eval}^*[\![ph, (), (ast_{arg} \dots), \widehat{scp}, \xi, \widehat{\Sigma}]\!] &= \langle (val_{arg} \dots), \widehat{\Sigma}_2 \rangle \end{aligned}$$

$$\text{eval}_{ph}[\![val, \widehat{scp}, \xi, \widehat{\Sigma}]\!] \quad 2 \quad = \langle val, \widehat{\Sigma} \rangle$$

unstop : transform \rightarrow transform

$\text{unstop}[\text{STOP}(\text{transform})] = \text{transform}$

$\text{unstop}[\text{transform}] = \text{transform}$

$\delta(\mathbf{stx-e}, \mathbf{STX}(\text{val}, \text{ctx})) = \text{val}$

$\delta(\mathbf{mk-stx}, \text{atom}, \mathbf{STX}(\text{val}, \text{ctx})) = \mathbf{STX}(\text{atom}, \text{ctx})$

$\delta(\mathbf{mk-stx}, \mathbf{LIST}(\text{stx}, \dots), \mathbf{STX}(\text{val}, \text{ctx})) = \mathbf{STX}(\mathbf{LIST}(\text{stx}, \dots), \text{ctx})$

parse : $ph \text{ stx } \Sigma \rightarrow \text{ast}$

$\text{parse}_{ph}[\mathbf{STX}(\mathbf{LIST}(\text{id}_{\text{lambda}}, \text{id}_{\text{arg}}, \text{stx}_{\text{body}}), \text{ctx}), \Sigma] = \mathbf{FUN}(\mathbf{VAR}(\text{resolve}_{ph}[\text{id}_{\text{arg}}, \Sigma]), \text{parse}_{ph}[\text{stx}_{\text{body}}, \Sigma])$

subject to $\text{resolve}_{ph}[\text{id}_{\text{lambda}}, \Sigma] = \text{lambda}$

$\text{parse}_{ph}[\mathbf{STX}(\mathbf{LIST}(\text{id}_{\text{quote}}, \text{stx}), \text{ctx}), \Sigma] = \text{strip}[\text{stx}]$

subject to $\text{resolve}_{ph}[\text{id}_{\text{quote}}, \Sigma] = \text{quote}$

$\text{parse}_{ph}[\mathbf{STX}(\mathbf{LIST}(\text{id}_{\text{syntax}}, \text{stx}), \text{ctx}), \Sigma] = \text{stx}$

subject to $\text{resolve}_{ph}[\text{id}_{\text{syntax}}, \Sigma] = \text{syntax}$

$\text{parse}_{ph}[\mathbf{STX}(\mathbf{LIST}(\text{stx}_{\text{rator}}, \text{stx}_{\text{rand}}, \dots), \text{ctx}), \Sigma] = \mathbf{APP}(\text{parse}_{ph}[\text{stx}_{\text{rator}}, \Sigma], \text{parse}_{ph}[\text{stx}_{\text{rand}}, \Sigma], \dots)$

$\text{parse}_{ph}[\text{id}, \Sigma] = \mathbf{VAR}(\text{resolve}_{ph}[\text{id}, \Sigma])$

resolve : $ph \text{ id } \Sigma \rightarrow \text{name}$

$\text{resolve}_{ph}[\mathbf{STX}(\text{'name'}, \text{ctx}), \Sigma] = \text{name}_{\text{biggest}}$

subject to $\Sigma(\text{name}) = \{\overline{scp}_{\text{bind}} \leftarrow \text{name}_{\text{bind}}, \dots\}$,

$\text{biggest-subset}[\text{ctx}(\text{ph}), \{\overline{scp}_{\text{bind}}, \dots\}] = \overline{scp}_{\text{biggest}}$,

$\{\overline{scp}_{\text{bind}} \leftarrow \text{name}_{\text{bind}}, \dots\}(\overline{scp}_{\text{biggest}}) = \text{name}_{\text{biggest}}$

$\text{resolve}_{ph}[\mathbf{STX}(\text{'name'}, \text{ctx}), \Sigma] = \text{name}$

biggest-subset : $\overline{scp} \{ \overline{scp}, \dots \} \rightarrow \overline{scp}$

$\text{biggest-subset}[\overline{scp}_{\text{ref}}, \{\overline{scp}_{\text{bind}}, \dots\}] = \overline{scp}_{\text{biggest}}$

subject to $\overline{scp}_{\text{biggest}} \subseteq \overline{scp}_{\text{ref}}, \overline{scp}_{\text{biggest}} \in \{\overline{scp}_{\text{bind}}, \dots\}$,

$\overline{scp}_{\text{bind}} \subseteq \overline{scp}_{\text{ref}} \Rightarrow \overline{scp}_{\text{bind}} \subseteq \overline{scp}_{\text{biggest}}$

strip : $\text{stx} \rightarrow \text{val}$

$\text{strip}[\mathbf{STX}(\text{atom}, \text{ctx})] = \text{atom}$

$\text{strip}[\mathbf{STX}(\mathbf{LIST}(\text{stx}, \dots), \text{ctx})] = \mathbf{LIST}(\text{strip}[\text{stx}], \dots)$

$$\begin{aligned}
& \text{expand} : ph \ stx \ \xi \ \widehat{\Sigma} \rightarrow \langle stx, \widehat{\Sigma} \rangle \\
& \text{expand}_{ph}[\text{STX}(\text{LIST}(id_{stop}, stx, \dots), ctx), \xi, \widehat{\Sigma}] = \langle \text{STX}(\text{LIST}(id_{stop}, stx, \dots), ctx), \widehat{\Sigma} \rangle \\
& \text{subject to } \widehat{\Sigma} = \langle \Sigma, _ , _ \rangle, \xi(\text{resolve}_{ph}[id_{stop}, \Sigma]) = \text{STOP}(_) \\
& \text{expand}_{ph}[\text{STX}(\text{LIST}(id_{lam}, id_{arg}, stx_{body}), ctx), \xi, \langle \Sigma, \overline{scp}_p, \overline{scp}_u \rangle] \\
& = \langle \text{STX}(\text{LIST}(id_{lam}, id_{new}, stx_{body2}), ctx), \langle \Sigma_4, \overline{scp}_p, \overline{scp}_u \rangle \rangle \\
& \text{subject to } \text{resolve}_{ph}[id_{lam}, \Sigma] = \text{lambda}, \text{alloc-name}[\Sigma] = \langle name_{new}, \Sigma_l \rangle, \\
& \quad \text{alloc-scope}[\Sigma_l] = \langle scp_{new}, \Sigma_2 \rangle, \text{add}_{ph}[id_{arg}, scp_{new}] = id_{new}, \\
& \quad \Sigma_2 + \{id_{new} \rightarrow name_{new}\} = \Sigma_3, \xi + \{name_{new} \rightarrow \text{VAR}(id_{new})\} = \xi_{new}, \\
& \quad \text{expand}_{ph}[\text{add}_{ph}[stx_{body}, scp_{new}], \xi_{new}, \langle \Sigma_3, \{scp_{new}\} \cup \overline{scp}_p, \emptyset \rangle] = \langle stx_{body2}, \langle \Sigma_4, _ , _ \rangle \rangle \\
& \text{expand}_{ph}[\text{STX}(\text{LIST}(id_{quote}, stx), ctx), \xi, \widehat{\Sigma}] = \langle \text{STX}(\text{LIST}(id_{quote}, stx), ctx), \widehat{\Sigma} \rangle \\
& \text{subject to } \widehat{\Sigma} = \langle \Sigma, _ , _ \rangle, \text{resolve}_{ph}[id_{quote}, \Sigma] = \text{quote} \\
& \text{expand}_{ph}[\text{STX}(\text{LIST}(id_{syntax}, stx), ctx), \xi, \widehat{\Sigma}] \\
& = \langle \text{STX}(\text{LIST}(id_{syntax}, stx_{pruned}), ctx), \widehat{\Sigma} \rangle \\
& \text{subject to } \widehat{\Sigma} = \langle \Sigma, \overline{scp}_p, \overline{scp}_u \rangle, \text{resolve}_{ph}[id_{syntax}, \Sigma] = \text{syntax}, \text{prune}_{ph}[stx, \overline{scp}_p] = stx_{pruned} \\
& \text{expand}_{ph}[\text{STX}(\text{LIST}(id_{ls}, id, stx_{rhs}, stx_{body}), ctx), \xi, \langle \Sigma, \overline{scp}_p, \overline{scp}_u \rangle] = \langle stx_{result}, \langle \Sigma_6, \overline{scp}_p, \overline{scp}_u \rangle \rangle \\
& \text{subject to } \text{resolve}_{ph}[id_{ls}, \Sigma] = \text{let-syntax}, \text{alloc-name}[\Sigma] = \langle name_{new}, \Sigma_l \rangle, \\
& \quad \text{alloc-scope}[\Sigma_l] = \langle scp_{new}, \Sigma_2 \rangle, \text{add}_{ph}[id, scp_{new}] = id_{new}, \\
& \quad \Sigma_2 + \{id_{new} \rightarrow name_{new}\} = \Sigma_3, \\
& \quad \text{expand}_{ph+1}[stx_{rhs}, \xi_{primitives}, \langle \Sigma_3, \emptyset, \emptyset \rangle] = \langle stx_{exp}, \langle \Sigma_4, _ , _ \rangle \rangle, \\
& \quad \text{eval}_{ph}[\text{parse}_{ph+1}[stx_{exp}, \Sigma_4], \bullet, \xi, \langle \Sigma_4, \overline{scp}_p, \emptyset \rangle] = \langle val_{exp}, \langle \Sigma_5, _ , _ \rangle \rangle, \\
& \quad \xi + \{name_{new} \rightarrow val_{exp}\} = \xi_{new}, \text{add}_{ph}[stx_{body}, scp_{new}] = stx_{body2}, \\
& \quad \text{expand}_{ph}[stx_{body2}, \xi_{new}, \langle \Sigma_5, \{scp_{new}\} \cup \overline{scp}_p, \emptyset \rangle] = \langle stx_{result}, \langle \Sigma_6, _ , _ \rangle \rangle \\
& \text{expand}_{ph}[stx_{macapp}, \xi, \langle \Sigma, \overline{scp}_p, \overline{scp}_u \rangle] = \langle stx_{result}, \widehat{\Sigma}_5 \rangle \\
& \text{subject to } stx_{macapp} = \text{STX}(\text{LIST}(id_{mac}, stx_{arg}, \dots), ctx), \xi(\text{resolve}_{ph}[id_{mac}, \Sigma]) = \text{val}, \\
& \quad \text{alloc-scope}[\Sigma] = \langle scp_u, \Sigma_2 \rangle, \text{alloc-scope}[\Sigma_2] = \langle scp_i, \Sigma_3 \rangle, \\
& \quad \langle \Sigma_3, \{scp_u\} \cup \overline{scp}_p, \{scp_u\} \cup \overline{scp}_u \rangle = \widehat{\Sigma}_3, \\
& \quad \text{eval}_{ph}[\text{APP}(val, \text{flip}_{ph}[\text{add}_{ph}[stx_{macapp}, scp_u], scp_i]), scp_i, \xi, \widehat{\Sigma}_3] = \langle stx_{exp}, \widehat{\Sigma}_4 \rangle, \\
& \quad \text{expand}_{ph}[\text{flip}_{ph}[stx_{exp}, scp_i], \xi, \widehat{\Sigma}_4] = \langle stx_{result}, \widehat{\Sigma}_5 \rangle \\
& \text{expand}_{ph}[\text{STX}(\text{LIST}(stx_{rtor}, stx_{rnd}, \dots), ctx), \xi, \langle \Sigma, \overline{scp}_p, \overline{scp}_u \rangle] \\
& = \langle \text{STX}(\text{LIST}(stx_{exptror}, stx_{expnd}, \dots), ctx), \langle \Sigma_l, \overline{scp}_p, \overline{scp}_u \rangle \rangle \\
& \text{subject to } \text{expand}^*_{ph}(_, (stx_{rtor} \ stx_{rnd} \ \dots), \xi, \langle \Sigma, \overline{scp}_p, \emptyset \rangle) = \langle (stx_{exptror} \ stx_{expnd} \ \dots), \Sigma_l \rangle \\
& \text{expand}_{ph}[id, \xi, \widehat{\Sigma}] = \langle id_{new}, \widehat{\Sigma} \rangle \\
& \text{subject to } \widehat{\Sigma} = \langle \Sigma, _ , _ \rangle, \xi(\text{resolve}_{ph}[id, \Sigma]) = \text{VAR}(id_{new}) \\
& \text{expand}^* : ph \ (stx \ \dots) \ (stx \ \dots) \ \xi \ \widehat{\Sigma} \rightarrow \langle (stx \ \dots), \Sigma \rangle \\
& \text{expand}^*_{ph}((stx_{done} \ \dots), (_), \xi, \langle \Sigma, _ , _ \rangle) = \langle (stx_{done} \ \dots), \Sigma \rangle \\
& \text{expand}^*_{ph}((stx_{done} \ \dots), (stx_0 \ stx_1 \ \dots), \xi, \langle \Sigma, \overline{scp}_p, \emptyset \rangle) \\
& = \text{expand}^*_{ph}((stx_{done} \ \dots \ stx_{done0}), (stx_1 \ \dots), \xi, \langle \Sigma_2, \overline{scp}_p, \emptyset \rangle) \\
& \text{subject to } \text{expand}_{ph}[stx_0, \xi, \langle \Sigma, \overline{scp}_p, \emptyset \rangle] = \langle stx_{done0}, \langle \Sigma_2, _ , _ \rangle \rangle
\end{aligned}$$

prune : $ph\ stx\ \overline{scp} \rightarrow stx$

$$\text{prune}_{ph}[\![\mathbf{STX}(atom, ctx), \overline{scp}_p]\!] = \mathbf{STX}(atom, ctx + \{ph \rightarrow ctx(ph) \setminus \overline{scp}_p\})$$

$$\text{prune}_{ph}[\![\mathbf{STX}(\mathbf{LIST}(stx, \dots), ctx), \overline{scp}_p]\!] = \mathbf{STX}(\mathbf{LIST}(stx_{pruned}, \dots), ctx + \{ph \rightarrow ctx(ph) \setminus \overline{scp}_p\})$$

subject to $\text{prune}_{ph}[\![stx, \overline{scp}_p]\!], \dots = stx_{pruned}, \dots$

add : $ph\ stx\ scp \rightarrow stx$

$$\text{add}_{ph}[\![\mathbf{STX}(atom, ctx), scp]\!] = \mathbf{STX}(atom, ctx + \{ph \rightarrow \{scp\} \cup ctx(ph)\})$$

$$\text{add}_{ph}[\![\mathbf{STX}(\mathbf{LIST}(stx, \dots), ctx), scp]\!] = \mathbf{STX}(\mathbf{LIST}(\text{add}_{ph}[\![stx, scp]\!], \dots), ctx + \{ph \rightarrow \{scp\} \cup ctx(ph)\})$$

flip : $ph\ stx\ scp \rightarrow stx$

$$\text{flip}_{ph}[\![\mathbf{STX}(atom, ctx), scp]\!] = \mathbf{STX}(atom, ctx + \{ph \rightarrow scp \oplus ctx(ph)\})$$

$$\text{flip}_{ph}[\![\mathbf{STX}(\mathbf{LIST}(stx, \dots), ctx), scp]\!] = \mathbf{STX}(\mathbf{LIST}(\text{flip}_{ph}[\![stx, scp]\!], \dots), ctx + \{ph \rightarrow scp \oplus ctx(ph)\})$$